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Statistic Model for Galactic Cosmic Rays

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This paper supposes that all cosmic ray (CR) particles of energy below 3×10^{18}eV mainly originate and accelerate in an individual explosion of the supernova (SN) in the Galaxy; with the use of an isotropic diffusion propagation model, the non-steady state density distribution of iron nuclei is investigated. Considering the effect of extragalactic CRs and the variety of the galactic CR nuclei, the statistic model of galactic CRs with a reasonable distribution of the galactic SNs in space and time can account for the spectrum of CRs in the energy range of $10^{12} - 10^{20}$eV quite well.

Key words: primary cosmic rays, cosmic ray composition, cosmic ray spectra, cosmic ray propagation.

1. INTRODUCTION

Since the discovery of cosmic rays, their origin, acceleration, and propagation have raised many interesting questions. While the answers to these questions remain uncertain, recently people have obtained more and more information about cosmic ray composition, spectrum, lifetime, and anisotropy [1-6], and some theories or ideas have been proposed [7-10], though no single theory has been commonly accepted.

To date, the main observed features of CRs may be summarized as follows:

(1) The CR spectrum is characterized by a power law with an index about 2.7 between 10^{10}eV and 10^{14}eV. The spectrum consists of protons and some heavier nuclei, such as helium, CNO, S-Ne,

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Fe, etc. The percentages of p, He, C-O, Ne-S, and $Z \geq 17$ were found to be 12 ± 9, 25 ± 14, 26 ± 12, 15 ± 8, and 21 ± 10, respectively [1,5]. The heavy nuclei have slightly flatter spectra.

(2) At about 3×10^{15}eV, the so-called knee, the index of the CR spectrum has a great change from 2.7 to 3.2, and there is some evidence of the compositional change.

(3) Above 3×10^{15}eV until 3×10^{18}eV, the spectrum becomes steeper with an index of 3.2. Recently the Fly's Eye detector obtained a new result in the energy range of $3 \times 10^{17} - 3 \times 10^{18}$eV [2]. It was deduced that the CRs are dominated by the heavy nuclei (mainly iron nuclei), and the index is about 3.3 in the observed energy range.

(4) Beyond 3×10^{18}eV, the spectrum flattens again with an index of 2.7.

(5) The lifetime of CRs was estimated to be about 10^8 years for the particles with energy below 10^{14}eV, which is based principally on the amount of matter traversed in the interstellar space by nuclei with medium mass under the leaky box model. But from the data of meteorites and lunar rocks, the lifetime of CRs is about 2×10^9 years [11].

(6) Measured from the total CR flux in the energy range of $10^{12} - 10^{15}$eV, the upper limit of CR anisotropy is about 5×10^{-4}, and it is likely that the Compton-obtaining anisotropy is dominant in the considered energy range [12]. There are also some evidences of the anisotropy increasing with the energy beyond 10^{15}eV [13], but it is absent of good significance, especially in the ultrahigh energy band. The anisotropic result from the Fly's Eye with energy above 3×10^{17}eV is not significantly deviated from isotropy [14].

In the general opinion, the CRs of $10^{10} - 10^{18}$eV are mainly of the galactic origin, and above 10^{18}eV they could be of extragalactic origin. People also like to accept the opinion that cosmic rays are transported in a diffusionary way. But in what energy region the diffusion mechanism works is a problem to be examined. A different diffusion mechanism derives different dependence of the diffusion coefficient on the energy. Generally speaking, propagation always makes the spectrum steeper than the primary. Below we will start with the transport equation of the iron nucleus. Then using the statistic model of discrete sources of the Galaxy, we will derive the spectra of some nuclei and the spectrum of total CRs observed at the Earth.

