

# 正则形式的 $SU(N)$ 规范理论的约束关联动力学 (III) 等时关联 Green 函数的运动方程与二体关联近似\*

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## 摘 要

把多时关联 Green 函数的运动方程转变成等时关联 Green 函数的运动方程, 其中包括夸克和胶子的密度矩阵的运动方程以及顶角函数的运动方程。在二体关联截断近似下, 给出运动方程、高斯定律和 Ward 恒等式的明显表达式。

**关键词** 等时关联 Green 函数的运动方程, 二体关联近似。

## 1 引 言

在文献[1,2]中,我们已得到正则形式的  $SU(N)$  规范理论的约束关联动力学的完整方程组系列,其中包括夸克和胶子场的多时关联 Green 函数的运动方程<sup>[1]</sup>和规范不变性导致的高斯定律以及 Ward 恒等式等规范约束条件<sup>[2]</sup>。多时关联 Green 函数的运动方程共分为四类:第一、二类涉及夸克场  $\psi(x)$  及其共轭  $\bar{\psi}(x)$  对时间的微商,描述多点关联 Green 函数由于夸克场因相互作用而产生的随时间的演化;第三、四类涉及胶子场  $A_i^a(x)$  及其共轭动量密度  $\pi_i^a(x)$  对时间的微商,描述多点关联 Green 函数由于胶子场因相互作用而产生的随时间的演化。如果把上述四类运动方程对时间的微商扩充为对多点 Green 函数中每一点的夸克场或胶子场进行,则四类运动方程熔合为一类运动方程,描述多点关联 Green 函数中每一点的夸克场或胶子场因相互作用而产生的随时间的演化。这种包含  $N$  个时间微商的  $N$  点关联 Green 函数的运动方程虽然复杂,但却包含了相互作用导致 Green 函数随时间演化的全部信息。对于关联 Green 函数而言,这就是相互作用如何产生场的多点关联的信息。此外,由于不同阶的关联 Green 函数的运动方程相互耦合,这些方程组也提供了相互作用如何造成场的多点关联从低阶向高阶演化的信息。应当指出,在这种多时 Green 函数的运动方程中,相互作用传播的推迟效应和因

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果性得到了明显的表示。但是,由于我们选取时性规范时采取了一定的坐标系,因而丧失了公式的规范不变性和罗仑兹协变性。这是时性规范和正则量子化的简便性所应付出的代价。但是, T. D. Lee 等<sup>[3]</sup>已证明,这种时性规范与正则量子化的理论形式与一般的规范协变和罗仑兹协变的理论形式在物理上是等价的。

上述多时 Green 函数的理论形式,在一些场合是必要的,而在另一些场合却显得累赘。有一些理由要求人们简化出一种单时 Green 函数的理论:(1)可观测量的计算是在单时 Green 函数下进行的<sup>[4]</sup>;(2)高斯定律是用单时 Green 函数表述的<sup>[2]</sup>;(3)上文中 Ward 恒等式也是用单时 Green 函数表达的<sup>[2]</sup>;(4)广义密度矩阵的理论形式是多时 Green 函数理论的正规等时极限<sup>[5]</sup>;(5)单时 Green 函数的运动方程便于数值求解与分析<sup>[6,7]</sup>。

单时 Green 函数的运动方程,可以从多时间微商的多时 Green 函数的运动方程取等时极限得到。这正是本文的第一个目的。

在得到单时 Green 函数的运动方程以后,通过二体关联截断,就可以实现本文的第二个目的:得到二体关联运动方程、高斯定律和 Ward 恒等式的明显表达式。

## 2 正规等时极限和单时 Green 函数的运动方程

从多时 Green 函数的运动方程,推导单时 Green 函数的运动方程的步骤如下:(1)对于  $N$  点关联 Green 函数  $G_c^{(N)}$ , 首先建立  $N$  个对不同点进行时间微商的运动方程。(2)把这  $N$  个方程相加,得到一个包含  $N$  个时间微商之和的  $G_c^{(N)}$  的运动方程。(3)对上述方程取正规等时极限,即得单时 Green 函数的运动方程,其中包括关联密度矩阵和顶角函数的运动方程。

从正则形式的多时关联 Green 函数<sup>[1]</sup>,

$$G_c^{(l+m+2n)}(\hat{1}\cdots\hat{l};\hat{i}\cdots\hat{m};1\cdots n;1'\cdots n') \\ \equiv \text{Tr}\{T(\pi(\hat{1})\cdots\pi(\hat{l})A(\hat{i})\cdots A(\hat{m})\phi(1)\cdots\phi(n)\bar{\phi}(n')\cdots\bar{\phi}(1'))\rho\}_c, \quad (1)$$

可定义相应的单时 Green 函数或广义密度矩阵,

$$\rho_c^{(l+m+2n)}(1'\cdots n';1\cdots n;\hat{1}\cdots\hat{l};\hat{i}\cdots\hat{m}) \\ \equiv \text{Tr}\{T(\bar{\phi}(1')\cdots\bar{\phi}(n')\phi(n)\cdots\phi(1)\pi(\hat{1})\cdots\pi(\hat{l})A(\hat{i})\cdots A(\hat{m}))\rho\}_{c,\text{equaltime}}. \quad (2)$$

上述广义密度矩阵是多时 Green 函数的正规等时极限。正规时序定义为

$$t_{1'} > t_{2'} > \cdots > t_{n'} > t_1 > t_2 > \cdots > t_l > t_1 > t_2 > \cdots > t_m > t_n \\ > t_{n-1} > \cdots > t_1, \quad (3)$$

对(1)式取正规等时极限 (NETL, 即在正规时序下取等时极限), 考虑到夸克场的反对易性和胶子场的对易性,可得

$$G_c^{(l+m+2n)}(\hat{1}\cdots\hat{l};\hat{i}\cdots\hat{m};1\cdots n;1'\cdots n')|_{\text{NETL}} \\ = (-1)^n \rho_c^{(l+m+2n)}(1'\cdots n';1\cdots n;\hat{1}\cdots\hat{l};\hat{i}\cdots\hat{m}). \quad (4)$$

$\rho_c^{(l+m+2n)}$  的运动方程可从下式推得,

$$i \frac{d}{dt} \rho_c^{(l+m+2n)}(1'\cdots n';1\cdots n;\hat{1}\cdots\hat{l};\hat{i}\cdots\hat{m})$$

$$= (-1)^n \left\{ \sum_{i=1}^n i \frac{d}{dt_i} + \sum_{i=1'}^{n'} i \frac{d}{dt_i} + \sum_{p=1}^{\hat{l}} i \frac{d}{dt_p} + \sum_{q=1}^{\hat{m}} i \frac{d}{dt_q} \right\} \\ G_c^{(l+m+2n)}(\hat{1} \cdots \hat{l}; \hat{i} \cdots \hat{m}; 1 \cdots n; 1' \cdots n')_{\text{NETL}}. \quad (5)$$

