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研究简报

对称张量场共形协变方程的真空解

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摘 要

本文讨论了对称张量场共形协变方程的真空解.

在以前的工作中，我们导出了对称张量场 $h_{\mu\nu}$ 的共形协变的场方程[1,2]. 这一方程更为普遍的共形协变形式应为

$$\partial^2 h_{\mu\nu}-\frac{2}{3}(\partial_\mu\partial^\sigma h_{\nu\sigma}+\partial_\nu\partial^\sigma h_{\mu\sigma})+\frac{1}{3}\partial_\mu\partial_\nu h+\frac{1}{3}g_{\mu\nu}(\partial^\rho\partial^\sigma h_{\rho\sigma}-\lambda\partial^2 h)=0. \tag{1}$$

其中 $h=h_\mu{}^\mu$, λ 为不唯一确定的常数. 在文献[1]中取 $\lambda=1$. (1) 式的共形协变性是很明显的,因为方程中的 $\partial^2 h$ 项本身是共形协变的,所以 λ 的引入并不破坏源方程的共形协变性. 最近有人指出，λ 的存在对寻找方程的解可能是重要的[3]. 在这一短文中，我们来讨论方程(1)的求解,并对解的性质作分析.

与(1)式对应的拉氏量为

$$\mathscr{L}=-\frac{1}{2}(\partial_\sigma h_{\mu\nu})^2+\frac{2}{3}\partial_\sigma h_{\mu\nu}\partial^\nu h^{\mu\sigma}+\frac{1}{6}\lambda(\partial_\sigma h)^2-\frac{1}{3}\partial_\sigma h\partial_\mu h^{\mu\sigma}. \tag{2}$$

为了讨论解的真空性质[4]-[6],我们引入以下共形协变动量——能量张量[7]

$$\theta_{\mu\nu}=g_{\mu\nu}\mathscr{L}-\pi_\mu{}^{\alpha\beta}\partial_\nu h_{\alpha\beta}-\frac{1}{2}\partial^\lambda[(\pi_\lambda{}^{\alpha\beta}I_{\mu\nu\alpha\beta}{}^{\rho\sigma}+\pi_\mu{}^{\alpha\beta}I_{\nu\lambda\alpha\beta}{}^{\rho\sigma}+\pi_\nu{}^{\alpha\beta}I_{\mu\lambda\alpha\beta}{}^{\rho\sigma})h_{\rho\sigma}]$$
$$-\frac{1}{2}(\partial^2 R_{\mu\nu}+g_{\mu\nu}\partial^\lambda\partial^\rho R_{\lambda\rho})+\frac{1}{2}(\partial_\mu\partial^\rho R_{\rho\nu}+\partial_\nu\partial^\rho R_{\rho\mu})$$
$$-\frac{1}{6}(\partial_\mu\partial_\nu-g_{\mu\nu}\partial^2)R_\lambda{}^\lambda. \tag{3}$$

其中

$$\pi_\mu{}^{\alpha\beta}=\frac{\partial\mathscr{L}}{\partial\partial^\mu h_{\alpha\beta}}, \tag{4}$$

$$I_{\mu\nu\alpha\beta}{}^{\rho\sigma}=(g_{\mu\alpha}g_\nu{}^\rho-g_\mu{}^\rho g_{\alpha\nu})g_\beta{}^\sigma+(g_{\mu\beta}g_\nu{}^\sigma-g_\mu{}^\sigma g_{\beta\nu})g_\alpha{}^\rho, \tag{5}$$

$$R_{\mu\nu}=\frac{1}{3}\left[\frac{1}{2}g_{\mu\nu}h_{\alpha\beta}{}^2+2h_{\mu\rho}h^\rho{}_\nu-hh_{\mu\nu}+\frac{1}{2}(\lambda-1)g_{\mu\nu}h^2\right]. \tag{6}$$