2. TRANSPORT EQUATION OF IRON NUCLEI AND ITS SOLUTION

Supposing supernovas of the Galaxy are responsible for the galactic cosmic rays, each SN is treated as a point source in space and time. Taking isotropic diffusion approximation, we start with a phenomenological transport equation of relativistic particle propagation in the Galaxy [15].

$$\frac{\partial N}{\partial t} - D\nabla^2 N + \frac{\partial (NW)}{\partial E} + BN = Q, \tag{1}$$

where $N(r, t, E)$ is the density of considered nuclei at r and t with unit of particles/cm^3 · GeV, and D is the diffusion coefficient in cm^2/s. The third term in Eq. (1) is the energy loss caused by ionization and the fourth term is a negative source due to the inelastic interaction, Q is the source.

$$B = c n_H \sigma_{ine} (1 / \mathrm{s}), \tag{2}$$

where n_H is the density of interstellar medium, mainly the atomic and molecular hydrogen, being about $0.66 - 1\ \mathrm{cm}^{-3}$ [15,16], c the speed of light, and σ_{ine} is the inelastic cross section of the considered nuclei with an atomic number A. For the solution of Eq. (1) some statements are given as follows:

(1) We limit ourselves to treat the particles with energies above 10^{12}eV. Therefore they are free from the solar modulation and the energy loss can be neglected.

(2) From the accelerator's experiment [17] the p-p total cross sections $\sigma_{tot}^{p\text{-}p}$ and $\sigma_{ine}^{p\text{-}p} = 0.8\sigma_{tot}^{p\text{-}p}$

were obtained in the energy range of $10 - 1.5 \times 10^6$GeV. Fitting the data we have

$$\sigma_{inc}^{p\text{-}p} = 28.003 - 0.7297\ln EG + 0.2112\ln^2 EG \text{ (mb)}, \tag{3}$$

and extrapolate it up to the ultra-high energy of 3×10^{18}eV, where EG is the energy of nucleus in GeV. Taking

$$\sigma_{ine} = A^{2/3}\sigma_{inc}^{p\text{-}p}, \tag{4}$$

and assuming the inelastic interaction is the only process of the negative source term, then

$$B = 3 \times 10^{17} A^{2/3} \sigma_{inc}^{p\text{-}p} (1/\text{s}), \tag{5}$$

where $n_H = 1$ cm^{-3} has been used.

(3) We suppose that until 3×10^{18}eV for iron nuclei, the transport is still in a diffusionary way. According to the low anisotropy of CRs, the large content of secondary nuclei in cosmic rays and the existence of a big cosmic ray halo with a radius of $10 - 15$ kpc up to 10^{15}eV (1 kpc $= 3.086 \times 10^{21}$cm), we can understand that the diffusion does take place. But to what energy region is the diffusion mechanism possible and reasonable? We test the problem below. The Larmor radius of an iron nucleus with energy E in the interstellar magnetic field of H is

$$r_L = \frac{E}{300HZ} \text{ (cm)}, \tag{6}$$

where E in eV, $H = 3 \times 10^{-10}$T and r_L in cm. For an iron nucleus of 3×10^{18}eV, r_L is about $1/Z$ kpc. For a proton of 10^{17}eV, the Larmor radius is 1/27 kpc. All of them are much smaller than the size of the CR halo. Recently, A.Z. Dolginov and M.E. Katz [18] gave a comprehensive review of some new results of the dynamic theory of high energy charged particles propagated in a stochastic electromagnetic field of cosmic plasma. They supposed that if only magnetic fluctuations are important and the medium is homogeneous, the tensor coefficient of diffusion becomes a common variable, and the well-known diffusion equation (Eq. (1)) is obtained. They have derived the expression of the coefficient of diffusion in various cases of the medium. It involves various turbulent structures and motion in the cosmic medium and the magnetic field. But we prefer to treat the coefficient of diffusion, D, as a free parameter and take a reasonable value, because we have so little knowledge about the magnetic field of the Galaxy and interstellar medium that we cannot say which process is dominant in the propagation. We take below

$$D = 8 \times 10^{25} (EG/Z)^{0.58} \quad (\text{cm}^2/\text{s}), \tag{7}$$

It is noted that the coefficient of diffusion adopted here is not contradictory with that used in other papers [19-21].