运用文献[1]中多时 Green 函数的运动方程并代入上式, 经具体计算可得,

$$i \frac{d}{dt} \rho_c^{(N)}(1' \cdots n'; 1 \cdots n; \hat{1} \cdots \hat{l}; \hat{i} \cdots \hat{m}) \\ = \sum_{i=1}^n [(\alpha_i \nabla_i^2) \rho_c^{(N)}(1' \cdots n'; 1 \cdots n; \hat{1} \cdots \hat{l}; \hat{i} \cdots \hat{m}) \\ - \rho_c^{(N)}(1' \cdots n'; 1 \cdots n; \hat{1} \cdots \hat{l}; \hat{i} \cdots \hat{m})(\alpha_i \nabla_i^2)] \\ - ig \sum_{i_1=1}^n (\overrightarrow{\alpha_i T^b})_{i_1} [\rho_c^{(N+1)}(1' \cdots n'; 1 \cdots n; A_i^b(j_1), \hat{1} \cdots \hat{l}; \hat{i} \cdots \hat{m}) \\ + \text{ASM}(ijk) \rho_c^{(N_1)}(1' \cdots k'; 1 \cdots k; \hat{1} \cdots \hat{i}; \hat{i} \cdots \hat{j}) \rho_c^{(N-N_1+1)}((k+1)' \cdots \\ n'; k+1 \cdots n; (\widehat{i+1}) \cdots \hat{l}; A_i^b(j_1), (j+1) \cdots \hat{m})] \\ - ig \sum_{i_1=1}^n [\rho_c^{(N+1)}(1' \cdots n'; 1 \cdots n; A_i^b(j'_1), \hat{1} \cdots \hat{l}; \hat{i} \cdots \hat{m}) \\ + \text{ASM}(ijk) \rho_c^{(N_1)}(1' \cdots k'; 1 \cdots k; \hat{1} \cdots \hat{i}; \hat{i} \cdots \hat{j}) \\ \rho_c^{(N-N_1+1)}((k+1)' \cdots n'; k+1 \cdots n; A_i^b(j'_1), (\widehat{i+1}) \cdots \hat{l}; (j+1) \cdots \hat{m})] \\ \times (\overleftarrow{\alpha_i T^b})_{i'_1} + i \sum_{p=1}^{\hat{m}} \rho_c^{(N)}(1' \cdots n'; 1 \cdots n; \hat{1} \cdots \hat{l}; [\hat{1} \hat{2} \cdots \pi(\hat{p}), (p+1) \cdots \hat{m}]_{\hat{p}}) \\ + i \sum_{q=1}^{\hat{l}} \{ \nabla_q^2 \rho_c^{(N)}(1' \cdots n'; 1 \cdots n; [\hat{1} \hat{2} \cdots A_r^a(q), (q+1) \cdots \hat{l}]_{\hat{q}^a}; \hat{i} \cdots \hat{m}) \\ - \nabla_a^2 \rho_c^{(N)}(1' \cdots n'; 1 \cdots n; [\hat{1} \hat{2} \cdots A_r^a(q), (q+1) \cdots \hat{l}]_{\hat{q}^a}; \hat{i} \cdots \hat{m}) - g^2 f^{abc} f^{cde} \\ \rho_c^{(N+2)}(1' \cdots n'; 1 \cdots n; [\hat{1} \hat{2} \cdots A_i^b(q) A_i^c(q) A_i^d(q), (q+1) \cdots \hat{l}]_{\hat{q}^a}; \hat{i} \cdots \hat{m}) \\ + ig (T^a \gamma_r)_q \rho_c^{(N+1)}(1' \cdots n', q^+; 1 \cdots n, q; [\hat{1} \hat{2} \cdots \hat{l}]_{\hat{q}^a}; \hat{i} \cdots \hat{m}) + gf^{abc} [D_{a_r}^i(y) \\ \rho_c^{(N+1)}(1' \cdots n'; 1 \cdots n; [\hat{1} \hat{2} \cdots A_i^b(q) A_i^c(y), (q+1) \cdots \hat{l}]_{\hat{q}^a}; \hat{i} \cdots \hat{m})]_{y=q} + gf^{abc} \\ [D_{a_r}^i(y) \text{ASM}(ijk) \rho_c^{(N_1)}(1' \cdots k'; 1 \cdots k; [\hat{1} \hat{2} \cdots A_i^b(q), (q+1) \cdots \hat{l}]_{\hat{q}^a}; \hat{i} \cdots \hat{j}) \\ \rho_c^{(N-N_1+1)}((k+1)' \cdots n'; k+1 \cdots n; A_i^c(y), (\widehat{i+1}) \cdots \hat{l}; (j+1) \cdots \hat{m})]_{y=q} \\ + g^2 f^{abc} f^{cde} P(q_r^c, q_i^d, q_i^b) \text{ASM}(ijk) \\ \rho_c^{(N_1+1)}(1' \cdots k'; 1 \cdots k; [\hat{1} \hat{2} \cdots A_i^b(q) A_i^c(q), (q+1) \cdots \hat{l}]_{\hat{q}^a}; \hat{i} \cdots \hat{j}) \\ \rho_c^{(N-N_1+1)}((k+1)' \cdots n'; k+1 \cdots n; A_i^c(q), (\widehat{i+1}) \cdots \hat{l}; (j+1) \cdots \hat{m}) \\ - ig (\overrightarrow{T^a \gamma_r})_q \text{ASM}(ijk) \rho_c^{(N-1)}(1' \cdots k'; 1 \cdots k-1, q; [\hat{1} \cdots \hat{i}]_{\hat{q}^a}; \hat{i} \cdots \hat{j}) \\ \rho_c^{(N-N_1+2)}(q^+(k+1)' \cdots n'; k \cdots n; (\widehat{i+1}) \cdots \hat{l}; (j+1) \cdots \hat{m})$$

$$\begin{aligned}
& -g^2 f^{abc} f^{cde} \text{ASM}(ijk) \sum_{i_1=0}^{l-i} \sum_{j_1=0}^{m-j} \sum_{k_1=0}^{n-k} \\
& \rho_c^{(N)}(1' \cdots k'; 1 \cdots k; [\hat{1} \cdots A_i^b(q), (q+1) \cdots \hat{i}]_{\hat{q}^a}; \hat{i} \cdots j) \\
& \rho_c^{(L)}((k+1)' \cdots (k+k_1)'; k+1 \cdots k+k_1; A_i^d(q), (\hat{i}+1) \cdots (\hat{i}+i_1); \\
& (j+1) \cdots (j+j_1)) \rho_c^{(M)}((k+k_1+1)' \cdots n'; k+k_1+1 \cdots n; \\
& A_i^e(q), (\hat{i}+i_1+1) \cdots \hat{l}; (j_1+j+1) \cdots \hat{m}), \quad (6)
\end{aligned}$$

