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$\Delta I = 2$ Energy Staggering in Superdeformed Bands in ^{149}Gd(b1) and ^{153}Dy(b1)

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The particle-rotor model is extended to describe the odd A superdeformed nuclear states in the $A \sim 150$ mass region. The calculated $\Delta I = 2$ staggering in superdeformed bands in ^{149}Gd(b1) and ^{153}Dy(b1) is compared with the observed data for the first time.

Key words: nuclear structure, superdeformation, particle-rotor model, C_4-symmetry, $\Delta I = 2$ staggering.

1. INTRODUCTION

The observation of $\Delta I = 2$ staggering in superdeformed (SD) rotational spectra of ^{149}Gd(b1) [1], in which states with $\Delta I = 4$ exhibit a systematic energy displacement with respect to a smooth reference, is one of the significant achievements in the study of superdeformed nuclei. Until now, $\Delta I = 2$ staggering has been observed in the SD bands in the $A \sim 150$, 190, and 130 mass regions [1–4]. A considerable amount of effort has been spent on understanding its physical implication based on various theoretical ideas. Among these, there are two contrary ideas and to interpret this phenomenon.

In Refs. [5, 6], the authors suggested that $\Delta I = 2$ staggering shows that there exists a C_4-symmetry of the nuclear Hamiltonian. In fact, calculations performed with an effective Hamiltonian invariant under this symmetry have been able to reproduce the main features of $\Delta I = 2$ staggering in the rotational spectra of SD bands. On the other hand, calculations in Ref. [7] show that such

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staggering can arise naturally from the projected shell model without introducing a C_4-symmetric Hamiltonian. However, since the model in the former is too simple and the intrinsic structure of SD nuclei is not taken into account, there seems to be little hope of attempting a fit with the experimental data. The model in the latter can be used to fit with the observed data in principle. Unfortunately, to date, we have not found calculated results using them. In Ref. [8], the perturbed particle-rotor model (i.e., the particle-rotor model plus perturbation) has been presented and the general features of $\Delta I = 2$ staggering in the rotational spectra have been investigated. The distractive merits of this model are as follows: the model Hamiltonian is realistic without C_4-symmetry, the intrinsic structure of SD nuclei is taken into account and $\Delta I = 2$ energy staggering is investigated within the framework of the unified particle-rotor model, in which the additional perturbation is a natural result of the distortion of the rotor. Therefore, this model can be used to fit with the experimental data immediately. The main purpose of this paper is to investigate the superdeformed rotational spectra of ^{149}Gd(b1) and ^{153}Dy(b1) using the perturbed particle-rotor model and to compare our results with the observed data.

2. OUTLINES OF THE MODEL

The brief description of the perturbed particle-rotor model is presented for the convenience of the discussion. The detailed formulas of this model can be found in Refs. [8–10].

The model Hamiltonian is written as

$$H = H_{\rm PR} + H_{\rm c}\,, \tag{1}$$

where $H_{\rm PR}$ is a triaxial-particle-rotor model Hamiltonian which describes the smooth change of rotational energies with increasing total angular momentum I and $H_{\rm c}$ is a perturbation term which disturbs the regular part of the rotational energies due to the distortion of the rotor and leads to the mixing of $\Delta K = 4$ bands.

The particle-rotor model Hamiltonian is given by

$$H_{\rm PR} = H_{\rm tro} + H_{\rm intr}\,, \tag{2}$$

where $H_{\rm rot}$ is rotor Hamiltonian

$$H_{\rm rot} = \sum_{k=1}^{3} \frac{\hbar^2}{2J_k}(I_k - j_k)^2 = \sum_{k=1}^{3} A_k (I_k - j_k)^2\,, \tag{3}$$

where J_k is the moment of inertia associated with rotation about the intrinsic kth axis, and $A_k = \hbar^2/(2J_k)$ is the inertial parameter. Since the superdeformed nucleus is an ideal rigid rotor and its energy spectra can be excellently described by ab formula [11], in the numerical calculations we take I-dependence of the moment of inertia [9]

$$J_k = J_{k0} f(I)\,, \qquad k = 1, 2, 3. \tag{4}$$

$$f(I) = \frac{1 + \sqrt{1 + bI(I+1)}}{2}\,, \tag{5}$$

where J_{k0} is the moment of inertia of the rigid rotor and b is an adjustable parameter. The formula of the intrinsic Hamiltonian $H_{\rm intr}$ can be found in Refs. [8, 9].