(4) Assuming that a point source at r_0 ejects particles of iron nuclei transiently with a spectrum of $N_0E^{-\gamma}$ at time $t = t_0$, we have

$$Q = N_0 E^{-\gamma} \delta(r - r_0)\,\delta(t - t_0) \quad (1 / \text{GeV}), \tag{8}$$

And we assume that the value of Q is the same for all supernovas. Transforming $N = \mathscr{L}\exp(-Bt)$ in Eq. (1), and using Green function we obtained the solution of Eq. (1) as follows:

$$N = \frac{N_0 E^{-\gamma}}{(4\pi Dt)^{1.5}} \exp\left(- \frac{r^2}{4Dt} - Bt \right) \quad (1 / \text{cm}^3 \cdot \text{GeV}), \tag{9}$$

Here we have taken $r_0 = t_0 = 0$, so that t is the age of the point source, and r is the radial distance with its origin at the source site.

Mathematically, Eq. (9) makes sense for any values of r and t, but the law of causality of information transport restricts that only those sources where $r/t \leq c$ contribute the observed cosmic ray flux. Calculating the density of iron nuclei using Eq. (9), Fig. 1 shows the density distribution of a source of 2×10^5 years old and with energy of 2×10^{15}eV. We can see that when $r \geq 1$ kpc the distribution is flat, i.e., $\nabla N = 0$. However, for iron nuclei of 2×10^{17}eV with same age, only when $r \geq 4$ kpc, one has $\nabla N = 0$. While for $t \geq 2 \times 10^7$ years and $E \geq 2 \times 10^{15}$eV the density of the iron nuclei is flat over the whole Galaxy. Here we have taken that $\gamma = 2.33$, $Z = 26$, $A = 56$, and $N_0^{\mathrm{Fe}} = 1.63 \times 10^{51}$.

Figure 2 shows the iron nucleus spectra for a supernova of 10^7 years at four different distances. Each spectrum is seen to have a turning point – the nearer the distance, the lower the turning point energy. The turning point of energy is about 5×10^{15}eV at $r = 5$ kpc. Here the function $I = \frac{cN}{4\pi}$ (cm · s · Sr · GeV)$^{-1}$ has been used.

3. STATISTIC MODEL OF GALACTIC COSMIC RAYS

According to the observed continuity of the spectrum between 3×10^{15}eV and 3×10^{18}eV, it is reasonable to assume that they are all of galactic origin [13,15,22]. Assuming SNs are distributed uniformly (1/100 years) in the galactic disk and in the time scale, the SN distribution density was now estimated.

$$n_0 = \frac{1}{100 \times 3.156 \times 10^7 \pi R_{15}^2} = 4.725 \times 10^{-56} \quad (1 / \mathrm{cm}^2 \cdot \mathrm{s}), \tag{10}$$

where $R_{15} = 15$ kpc. The observed cosmic ray density N at the Earth should be contributed by all supernovas, which may be divided into three sections characterized by two characteristic times t_1, t_2: $t_1 = R_5 / c = 1.63 \times 10^4$ years, $t_2 = R_{25} / c = 8.15 \times 10^4$ years, $R_5 = 5$ kpc, $R_{25} = 25$ kpc.