其中

$$\begin{aligned}
N &= l + m + 2n, \quad N_1 = i + j + 2k, \\
N_2 &= i_1 + j_1 + 2k_1, \quad L = N_2 + 1, \\
M &= N - N_1 - N_2 + 1, \quad (7)
\end{aligned}$$

$$\text{ASM}(ijk) = A_{[1' \cdots n']} S_{[\hat{1} \cdots \hat{i}]} \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n, \quad (8)$$

$$D_{ii}^i(y) = 2\nabla_y^i + \nabla_y^i P(x, y) - \nabla_y^i P(y_i^i, y_i^i), \quad (9)$$

$$P(x_i^i, x_i^d, x_i^b) = 1 + P(x_i^i, x_i^d) + P(x_i^i, x_i^b). \quad (10)$$

$P(x, y)$  为变量  $x, y$  的交换算符。  $1 \cdots n, 1' \cdots n', \hat{1} \cdots \hat{l}, \hat{i} \cdots \hat{m}$ , 表示相应的算符  $\phi(1) \cdots \phi(n), \bar{\phi}(1') \cdots \bar{\phi}(n'), \pi(\hat{1}) \cdots \pi(\hat{l}), A(\hat{i}) \cdots A(\hat{m})$ , 旋量指标和色指标包含在其中。  $A, S$  表示对其下标变量的反对称运算和对称运算, 重复的项目应当去掉<sup>[7]</sup>。  $[\hat{1} \cdots \hat{m}]_b$  表示在乘积  $A(\hat{1}) \cdots A(\hat{m})$  中去掉  $A(\hat{b})$ 。  $[\hat{1} \cdots A_i^b(\hat{q}) \cdots \hat{l}]_{\hat{q}^a}$  表示在乘积  $\pi(\hat{1}) \cdots \pi(\hat{l})$  中去掉  $\pi(\hat{q})$  代之以  $A_i^b(\hat{q})$ 。

方程组系列 (6) 是夸克和胶子的关联密度矩阵和顶角函数的运动方程, 是多时关联 Green 函数的运动方程的等时极限<sup>[7]</sup>。 这是耦合的非线性方程, 是时间的一阶和空间的二阶偏微分方程。  $N$  点关联的运动方程既与低点关联相耦合, 又与  $N+1$  点和  $N+2$  点关联相耦合, 非线性耦合度为三阶(即三个  $\rho_c$  的乘积)。 由于是时间的一阶方程, 所以只要知道  $\rho_c^N$  的初值, 方程组的解就确定了。

但是方程组 (6) 是一个无穷 series, 只有在截断近似下才能求解。 关联 Green 函数和关联密度矩阵的运动方程的最大优点是, 提供了一个很自然的按关联等级截断的系统方案, 而且每一种截断都导致一个非微扰的近似方程组<sup>[5,7]</sup>。 下节将讨论目前计算机可以求解的、超越平均场近似的四级关联截断下的二体关联动力学方程组。

### 3 二体关联动力学

二体关联动力学对应的截断近似是略去 4 点以上的关联, 即

$$\rho_c^{(N)} = 0, N \geq 5, \quad (11)$$

在上述近似下, 从方程组 (6) 可得夸克和胶子的截至 4 点的关联函数的运动方程组。

对于夸克, 有 2 点和 4 点密度矩阵的运动方程,

$$i \frac{d}{dt} \langle \bar{\phi}(1') \phi(1) \rangle_c = \langle \bar{\phi}(1') \alpha_i \bar{\nabla}_i^i \phi(1) \rangle_c - \langle \bar{\phi}(1') \alpha_i \bar{\nabla}_i^i \phi(1) \rangle_c.$$

$$-ig[\langle\bar{\phi}(1')(\alpha_s T^b)_1\phi(1)A_i^b(1)\rangle_c + \langle\bar{\phi}(1')(\alpha_s T^b)_{1'}\phi(1)A_i^b(1')\rangle_c + \langle\bar{\phi}(1')(\alpha_s T^b)_1\phi(1)\rangle_c\langle A_i^b(1)\rangle + \langle\bar{\phi}(1')(\alpha_s T^b)_{1'}\phi(1)\rangle_c\langle A_i^b(1')\rangle], \quad (12)$$

$$i\frac{d}{dt}\langle\bar{\phi}(1')\bar{\phi}(2')\phi(2)\phi(1)\rangle_c = \langle\bar{\phi}(1')\bar{\phi}(2')((\alpha_s\bar{\nabla}^c)_1 + (\alpha_s\bar{\nabla}^c)_2)\phi(2)\phi(1)\rangle_c - \langle\bar{\phi}(1')\bar{\phi}(2')((\alpha_s\bar{\nabla}^c)_{1'} + (\alpha_s\bar{\nabla}^c)_{2'})\phi(2)\phi(1)\rangle_c - igAS_{[1'2']}^{\{12\}}\{\langle\bar{\phi}(1')(\alpha_s T^b)_1\phi(1)\rangle_c\langle\bar{\phi}(2')\phi(2)A_i^b(1)\rangle_c + \langle\bar{\phi}(1')(\alpha_s T^b)_{1'}\phi(1)\rangle_c\langle\bar{\phi}(2')\phi(2)A_i^b(1')\rangle_c + \langle\bar{\phi}(1')\bar{\phi}(2')(\alpha_s T^b)_1\phi(2)\phi(1)\rangle_c\langle A_i^b(1)\rangle + \langle\bar{\phi}(1')\bar{\phi}(2')(\alpha_s T^b)_{1'}\phi(2)\phi(1)\rangle_c\langle A_i^b(1')\rangle\}.$$

对于胶子, 1 至 3 点密度矩阵的运动方程为, (13)

$$i\frac{d}{dt}\langle A_i^a(1)\rangle = i\langle\pi_i^a(1)\rangle, \quad (14)$$

$$i\frac{d}{dt}\langle\pi_i^a(1)\rangle = i\nabla_1^2\langle A_i^a(1)\rangle - i\nabla_1^i\nabla_i^a\langle A_i^a(1)\rangle + igf^{abc}[D_{ii}^c(y)(\langle A_i^b(1)A_i^c(y)\rangle_c + \langle A_i^b(1)\rangle\langle A_i^c(y)\rangle)]_{y=1} + ig^2f^{abc}f^{d'e}P(1_i^c, 1_i^d, 1_i^e)[\langle A_i^b(1)A_i^d(1)A_i^e(1)\rangle_c + \langle A_i^b(1)\rangle\langle A_i^d(1)\rangle\langle A_i^e(1)\rangle + \langle A_i^b(1)A_i^d(1)\rangle_c\langle A_i^e(1)\rangle] - g\langle\bar{\phi}(1)(T^a\gamma_i)\phi(1)\rangle_c, \quad (15)$$

$$i\frac{d}{dt}\langle A_{i_1}^a(1)A_{i_2}^a(2)\rangle_c = iS_{[12]in(12)order}\langle\pi_{i_1}^a(1)A_{i_2}^a(2)\rangle_c, \quad (16)$$