由(2)—(6)式,$\theta_{\mu\nu}$ 可写成以下明显表示式

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$$\theta_{\mu\nu} = g_{\mu\nu}\Big[-\frac{7}{18}\,(\partial_\rho h_{\alpha\beta})^2 - \frac{1}{3}\,(\partial^\alpha h_{\alpha\beta})^2 + \frac{1}{3}\,\partial^\rho h^{\alpha\beta}\partial_\alpha h_{\beta\rho}$$
$$+ \frac{1}{18}\lambda(\partial_\rho h)^2 + \frac{1}{3}\,\partial^\alpha h\partial^\beta h_{\beta\alpha}\Big] + \frac{2}{3}\,\partial^\alpha h_\mu{}^\beta\partial_\alpha h_{\nu\beta}$$
$$+ \frac{4}{3}\,\partial^\alpha h_{\mu\beta}\partial^\beta h_{\nu\alpha} - \frac{4}{3}\,\partial^\alpha h_{\mu\nu}\partial^\beta h_{\beta\alpha} + \frac{8}{9}\,\partial_\mu h^{\alpha\beta}\partial_\nu h_{\alpha\beta}$$
$$- \frac{2}{9}\lambda\partial_\mu h\partial_\nu h - \frac{4}{3}\,\partial_\mu h^{\alpha\beta}\partial_\alpha h_{\beta\nu} - \frac{4}{3}\,\partial_\nu h^{\alpha\beta}\partial_\alpha h_{\beta\mu}$$
$$+ \frac{2}{3}\,\partial_\mu h_\nu{}^\alpha\partial^\beta h_{\beta\alpha} + \frac{2}{3}\,\partial_\nu h_\mu{}^\alpha\partial^\beta h_{\beta\alpha} + \frac{1}{6}\,\partial_\mu h\partial^\alpha h_{\alpha\nu}$$
$$+ \frac{1}{6}\,\partial_\nu h\partial^\alpha h_{\alpha\mu} - \frac{1}{6}\,\partial^\alpha h\partial_\mu h_{\nu\alpha} - \frac{1}{6}\,\partial^\alpha h\partial_\nu h_{\mu\alpha}$$
$$+ g_{\mu\nu}\Big[h^{\alpha\beta}\Big(\frac{1}{9}\,\partial^2 h_{\alpha\beta} - \frac{2}{3}\,\partial_\beta\partial^\rho h_{\rho\alpha} + \frac{1}{2}\,\partial_\alpha\partial_\beta h$$
$$+ \frac{1}{6}\,g_{\alpha\beta}\partial^\lambda\partial^\rho h_{\lambda\rho} - \frac{1}{9}\,\lambda g_{\alpha\beta}\partial^2 h\Big)\Big]$$
$$+ h_\mu{}^\alpha\Big(\frac{1}{3}\,\partial^2 h_{\nu\alpha} - \frac{2}{3}\,\partial_\nu\partial^\beta h_{\beta\alpha} - \frac{1}{6}\,\partial_\nu\partial_\alpha h + \frac{2}{3}\,\partial_\alpha\partial^\beta h_{\beta\nu}\Big)$$
$$+ h_\nu{}^\alpha\Big(\frac{1}{3}\,\partial^2 h_{\mu\alpha} - \frac{2}{3}\,\partial_\mu\partial^\beta h_{\beta\alpha} - \frac{1}{6}\,\partial_\mu\partial_\alpha h + \frac{2}{3}\,\partial_\alpha\partial^\beta h_{\beta\mu}\Big)$$
$$+ h^{\alpha\beta}\Big(\frac{2}{3}\,\partial_\mu\partial_\alpha h_{\beta\nu} + \frac{2}{3}\,\partial_\nu\partial_\alpha h_{\beta\mu} - \frac{1}{9}\,\partial_\mu\partial_\nu h_{\alpha\beta} - \frac{4}{3}\,\partial_\alpha\partial_\beta h_{\mu\nu}\Big)$$
$$- \frac{1}{6}\,h_{\mu\nu}\partial^2 h + \frac{1}{6}\,h\partial^2 h_{\mu\nu} - \frac{1}{6}\,h\partial_\mu\partial^\alpha h_{\alpha\nu} - \frac{1}{6}\,h\partial_\nu\partial^\alpha h_{\alpha\mu}$$
$$+ \frac{1}{9}\,\lambda h\partial_\mu\partial_\nu h. \qquad (7)$$