$$N_1 = \int_0^{t_1} \mathrm{d}t \int_0^{Ct} n_0 \cdot 2\pi r \cdot N \cdot \mathrm{d}r; \tag{11}$$

$$N_2 = \int_{t_1}^{t_2} \mathrm{d}t \left[\int_0^{R_5} n_0 \cdot 2\pi r \cdot N \cdot \mathrm{d}r + \int_{R_5}^{Ct} n_0 \cdot 2\pi r_e \cdot N \cdot \mathrm{d}r \right]; \tag{12}$$

$$N_3 = \int_{t_2}^{T_s} \mathrm{d}t \left[\int_0^{R_5} n_0 \cdot 2\pi r \cdot N \cdot \mathrm{d}r + \int_{R_5}^{R_{25}} n_0 \cdot 2\pi r_e \cdot N \cdot \mathrm{d}r \right], \tag{13}$$

where r_e is the equivalent radius $r_e = \left[\frac{\alpha}{\pi} \right] r$, $\alpha = \cos^{-1} \left[\frac{r^2 - 125}{20r} \right]$. N_1 is contributed by those SNs with age younger than t_1, N_2 is from the SNs with ages between t_1 and t_2, and N_3 is from the SNs with ages elder than t_2 and up to 10^9 years. In Fig. 3 is shown the distance r_e as the function of r. From Fig. 3 we can see that $r/r_e \leq 4$ is between 5 and 20 kpc. That means if one simply takes $r_e = r$, it is equivalent to increase n_0 by a factor of 4. As a matter of fact, n_0 is really increased significantly between 5 and 20 kpc from the gamma ray astronomy observation [23,24]. In addition, $N \propto \exp(-r^2 / 4Dt)$, the contribution from the remote supernovas decreases rapidly, and the overestimation can be merged.

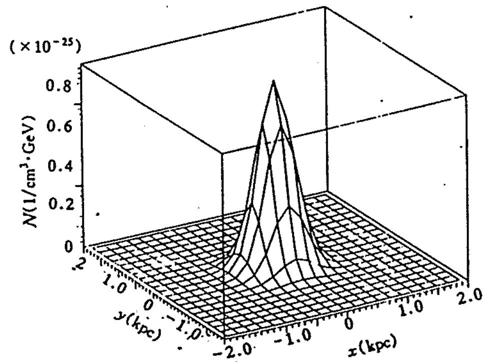

Fig. 1
The density distribution of iron nuclei of 2×10^6 GeV for a source 2×10^5 years old, which was calculated using Eq. (1).

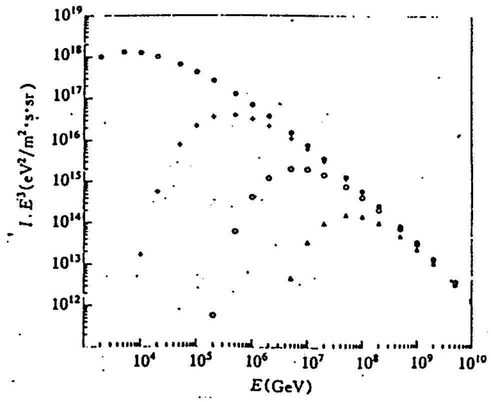

Fig. 2
The spectra of iron nuclei for a SN of 10^7 years old at different distance. ● r = 0.5kpc, + r = 2kpc, ○ r = 5kpc, ▲ r = 10kpc.

Therefore

$$N_1 + N_2 \approx \int_0^{t_2} \mathrm{d}t \int_0^{Ct} n_0 \cdot 2\pi r \cdot N \cdot \mathrm{d}r$$

$$= \frac{n_0 N_0 E^{-\gamma}}{\sqrt{D}} \left[\frac{1}{\sqrt{B}} \frac{1}{\sqrt{2\pi}} \int_0^{\sqrt{2Bt_2}} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x - \frac{1}{\sqrt{B'}} \frac{1}{\sqrt{2\pi}} \int_0^{\sqrt{2B't_2}} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x \right];$$

$$N_3 \approx \int_{t_2}^{T_9} \mathrm{d}t \int_0^{R_{25}} n_0 \cdot 2\pi r \cdot N \cdot \mathrm{d}r$$