$$i\frac{d}{dt}\langle\pi_{i_1}^a(1)A_{i_2}^a(2)\rangle_c = i\langle\pi_{i_1}^a(1)\pi_{i_2}^a(2)\rangle_c - g\langle\bar{\phi}(1)(T^a\gamma_{i_1})\phi(1)A_{i_2}^a(2)\rangle_c + i\nabla_1^2\langle A_{i_1}^a(1)A_{i_2}^a(2)\rangle_c - i\nabla_1^i\nabla_i^a\langle A_{i_1}^a(1)A_{i_2}^a(2)\rangle_c + igf^{abc}[D_{i_1 i_1}^c(y)(\langle A_{i_1}^b(1)A_{i_1}^c(y)A_{i_2}^a(2)\rangle_c + \langle A_{i_1}^b(1)A_{i_2}^a(2)\rangle_c\langle A_{i_1}^c(y)\rangle + \langle A_{i_1}^b(1)\rangle\langle A_{i_2}^a(2)A_{i_1}^c(y)\rangle_c)]_{y=1} + ig^2f^{abc}f^{d'e}P(1_{i_1}^c, 1_{i_1}^d, 1_{i_1}^e)[\langle A_{i_1}^b(1)A_{i_1}^d(1)A_{i_1}^e(1)A_{i_2}^a(2)\rangle_c + \langle A_{i_1}^c(1)A_{i_2}^a(2)\rangle_c\langle A_{i_1}^d(1)\rangle\langle A_{i_1}^e(1)\rangle + \langle A_{i_1}^b(1)A_{i_1}^d(1)A_{i_2}^a(2)\rangle_c\langle A_{i_1}^e(1)\rangle + \langle A_{i_1}^b(1)A_{i_1}^d(1)\rangle_c\langle A_{i_1}^e(1)A_{i_2}^a(2)\rangle_c], \quad (17)$$

$$i\frac{d}{dt}\langle\pi_{i_1}^a(1)\pi_{i_2}^a(2)\rangle_c = S_{[12]in(12)order}\{i\nabla_1^2\langle A_{i_1}^a(1)\pi_{i_2}^a(2)\rangle_c - i\nabla_1^i\nabla_i^a\langle A_{i_1}^a(1)\pi_{i_2}^a(2)\rangle_c - g\langle\bar{\phi}(1)(T^a\gamma_{i_1})\phi(1)\pi_{i_2}^a(2)\rangle_c + igf^{abc}[D_{i_1 i_1}^c(y)(\langle A_{i_1}^b(1)A_{i_1}^c(y)\pi_{i_2}^a(2)\rangle_c + \langle A_{i_1}^b(1)\pi_{i_2}^a(2)\rangle_c\langle A_{i_1}^c(y)\rangle + \langle A_{i_1}^b(1)\rangle\langle A_{i_1}^c(y)\pi_{i_2}^a(2)\rangle_c)]_{y=1} + ig^2f^{abc}f^{d'e}P(1_{i_1}^c, 1_{i_1}^d, 1_{i_1}^e)[\langle A_{i_1}^b(1)A_{i_1}^d(1)A_{i_1}^e(1)\pi_{i_2}^a(2)\rangle_c + \langle A_{i_1}^c(1)\rangle\langle A_{i_1}^d(1)A_{i_1}^e(1)\pi_{i_2}^a(2)\rangle_c + \langle A_{i_1}^b(1)A_{i_1}^d(1)\rangle_c\langle A_{i_1}^e(1)\pi_{i_2}^a(2)\rangle_c + \langle A_{i_1}^b(1)\rangle\langle A_{i_1}^d(1)\rangle\langle A_{i_1}^e(1)\pi_{i_2}^a(2)\rangle_c]\}, \quad (18)$$

$$i\frac{d}{dt}\langle A_{i_1}^a(1)A_{i_2}^a(2)A_{i_3}^a(3)\rangle_c = iS_{[123]in(123)order}\langle\pi_{i_1}^a(1)A_{i_2}^a(2)A_{i_3}^a(3)\rangle_c, \quad (19)$$

$$\begin{aligned}
i \frac{d}{dt} \langle \pi_{i_1}^{a_1}(1) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c &= i S_{[23] \ln(123) \text{ order}} \langle \pi_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&+ i \nabla_1^2 \langle A_{i_1}^{a_1}(1) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c - i \nabla_1^i \nabla_1^j \langle A_{i_1}^{a_1}(1) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&- g \langle \bar{\psi}(1) (T^{a_1 \gamma_{i_1}}) \psi(1) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c + i g f^{a_1 b c} [D_{i_1}^c(y) \\
&S_{[23]} (\langle A_{i_1}^b(1) A_{i_1}^c(y) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^b(1) \rangle \langle A_{i_1}^c(y) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&+ \langle A_{i_1}^b(1) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \langle A_{i_1}^c(y) \rangle + \langle A_{i_1}^b(1) A_{i_2}^{a_2}(2) \rangle_c \langle A_{i_1}^c(y) A_{i_3}^{a_3}(3) \rangle_c) ]_{y=1} \\
&+ i g^2 f^{a_1 b c} f^{e d c} P(1_{i_1}^e, 1_{i_2}^d, 1_{i_3}^b) S_{[23]} \{ \langle A_{i_1}^e(1) \rangle \langle A_{i_2}^d(1) A_{i_3}^{a_3}(3) \rangle_c \\
&+ \langle A_{i_1}^e(1) A_{i_2}^{a_2}(2) \rangle_c \langle A_{i_3}^d(1) A_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^e(1) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&\langle A_{i_2}^d(1) A_{i_3}^b(1) \rangle_c + \langle A_{i_2}^d(1) \rangle \langle A_{i_3}^b(1) \rangle \langle A_{i_1}^e(1) A_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&+ \langle A_{i_1}^e(1) \rangle \langle A_{i_2}^d(1) A_{i_3}^{a_3}(3) \rangle_c \langle A_{i_1}^e(1) A_{i_2}^{a_2}(2) \rangle_c \\
&+ \langle A_{i_1}^e(1) A_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^d(1) \rangle \langle A_{i_1}^e(1) A_{i_2}^{a_2}(2) \rangle_c \}, \quad (20)
\end{aligned}$$

$$\begin{aligned}
i \frac{d}{dt} \langle \pi_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c &= i \langle \pi_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \\
&+ [1 + P(1, 2)]_{\ln(123) \text{ order}} \{ i \nabla_1^2 \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&- i \nabla_1^i \nabla_1^j \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c - g \langle \bar{\psi}(1^+) (T^{a_1 \gamma_{i_1}}) \psi(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&+ i g f^{a_1 b c} D_{i_1}^c(y) [ \langle A_{i_1}^b(1) A_{i_1}^c(y) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c + P(1_{i_1}^b, y_{i_1}^c) (\langle A_{i_1}^c(y) \rangle \\
&\langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_1}^c(y) A_{i_3}^{a_3}(3) \rangle_c) ]_{y=1} + i g^2 f^{a_1 b c} f^{e d c} \\
&P(1_{i_1}^e, 1_{i_2}^d, 1_{i_3}^b) [ \langle A_{i_1}^e(1) \rangle \langle A_{i_2}^d(1) A_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \\
&\langle A_{i_2}^d(1) A_{i_3}^b(1) \rangle_c + \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_2}^d(1) A_{i_3}^b(1) A_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^e(1) A_{i_3}^{a_3}(3) \rangle_c \\
&\langle A_{i_2}^d(1) A_{i_3}^b(1) \pi_{i_2}^{a_2}(2) \rangle_c + \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) A_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^d(1) \rangle \langle A_{i_3}^b(1) \rangle \\
&+ \langle A_{i_1}^e(1) \rangle (\langle A_{i_2}^d(1) \pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_3}^b(1) A_{i_3}^{a_3}(3) \rangle_c \\
&+ \langle A_{i_2}^d(1) A_{i_3}^{a_3}(3) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c) ] \}, \quad (21)
\end{aligned}$$