现在我们假定场量 $h_{\mu\nu}$ 具有以下形式

$$h_{\mu\nu} = g_{\mu\nu}H(\tau) + \tau_\mu\tau_\nu G(\tau). \qquad (8)$$

其中 $\tau = \frac{1}{2}\,cx^2 + d_\mu x^\mu + l$，$\tau_\mu = \frac{\partial\tau}{\partial x^\mu}$[5][6]. 将(8)式代入(1)式，场方程可化为

$$(1-\lambda)[4\tau_\lambda\tau^\lambda\ddot{H} + 16c\dot{H} + (\tau_\lambda\tau^\lambda)^2\ddot{G} + 8c\tau_\lambda\tau^\lambda\dot{G} + 8c^2G] = 0. \qquad (9)$$

其中点表示对 τ 的微分．而动量——能量张量(7)式当代入场方程(9)式化简后具有以下形式

$$\theta_{\mu\nu} = (4\tau_\mu\tau_\nu - g_{\mu\nu}\tau_\lambda\tau^\lambda)\theta(\tau), \qquad (10)$$

$$\theta(\tau) = \frac{1}{9}(1-\lambda)\Big[8\dot{H}^2 + \frac{1}{2}\,(\tau_\lambda\tau^\lambda)^2\dot{G}^2 + 4\tau_\lambda\tau^\lambda\dot{H}\dot{G}$$
$$+ 16c\,\frac{1}{\tau_\lambda\tau^\lambda}\,\dot{H}H + 12c\dot{H}G + 4c\dot{G}H$$
$$+ 3c\tau_\lambda\tau^\lambda\dot{G}G + 8c^2\,\frac{1}{\tau_\lambda\tau^\lambda}\,HG + 4c^2G^2\Big]. \qquad (11)$$

很容易看到，当 $\lambda = 1$ 时，(9)式和(10)(11)式对任意函数 $H(\tau)$、$G(\tau)$．恒等于零．在这

种情况下，无法确定场量 $h_{\mu\nu}$. 所以在以下求解讨论中，我们排除 $\lambda = 1$ 的情况. 另外(9)式显然不足以同时确定 $H(\tau)$ 和 $G(\tau)$，所以我们考虑(9)式的特解，即认为 $H(\tau)$ 和 $G(\tau)$ 分别满足

$$\tau_\lambda \tau^\lambda \ddot{H} + 4c\dot{H} = 0, \tag{12a}$$

$$(\tau_\lambda \tau^\lambda)^2 \ddot{G} + 8c\tau_\lambda \tau^\lambda \dot{G} + 8c^2 G = 0. \tag{12b}$$

将 $\tau_\lambda \tau^\lambda = 2c\left(\tau + \frac{d^2}{2c} - l\right)$ 代入(12)式，经过整理，很容易求得

$$H = \frac{C_1}{\tau_\lambda \tau^\lambda} + C_2, \tag{13a}$$

$$G = \frac{D_1}{(\tau_\lambda \tau^\lambda)^2} + \frac{D_2}{\tau_\lambda \tau^\lambda}. \tag{13b}$$

其中 C_1、C_2、D_1、D_2 为积分常数. 将(13)式代入(11)式，则有

$$\theta(\tau) = \frac{2(\lambda - 1)c^2}{9(\tau_\lambda \tau^\lambda)^3}(D_1 + 4C_1)(D_2 + 4C_2). \tag{14}$$

(14)式正是 meronlike 解的形式，是一种真空解[6,8].

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VACUUM SOLUTIONS OF CONFORMALLY COVARIANT EQUATION FOR SYMMETRIC TENSOR FIELD

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Abstract

Vacuum solutions of conformally covariant equation for symmetric tensor field are discussed.