$$= \frac{n_0 N_0 E^{-\gamma}}{\sqrt{DB}} \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2Bt_2}}^{\sqrt{2BT_9}} \exp\left(-\frac{x^2}{2}\right) \mathrm{d}x - \frac{n_0 N_0 E^{-\gamma}}{\sqrt{4\pi D}} \int_{t_2}^{T_9} t^{-0.5} \exp\left(-Bt - \frac{R_{25}^2}{4Dt}\right) \mathrm{d}t,$$

where $B' = B + \frac{c^2}{4D}$. In the considered energy band, we have $\sqrt{2BT_9} \geq 8.2$ and $\sqrt{2B' t_2} \geq 9.3$. Therefore, in the first-order approximate, we have:

$$N = \sum_{i=1}^{3} N_i = \frac{n_0 N_0 E^{-\gamma}}{\sqrt{4D}} \left[\frac{1}{\sqrt{B}} - \frac{1}{\sqrt{B'}} \right]. \tag{14}$$

4. PRIMARY INJECTION SPECTRUM

Though the shock acceleration as applied to supernova explosion has been sufficiently studied, it is certainly not the only acceleration mechanism. Until now there has been no well-accepted acceleration mechanism for ultra high energy CRs, although there have been several interesting suggestions [25,26]. Our propagation model prefers an acceleration mechanism with a short time scale and not far from the former stellars.

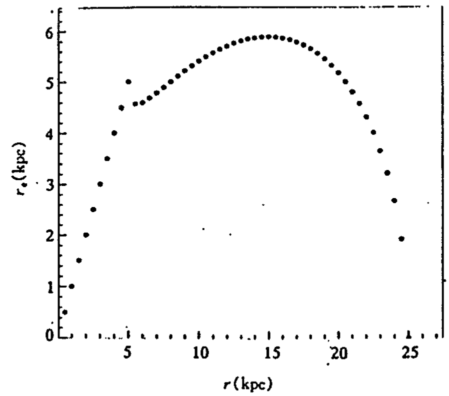

Fig. 3
The r_e as a function of r.

Referring to Peter L. Biermann [26] and some experimental data [1-6], for cosmic rays below 700Z TeV we put a set of the indices into injection spectra as follows:

$$\gamma^{P} = 2.4,\ \gamma^{He} = \gamma^{CNO} = \gamma^{Ne-S} = \gamma^{Fe} = 2.33;$$

Between 700Z TeV and 100Z PeV, they are:

$$\gamma^{P} = 2.7,\ \gamma^{He} = \gamma^{CNO} = \gamma^{Ne-S} = \gamma^{Fe} = 2.63.$$

Taking the total $N_0 = 1.63 \times 10^{52}$, the relative ratios of different nucleus species are obtained as below:

$$N_0^{P}:\ N_0^{He}:\ N_0^{CNO}:\ N_0^{Ne-S}:\ N_0^{Fe} = 0.55:\ 0.15:\ 0.1:\ 0.1:\ 0.1.$$

In this case, the released energy of relativistic particles for a SN explosion is about 4.5×10^{48} ergs.

5. CONCLUSION AND DISCUSSION

Supposing that Eqs. (9) and (14) are valid for protons and all CR nuclei, we may evaluate N from Eq. (14) for protons, He, CNO, Ne-S, and Fe nuclei individually. Furthermore, we assume that cosmic rays are of local isotropy, so that the flux of CRs is given by

$$I = \frac{cN}{4\pi}\ (\mathrm{cm^2 \cdot s \cdot sr \cdot GeV^{-1}}), \qquad (15)$$

In Fig. 4 are shown these spectra individually, the all-particle spectrum and the spectrum above 3×10^{18}eV detected by the Fly's Eye [2], which could be of an extragalactic origin. For comparison, in Fig. 5 are shown some experimental results [5].

We can see that the expected spectra account for the observed data quite well in the energy range of $10^{12} - 3 \times 10^{18}$eV. Above 3×10^{18}eV it is believed that CRs come from an extragalactic origin. The relative ratio below 10^{15}eV is also shown to agree with the data measured by JACEE [1] and other experiment [5]. After 5×10^{15}eV, the heavier nucleus component increases, at about 2×10^{17}eV, the iron nucleus group (including CNO) is the dominant composition, and then, because of extragalactic cosmic rays (mainly protons), the protons increase again up to the highest energy.