$$\begin{aligned}
i \frac{d}{dt} \langle \pi_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c &= [1 + P(1, 2) + P(1, 3)]_{\ln(123) \text{ order}} \\
&\{ i [ \nabla_1^2 \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c - \nabla_1^i \nabla_1^j \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c ] \\
&- g \langle \bar{\psi}(1^+) (T^{a_1 \gamma_{i_1}}) \psi(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \\
&+ i g f^{a_1 b c} D_{i_1}^c(y) [ \langle A_{i_1}^b(1) A_{i_1}^c(y) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c + P(1_{i_1}^b, y_{i_1}^c) (\langle A_{i_1}^c(y) \rangle \\
&\langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_1}^c(y) \pi_{i_3}^{a_3}(3) \rangle_c) ]_{y=1} + i g^2 f^{a_1 b c} f^{e d c} \\
&P(1_{i_1}^e, 1_{i_2}^d, 1_{i_3}^b) [ \langle A_{i_1}^e(1) \rangle \langle A_{i_2}^d(1) A_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \\
&\langle A_{i_2}^d(1) A_{i_3}^b(1) \rangle_c + \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_2}^d(1) A_{i_3}^b(1) \pi_{i_3}^{a_3}(3) \rangle_c + \langle A_{i_1}^e(1) \pi_{i_3}^{a_3}(3) \rangle_c \\
&\langle A_{i_2}^d(1) A_{i_3}^b(1) \pi_{i_2}^{a_2}(2) \rangle_c + \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^d(1) \rangle \langle A_{i_3}^b(1) \rangle \\
&+ \langle A_{i_1}^e(1) \rangle (\langle A_{i_2}^d(1) \pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_3}^b(1) \pi_{i_3}^{a_3}(3) \rangle_c \\
&+ \langle A_{i_2}^d(1) \pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c) ] \}. \quad (22)
\end{aligned}$$

3 点顶角函数的运动方程为,

$$\begin{aligned}
i \frac{d}{dt} \langle \bar{\psi}(1') \psi(1) A(2) \rangle_c &= \langle \bar{\psi}(1') \alpha_r \bar{\nabla}_1^r \psi(1) A(2) \rangle_c - \langle \bar{\psi}(1') \alpha_r \bar{\nabla}_1^r \psi(1) A(2) \rangle_c \\
&- i g [ \langle \bar{\psi}(1') (\alpha_r T^b)_1 \psi(1) A_r^b(1) A(2) \rangle_c + \langle \bar{\psi}(1') (\alpha_r T^b)_1 \psi(1) A_r^b(1') A(2) \rangle_c \\
&+ \langle \bar{\psi}(1') (\alpha_r T^b)_1 \psi(1) A(2) \rangle_c \langle A_r^b(1) \rangle
\end{aligned}$$

$$\begin{aligned}
 & + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) A(2) \rangle_c \langle A_i^b(1') \rangle \\
 & + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) \rangle_c \langle A_i^b(1) A(2) \rangle_c \\
 & + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) \rangle_c \langle A_i^b(1') A(2) \rangle_c ] \\
 & + i \langle \bar{\phi}(1') \phi(1) \pi(2) \rangle_c, \tag{23}
 \end{aligned}$$

$$\begin{aligned}
 i \frac{d}{dt} \langle \bar{\phi}(1') \phi(1) \pi_i^a(2) \rangle_c & = \langle \bar{\phi}(1') \alpha_i \bar{\nabla}_1^i \phi(1) \pi_i^a(2) \rangle_c - \langle \bar{\phi}(1') \alpha_i \bar{\nabla}_1^i \phi(1) \pi_i^a(2) \rangle_c \\
 & - ig [\langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) \pi_i^a(2) A_i^b(1) \rangle_c + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) A_i^b(1') \pi_i^a(2) \rangle_c \\
 & + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) \pi_i^a(2) \rangle_c \langle A_i^b(1) \rangle + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) \pi_i^a(2) \rangle_c \langle A_i^b(1') \rangle \\
 & + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) \rangle_c \langle \pi_i^a(2) A_i^b(1) \rangle_c + \langle \bar{\phi}(1')(\alpha_i T^b)_1 \phi(1) \rangle_c \\
 & \langle A_i^b(1') \pi_i^a(2) \rangle_c] + i \nabla_2^i \langle \bar{\phi}(1') \phi(1) A_i^a(2) \rangle_c - i \nabla_2^i \nabla_2^j \langle \bar{\phi}(1') \phi(1) A_i^a(2) \rangle_c \\
 & + ig f^{abc} [D_{ii}^i(y) \langle \bar{\phi}(1') \phi(1) A_i^b(2) A_i^c(y) \rangle_c + \langle A_i^b(2) \rangle \langle \bar{\phi}(1') \phi(1) A_i^c(y) \rangle_c \\
 & + \langle \bar{\phi}(1') \phi(1) A_i^b(2) \rangle_c \langle A_i^c(y) \rangle_c],_{y=2} \\
 & + ig^2 f^{abc} f^{dce} P(2_i^a, 2_j^d, 2_k^e) [\langle A_i^b(2) A_j^c(2) \rangle_c \langle \bar{\phi}(1') \phi(1) A_k^e(2) \rangle_c \\
 & + \langle \bar{\phi}(1') \phi(1) A_i^b(2) A_j^c(2) \rangle_c \langle A_k^e(2) \rangle_c \\
 & + \langle A_i^b(2) \rangle \langle A_j^c(2) \rangle \langle \bar{\phi}(1') \phi(1) A_k^e(2) \rangle_c] \\
 & - g \langle \bar{\phi}(1') \bar{\phi}(2^+) (T^a \gamma_i)_2 \phi(2) \phi(1) \rangle_c \\
 & + g (T^a \gamma_i)_2 \langle \bar{\phi}(2^+) \phi(1) \rangle_c \langle \bar{\phi}(1') \phi(2) \rangle_c, \tag{24}
 \end{aligned}$$

其中

$$D_{ii}^i(y) = 2 \nabla_y^i + \nabla_y^i P(1, y) - \nabla_y^i P(y_i^i, y_i^i), \tag{25}$$

$$P(1_i^a, 1_j^b, 1_k^c) = 1 + P(1_i^a, 1_j^b) + P(1_i^a, 1_k^c). \tag{26}$$

$P(x, y)$  是变量  $x, y$  的交换运算。 $D_{ii}^i(y)$  和  $P(1_i^a, 1_j^b, 1_k^c)$  的运算都要去掉重复项。

为了使二体关联动力学方程组完备, 还需 4 点胶子密度矩阵的运动方程和 4 点顶角的运动方程(见附录)。

由于规范场的非物理自由度的存在, 上述单时 Green 函数并不独立, 它们之间存在着由规范不变性导致的约束条件, 包括高斯定律与 Ward 恒等式<sup>[2]</sup>。

高斯定律是最低阶的约束条件, 它们是

$$\begin{aligned}
 \frac{1}{g} \nabla_x^i \langle \pi_j^a(x) \rangle + \langle \phi^\dagger(x) T^a \phi(x) \rangle_c + \frac{1}{2} f^{abc} \langle \pi_j^c(x) A_j^b(x) + A_j^b(x) \pi_j^c(x) \rangle_c \\
 + f^{abc} \langle A_j^b(x) \rangle \langle \pi_j^c(x) \rangle = 0, \tag{27}
 \end{aligned}$$