The perturbation is expressed as [8]

$$H_{\rm c} = h_4(I_+^4 + I_-^4) + h_0(I_+^2 I_-^2 + I_-^2 I_+^2 + I_+ I_- I_- I_+ + I_+ I_- I_+ I_- + I_- I_+ I_+ I_- + I_- I_+ I_- I_+). \tag{6}$$

where h_0 and h_4 are perturbation parameters. To make discussion consistent with Refs. [5, 6] parameters h_0 and h_4 are rewritten by B_1 and B_2

$$h_0 = \frac{1}{6}\left(\frac{B_1}{2} + B_2\right),$$
$$h_4 = \frac{B_1}{4}. \tag{7}$$

The perturbation term relates to the deviation from the axially symmetric rotor [8] and B_1 and B_2 depend on the square of the moment of inertia. Thus the perturbation intensity parameters B_1 and B_2 used in the calculation are also slowly changed with increasing angular momentum I:

$$B_1 = \frac{A_{30}}{C_1 f^2(I)},$$
$$B_2 = C_2 B_1, \tag{8}$$

where $A_{30} = \hbar^2/(2J_{30})$, $f(I)$ is expressed by Eq. (5) and C_1 and C_2 are two constants.

3. RESULTS AND DISCUSSION

In Refs. [9, 12], the particle-rotor model has been used to describe the odd A superdeformed nuclear states in the $A \sim 190$ mass region. In Ref. [8], this model is also used to investigate $\Delta I = 2$ staggering in the superdeformed rotational spectra and the dependence of $\Delta I = 2$ staggering on the perturbation intensity and the intrinsic properties of nuclei. In this paper, the particle-rotor model is extended to describe the odd A superdeformed nuclear states in the $A \sim 150$ mass region and the calculations have been performed for ^{149}Gd(b1) and ^{153}Dy(b1) and compared with the experimental data. The last neutron occupies the ν[761 3/2] and ν[770 1/2] orbital for ^{153}Dy(b1) and ^{149}Gd(b1), respectively. Therefore, the single $j = 15/2$ particle-rotor model is used in the calculations. In order to compare with the observed data we assume that the exit spins are $I_0 = 63/2$ and $I_0 = 51/2$ [13], corresponding to transition energy $E_\gamma(I_0 + 2 \to I_0) = 721.4$ keV and 617.8 keV, for ^{153}Dy(b1) and ^{149}Gd(b1), respectively. The exit spin assignments for ^{153}Dy(b1) and ^{149}Gd(b1) are identical with the values in Ref. [14]. In the following, the discussion will concentrate on the calculated results in ^{153}Dy(b1), while only a brief description is given for ^{149}Gd(b1).

The γ-transition energies as a function of the total angular momentum I for ^{153}Dy(b1) are given in Fig. 1 and the dynamic moments of inertia $J^{(2)}$ are presented in Fig. 2. In the calculations, the axially symmetric deformation $\gamma = 0°$ is assumed. Since the appropriate distortion of the rotor is important for the occurrence of the $\Delta I = 2$ energy staggering [9] $J_{30} \neq 0$ is used in the calculations. κ is an energy unit which is determined by using normalization $E_\gamma(I_{cal} = 71/2) = E_\gamma(I_{exp} = 71/2) = 765.9$ keV. Thus $\kappa = 5.4719$ MeV is obtained, which is consistent with the value $\kappa = 5$–6 MeV estimated by the cranking model in Ref. [15]. From Figs. 1 and 2 it is seen that: (1) In the observed range the calculated γ-transition energies coincide with the experimental values quite well. It shows that the particle-rotor model can be used to describe the odd A superdeformed nuclear states in the $A \sim 150$ mass region. (2) Since the exit spins of the SD bands in the $A \sim 150$ mass region are much higher then those in the $A \sim 190$ mass region and related to the spin alignments (especially where high j intruder states are involved, such as $\nu j_{15/2}$), many authors doubt the reliability of the exit spin assignments in SD bands through the theoretical models. The agreement between the calculated and observed values shows that the assumption of the exit spin assignment for ^{153}Dy(b1) is correct. (3) The