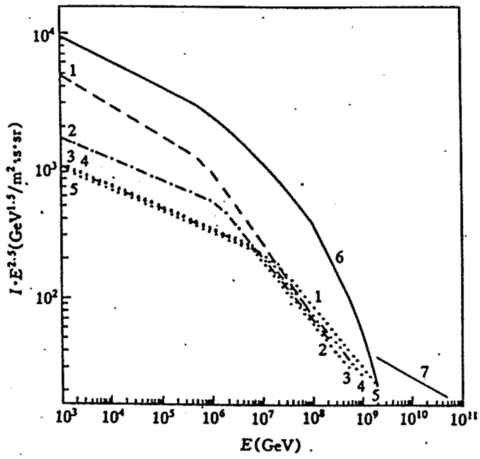

Fig. 4
The expected spectra of individual nuclei and the all-particle spectrum.
1: ---- p; 2: -·-·- He; 3: ······ CNO; 4: —··· Ne-S; 5: ······ Fe; 6: —— the all-particle spectrum; 7 —— the spectrum of extragalactic CRs observed by the Fly's Eye.

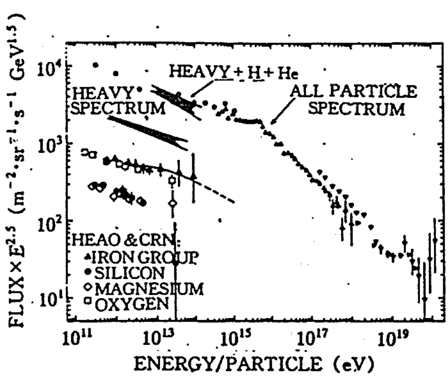

Fig. 5
The observed energy spectra.
The solid line describes the spectrum of iron nuclei expected by the leaky box model. The broken line indicates its extrapolated spectrum up to 10^{15}eV. The all-particle spectrum is cited from: ○ Grigorov *et al.*, 1971; △ Gara *et al.*, 1983; ▷ Diminstein *et al.*, 1982; ▽ Bower *et al.*, 1981.

Although we have considered the contribution of all supernovas in the Galaxy as long as they satisfy the causality law of information transport, i.e., $r/t \leq c$, only a relatively small number of supernovas contribute a major part of the CR flux at Earth at any time because of the combined effects of diffusion propagation and inelastic interaction. Those point sources which are close to Earth should make a great contribution. Unfortunately, very less information about the ages is now available, so we must employ the statistical model.

Our model is different from the leaky box model which supposes that N is constant through the whole system (the Galaxy or some small volume). In this case, certain quantities are averaged over the whole system, and the knee of CR spectrum is caused by the CR escape from the system, which occurs when the CR rigidity is greater than a critical value.

In our model, each SN contributes the CRs in a certain space point with a certain age, density, and gradient of density. The observed CR flux at Earth is contributed by all possible SNs. Therefore the age of CRs near Earth should not be only one value. At a different section of the CR spectrum, there should be a dominate age of CRs. For example, below 10^{14}eV the ages of CRs may be dominated by elder CRs, because young CRs cannot arrive at Earth from remote distances. In this case the gradient of CRs is nearly zero, which is in agreement with the observed low anisotropy of 5×10^{-4} in this energy range. As to the knee of the spectrum, it may be caused by a different acceleration mechanism with a certain index at a different energy band and this needs to be studied in more detail. This is similar to the idea of Ref. [26]. We can also think that the leaky box model may be a special version of the diffusion model. If the velocity of diffusion is very rapid, the density of CRs in the whole system should reach a smooth distribution sooner. But it is not possible to find a compatible statement for the knee of spectrum between those two models.

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