这是胶子场的 1 点和 2 点 Green 函数与夸克场 2 点 Green 函数之间的关联, 代表色荷守恒所产生的约束条件。

在二体关联动力学近似下, 二级 Ward 恒等式为

$$\langle g^a(x) g^{a'}(x') \rangle - \langle g^{a'}(x') g^a(x) \rangle = 0, \tag{28}$$

其中

$$\begin{aligned}
 g^a(x) & = \frac{1}{g} \nabla_x^i \pi_j^a(x) + \frac{1}{2} f^{abc} (\pi_j^c(x) A_j^b(x) + A_j^b(x) \pi_j^c(x)) \\
 & + \phi^\dagger(x) T^a \phi(x), \tag{29}
 \end{aligned}$$

上述 Ward 恒等式代表了 2 点、3 点和 4 点 Green 函数之间的约束条件。

考虑到 Gauss 定律, Ward 恒等式可写成

$$\langle g^a(x)g^{a'}(x') \rangle_{c(xx')} - \langle g^{a'}(x')g^a(x) \rangle_{c(xx')} = 0, \quad (30)$$

因为:

$$\begin{aligned} \langle g^a(x)g^{a'}(x') \rangle &= \langle g^a(x) \rangle \langle g^{a'}(x') \rangle + \langle g^a(x)g^{a'}(x') \rangle_{c(xx')} \\ &= \langle g^a(x)g^{a'}(x') \rangle_{c(xx')}, \end{aligned} \quad (31)$$

其中下指标  $c(xx')$  表示  $x-x'$  关联。在(30)式只存在  $x$  与  $x'$  的关联项。

至此, 我们给出了二体关联动力学方程及其规范约束条件。上述方程组保持费米子数、线性动量和角动量、以及能量守恒。对于高斯定律和二级 Ward 恒等式, 如果初始物理条件满足, 则运动方程导致的随时间演化的 Green 函数也满足。

但是, 上述方程在数值求解时还遇到如下的问题: 在包含  $A(x)$  和  $\pi(x')$  的混合 Green 函数中, 交换  $A(x)$  与  $\pi(x')$  时, 按照量子化条件会出现  $\delta(x-x')$  这样的无穷大, 这给方程的求解和初始条件的设置带来困难。这是  $SU(N)$  规范理论发散问题在运动方程中的表现。为了使上述方程能够数值求解, 必须运用适当的方法将二体运动方程重整化。这也正是下一篇文章所要解决的问题。

## 4 结论与讨论

基于多时 Green 函数的运动方程, 在正规等时极限下, 得到了单时 Green 函数的运动方程, 其中包括广义密度矩阵的运动方程和顶角函数的运动方程。这是时间一阶、空间二阶、非线性度为三次的耦合的偏微分方程组的无穷系列。这个方程组遵从所有的守恒定律与规范约束条件。该方程组要能够实际应用求解, 必须截断。关联动力学恰好提供了一个按关联等级而不是按相互作用强度截断的方案。而有趣的是, 按关联等级截断的方案也与守恒定律和高斯定律相容。由于高斯算子是 2 点 Green 函数, 即使在平均场近似下, 高斯定律也得到满足。保持所有守恒定律和高斯定律的最小截断近似是保留 4 点 Green 函数的二体关联动力学。由于它超越了平均场近似, 是非微扰理论, 适用于强耦合问题, 而且目前的计算机有可能对它数值求解, 因此二体约束关联动力学就成为有希望的近似。

### 附录 4 点密度矩阵与顶角函数的运动方程

胶子的 4 点密度矩阵的运动方程为

$$\begin{aligned} i \frac{d}{dt} \langle A_{i_1}^{a_1}(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4) \rangle_c \\ = i S_{[1234] \text{ jin}(1234) \text{ order}} \langle \pi_{i_1}^{a_1}(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4) \rangle_c, \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} i \frac{d}{dt} \langle \pi_{i_1}^{a_1}(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4) \rangle_c \\ = i S_{[1234] \text{ jin}(1234) \text{ order}} \langle \pi_{i_1}^{a_1}(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4) \rangle_c \\ + i \nabla_i^1 \langle A_{i_1}^{a_1}(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4) \rangle_c - i \nabla_i^1 \nabla_i^2 \langle A_{i_1}^{a_1}(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4) \rangle_c. \end{aligned}$$



$$\begin{aligned}
& + igf^{abc}\{D_{i_1}^i(y)[1 + P(1_i^b, y_{i_1}^c)]S_{[234]}[\langle A_i^b(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^c(y)\rangle_c \\
& + \langle A_i^b(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)\rangle_c \langle A_{i_1}^c(y)A_{i_4}^{a_4}(4)\rangle_c]\}_{y=1} + ig^2f^{abc}f^{cde} \\
& P(1_i^e, 1_i^d, 1_i^b)S_{[234]}[\langle A_i^b(1)\rangle_c \langle A_i^d(1)\rangle_c \langle A_{i_1}^e(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& + \langle A_i^b(1)\rangle_c \langle A_i^d(1)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)\rangle_c \\
& + \langle A_i^b(1)A_{i_3}^{a_3}(3)\rangle_c \langle A_i^d(1)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)A_{i_2}^{a_2}(2)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)\rangle_c \langle A_{i_1}^e(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)A_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)A_{i_2}^{a_2}(2)\rangle_c], \tag{A.2}
\end{aligned}$$

$$\begin{aligned}
& i \frac{d}{dt} \langle \pi_{i_1}^{a_1}(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& = iS_{[34]} \ln(1234) \text{order} \langle \pi_{i_1}^{a_1}(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& + S_{[12]} \ln(1234) \text{order} \{i\nabla^i \langle A_{i_1}^{a_1}(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& - i\nabla^i \langle A_{i_1}^{a_1}(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \} \\
& + igf^{abc}[D_{i_1}^i(y)[1 + P(1_i^b, y_{i_1}^c)]S_{[34]}[\langle A_i^b(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^c(y)\rangle_c \\
& + \langle A_i^b(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)\rangle_c \langle A_{i_1}^c(y)A_{i_4}^{a_4}(4)\rangle_c \\
& + \langle A_i^b(1)\pi_{i_2}^{a_2}(2)\rangle_c \langle A_{i_1}^c(y)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c]\}_{y=1} \\
& + ig^2f^{abc}f^{cde}S_{(1_i^e, 1_i^d, 1_i^b)} S_{[34]}[\langle A_i^b(1)\rangle_c \langle A_i^d(1)\rangle_c \langle A_{i_1}^e(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& + \langle A_i^b(1)A_{i_3}^{a_3}(3)\rangle_c \langle A_i^d(1)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)\pi_{i_2}^{a_2}(2)\rangle_c \\
& + \langle A_i^b(1)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \langle A_i^d(1)\rangle_c \langle A_{i_1}^e(1)\pi_{i_2}^{a_2}(2)\rangle_c \\
& + \langle A_i^b(1)\rangle_c \langle A_i^d(1)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)\rangle_c \langle A_{i_1}^e(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)\pi_{i_2}^{a_2}(2)\rangle_c \langle A_{i_1}^e(1)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)\pi_{i_2}^{a_2}(2)A_{i_3}^{a_3}(3)\rangle_c \langle A_{i_1}^e(1)A_{i_4}^{a_4}(4)\rangle_c \\
& + \langle A_i^d(1)A_i^b(1)A_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c \langle A_{i_1}^e(1)\pi_{i_2}^{a_2}(2)\rangle_c], \tag{A.3}
\end{aligned}$$

$$i \frac{d}{dt} \langle \pi_{i_1}^{a_1}(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3)A_{i_4}^{a_4}(4)\rangle_c = i \langle \pi_{i_1}^{a_1}(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3)\pi_{i_4}^{a_4}(4)\rangle_c$$

$$\begin{aligned}
& + S_{\Gamma_{1234}\text{jin}(1234)} \text{order} \{ i \nabla_i^2 \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \\
& - i \nabla_i^2 \nabla_i^2 \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \\
& + i g f^{a_1 b c} [ D_{i_1}^c(y) (1 + P(1_{i_2}^b, y_{i_1}^c)) S_{\Gamma_{1234}} (\langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^c(y) \rangle \\
& + \langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_1}^c(y) A_{i_4}^{a_4}(4) \rangle_c \\
& + \langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) A_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^c(y) \pi_{i_3}^{a_3}(3) \rangle_c ]_{y=1} \\
& + i g^2 f^{a_1 b c} f^{c d e} S_{(\Gamma_{i_1}^a, \Gamma_{i_2}^d, \Gamma_{i_3}^e)} S_{\Gamma_{1234}} [ \langle A_{i_1}^b(1) \rangle_c \langle A_{i_2}^d(1) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \\
& + \langle A_{i_1}^b(1) \pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^d(1) A_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c \\
& + \langle A_{i_1}^b(1) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \langle A_{i_2}^d(1) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c \\
& + \langle A_{i_1}^b(1) \rangle_c \langle A_{i_2}^d(1) A_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) A_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) \pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_1}^e(1) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_1}^e(1) A_{i_4}^{a_4}(4) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) \pi_{i_3}^{a_3}(3) A_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c ] \}, \tag{A.4} \\
& i \frac{d}{dt} \langle \pi_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \\
& = S_{\Gamma_{1234}\text{jin}(1234)} \text{order} \{ i \nabla_i^2 \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \\
& - i \nabla_i^2 \nabla_i^2 \langle A_{i_1}^{a_1}(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \\
& + i g f^{a_1 b c} [ D_{i_1}^c(y) [ 1 + P(1_{i_2}^b, y_{i_1}^c) ] S_{\Gamma_{1234}} (\langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^c(y) \rangle \\
& + \langle A_{i_1}^b(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_1}^c(y) \pi_{i_4}^{a_4}(4) \rangle_c ]_{y=1} \\
& + i g^2 f^{a_1 b c} f^{c d e} S_{(\Gamma_{i_1}^a, \Gamma_{i_2}^d, \Gamma_{i_3}^e)} S_{\Gamma_{1234}} [ \langle A_{i_1}^b(1) \rangle_c \langle A_{i_2}^d(1) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \\
& + \langle A_{i_1}^b(1) \pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^d(1) \pi_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c \\
& + \langle A_{i_1}^b(1) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \langle A_{i_2}^d(1) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) \pi_{i_3}^{a_3}(3) \pi_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \rangle_c \\
& + \langle A_{i_1}^b(1) A_{i_2}^d(1) \pi_{i_4}^{a_4}(4) \rangle_c \langle A_{i_1}^e(1) \pi_{i_2}^{a_2}(2) \pi_{i_3}^{a_3}(3) \rangle_c ] \}. \tag{A.5}
\end{aligned}$$

四顶角函数的运动方程为

$$\begin{aligned}
 i \frac{d}{dt} \langle \bar{\psi}(1') \psi(1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c &= i S_{[23] \text{ in } (23) \text{ order}} \langle \bar{\psi}(1') \psi(1) \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \\
 &+ \langle \bar{\psi}(1') \alpha_i \nabla_i^a \psi(1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c - \langle \bar{\psi}(1') \alpha_i \nabla_i^a \psi(1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \\
 &- i g S_{[23]} [\langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \langle A_i^b (1) \rangle \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \langle A_i^b (1') \rangle \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) A_{i_2}^{a_2} (2) \rangle_c \langle A_i^b (1) A_{i_3}^{a_3} (3) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) A_{i_2}^{a_2} (2) \rangle_c \langle A_i^b (1') A_{i_3}^{a_3} (3) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \rangle_c \langle A_i^b (1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \rangle_c \langle A_i^b (1') A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c], \tag{A.6} \\
 i \frac{d}{dt} \langle \bar{\psi}(1') \psi(1) \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c &= i \langle \bar{\psi}(1') \psi(1) \pi_{i_2}^{a_2} (2) \pi_{i_3}^{a_3} (3) \rangle_c \\
 &+ \langle \bar{\psi}(1') \alpha_i \nabla_i^a \psi(1) \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c - \langle \bar{\psi}(1') \alpha_i \nabla_i^a \psi(1) \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \\
 &- i g [\langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \langle A_i^b (1) \rangle \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \langle A_i^b (1') \rangle \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \pi_{i_2}^{a_2} (2) \rangle_c \langle A_i^b (1) A_{i_3}^{a_3} (3) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \pi_{i_2}^{a_2} (2) \rangle_c \langle A_i^b (1') A_{i_3}^{a_3} (3) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) A_{i_3}^{a_3} (3) \rangle_c \langle \pi_{i_2}^{a_2} (2) A_i^b (1) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) A_{i_3}^{a_3} (3) \rangle_c \langle A_i^b (1') \pi_{i_2}^{a_2} (2) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \rangle_c \langle \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) A_i^b (1) \rangle_c \\
 &+ \langle \bar{\psi}(1') (\alpha_i T^b)_i \psi(1) \rangle_c \langle A_i^b (1') \pi_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c] \\
 &+ i \nabla_i^a \langle \bar{\psi}(1') \psi(1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c - i \nabla_{i_2}^a \nabla_{i_3}^b \langle \bar{\psi}(1') \psi(1) A_{i_2}^{a_2} (2) A_{i_3}^{a_3} (3) \rangle_c \\
 &+ i g^2 f^{abc} [D_{i_2}^a (y) (\langle \bar{\psi}(1') \psi(1) A_{i_2}^b (2) A_{i_3}^c (3) \rangle_c \langle A_{i_2}^a (y) \rangle \\
 &+ \langle \bar{\psi}(1') \psi(1) A_{i_2}^b (2) \rangle_c \langle A_{i_3}^c (3) A_{i_2}^a (y) \rangle_c \\
 &+ \langle A_{i_2}^b (2) A_{i_3}^c (3) \rangle_c \langle \bar{\psi}(1') \psi(1) A_{i_2}^a (y) \rangle_c \\
 &+ \langle A_{i_2}^b (2) \rangle_c \langle \bar{\psi}(1') \psi(1) A_{i_2}^a (y) A_{i_3}^c (3) \rangle_c]_{y=2} \\
 &+ i g^2 f^{abc} f^{cde} P(2_i^a, 2_i^b, 2_i^c) [\langle A_{i_2}^b (2) \rangle_c \langle A_{i_2}^d (2) \rangle_c \langle \bar{\psi}(1') \psi(1) A_{i_2}^e (2) A_{i_3}^c (3) \rangle_c \\
 &+ (\langle A_{i_2}^b (2) A_{i_3}^c (3) \rangle_c \langle A_{i_2}^d (2) \rangle_c + \langle A_{i_2}^b (2) \rangle_c \langle A_{i_2}^d (2) A_{i_3}^c (3) \rangle_c)
 \end{aligned}$$

$$\begin{aligned}
& \langle \bar{\psi}(1')\psi(1)A_{i_2}^a(2) \rangle_c + \langle A_{i_2}^b(2)A_{i_2}^d(2) \rangle_c \langle \bar{\psi}(1')\psi(1)A_{i_2}^a(2)A_{i_3}^{a_3}(3) \rangle_c \\
& + \langle A_{i_2}^b(2)A_{i_2}^d(2)A_{i_3}^{a_3}(3) \rangle_c \langle \bar{\psi}(1')\psi(1)A_{i_2}^a(2) \rangle_c \\
& + \langle \bar{\psi}(1')\psi(1)A_{i_2}^b(2)A_{i_2}^d(2) \rangle_c \langle A_{i_2}^a(2)A_{i_3}^{a_3}(3) \rangle_c ] \\
& + g(T^{a_2}\gamma_{i_2})_2 \langle \bar{\psi}(2^+)\psi(1)A_{i_3}^{a_3}(3) \rangle_c \langle \bar{\psi}(1')\psi(2) \rangle_c \\
& + g(T^{a_2}\gamma_{i_2})_2 \langle \bar{\psi}(2^+)\psi(1) \rangle_c \langle \bar{\psi}(1')\psi(2)A_{i_3}^{a_3}(3) \rangle_c, \tag{A.7}
\end{aligned}$$

$$\begin{aligned}
& i \frac{d}{dt} \langle \bar{\psi}(1')\psi(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c = S_{[233]in(23)} \text{order} \\
& \{ \langle \bar{\psi}(1')\alpha_i \bar{\nabla}_i^j \psi(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c - \langle \bar{\psi}(1')\alpha_i \bar{\nabla}_i^j \psi(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c \\
& - ig[\langle \bar{\psi}(1')(\alpha_i T^b)_i \psi(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^b(1) \rangle \\
& + \langle \bar{\psi}(1')(\alpha_i T^b)_i \psi(1)\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^b(1') \rangle \\
& + \langle \bar{\psi}(1')(\alpha_i T^b)_i \psi(1)\pi_{i_2}^{a_2}(2) \rangle_c \langle \pi_{i_3}^{a_3}(3)A_{i_2}^b(1) \rangle_c \\
& + \langle \bar{\psi}(1')(\alpha_i T^b)_i \psi(1)\pi_{i_2}^{a_2}(2) \rangle_c \langle A_{i_2}^b(1')\pi_{i_3}^{a_3}(3) \rangle_c \\
& + \langle \bar{\psi}(1')(\alpha_i T^b)_i \psi(1) \rangle_c \langle \pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3)A_{i_2}^b(1) \rangle_c \\
& + \langle \bar{\psi}(1')(\alpha_i T^b)_i \psi(1) \rangle_c \langle A_{i_2}^b(1')\pi_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c ] \\
& + i\nabla_i^j \langle \bar{\psi}(1')\psi(1)A_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c - i\nabla_i^j \langle \bar{\psi}(1')\psi(1)A_{i_2}^{a_2}(2)\pi_{i_3}^{a_3}(3) \rangle_c \\
& + igf^{a_2 b c} [D_{i_2}^{i_2}(y) \langle \bar{\psi}(1')\psi(1)A_{i_2}^b(2)\pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^c(y) \rangle \\
& + \langle \bar{\psi}(1')\psi(1)A_{i_2}^b(2) \rangle_c \langle A_{i_2}^c(y)\pi_{i_3}^{a_3}(3) \rangle_c \\
& + \langle A_{i_2}^b(2)\pi_{i_3}^{a_3}(3) \rangle_c \langle \bar{\psi}(1')\psi(1)A_{i_2}^c(y) \rangle_c \\
& + \langle A_{i_2}^b(2) \rangle_c \langle \bar{\psi}(1')\psi(1)A_{i_2}^c(y)\pi_{i_3}^{a_3}(3) \rangle_c ]_{y=2} \\
& + ig^2 f^{a_2 b c} f^{c d e} P(2_{i_2}^a, 2_{i_2}^d, 2_{i_2}^e) [\langle A_{i_2}^b(2) \rangle_c \langle A_{i_2}^d(2) \rangle_c \langle \bar{\psi}(1')\psi(1)A_{i_2}^a(2)\pi_{i_3}^{a_3}(3) \rangle_c \\
& + (\langle A_{i_2}^b(2)\pi_{i_3}^{a_3}(3) \rangle_c \langle A_{i_2}^d(2) \rangle_c + \langle A_{i_2}^b(2) \rangle_c \langle A_{i_2}^d(2)\pi_{i_3}^{a_3}(3) \rangle_c) \\
& \langle \bar{\psi}(1')\psi(1)A_{i_2}^a(2) \rangle_c + \langle A_{i_2}^b(2)A_{i_2}^d(2) \rangle_c \langle \bar{\psi}(1')\psi(1)A_{i_2}^a(2)\pi_{i_3}^{a_3}(3) \rangle_c \\
& + \langle A_{i_2}^b(2)A_{i_2}^d(2)\pi_{i_3}^{a_3}(3) \rangle_c \langle \bar{\psi}(1')\psi(1)A_{i_2}^a(2) \rangle_c \\
& + \langle \bar{\psi}(1')\psi(1)A_{i_2}^b(2)A_{i_2}^d(2) \rangle_c \langle A_{i_2}^a(2)\pi_{i_3}^{a_3}(3) \rangle_c ] \\
& + g(T^{a_2}\gamma_{i_2})_2 \langle \bar{\psi}(2^+)\psi(1)\pi_{i_3}^{a_3}(3) \rangle_c \langle \bar{\psi}(1')\psi(2) \rangle_c \\
& + g(T^{a_2}\gamma_{i_2})_2 \langle \bar{\psi}(2^+)\psi(1) \rangle_c \langle \bar{\psi}(1')\psi(2)\pi_{i_3}^{a_3}(3) \rangle_c. \tag{A.8}
\end{aligned}$$

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## Constrained Correlation Dynamics of $SU(N)$ Gauge Theories in Canonical Form (III) Equal Time Limit and Two-body Correlation Dynamics

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### Abstract

The equations of motion for multi-time correlation Green's functions have been transformed into those for equal-time correlation Green's functions, which include the equations of motion for quark's and gluon's density matrices as well as vertex functions. In two-body correlation truncation approximation, we present the formalism for the equations of motion, Gauss law and Ward identities explicitly.

**Key words** equations of motion for single-time correlation Green's functions, two-body correlation truncation approximation.