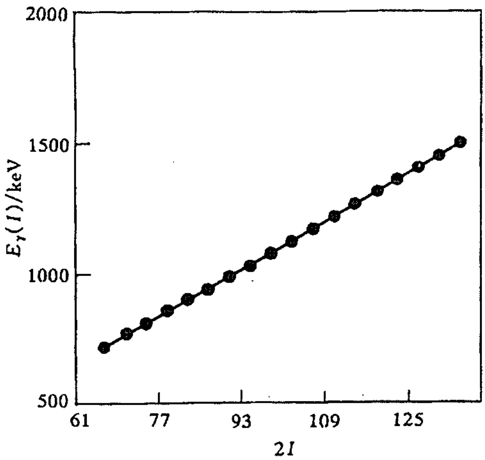

Fig. 1

Calculated γ-transition energies E_γ in ^{153}Dy(b1) are compared with the observed ones.

Parameters used are: $\gamma = 0°$, $\lambda = 0.71\kappa$, $\Delta = 0.045\kappa$, $J_{10} = J_{20} = 750/\kappa$, $J_{30} = 200/\kappa$, $b = -7.0 \times 10^{-6}$, $B_1 = A_{30}/(80 \cdot f^2)$, $B_2 = -1.8 \times 10^{-4} B_1$. Solid line: calculated values; points: experimental data.

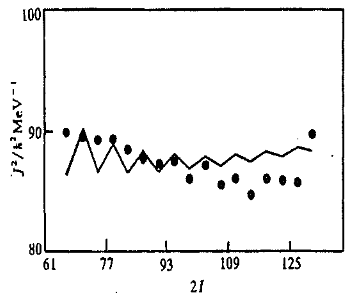

Fig. 2

Calculated dynamic moments of inertia $J^{(2)}$ in ^{153}Dy(b1) are compared with the observed data.

Parameters used are: $\gamma = 0°$, $\lambda = 0.71\kappa$, $\Delta = 0.045\kappa$, $J_{10} = J_{20} = 750/\kappa$, $J_{30} = 200/\kappa$, $b = -7.0 \times 10^{-6}$, $B_1 = A_{30}/(80 \cdot f^2)$, $B_2 = -1.8 \times 10^{-4} B_1$. Solid line: calculated values; points: experimental data.

calculated dynamic moments of inertia $J^{(2)}$ reproduce the primary feature of the observed values, namely, in a large region of the angular momentum $J^{(2)}$ its values are almost constant. The fluctuation of $J^{(2)}$ values reflect the existence of $\Delta I = 2$ energy staggering. In Fig. 3, $\Delta I = 2$ energy staggering as a function of the total angular momentum I for ^{153}Dy(b1) is given. The magnitude of $\Delta I = 2$ staggering is defined as the energy difference between two successive γ-transition energies $\Delta I = 2\Delta E_\gamma(I) = E_\gamma(I)$

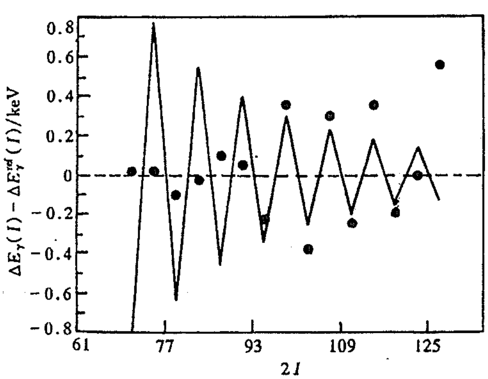

Fig. 3

$\Delta I = 2$ staggering as a function of the total angular momentum I for ^{153}Dy(b1).

Parameters used are: $\gamma = 0°$, $\lambda = 0.71\kappa$, $\Delta = 0.045\kappa$, $J_{10} = J_{20} = 750/\kappa$, $J_{30} = 200/\kappa$, $b = -7.0 \times 10^{-6}$, $B_1 = A_{30}/(80 \cdot f^2)$, $B_2 = -1.8 \times 10^{-4} B_1$. Solid line: calculated values; points: experimental data.

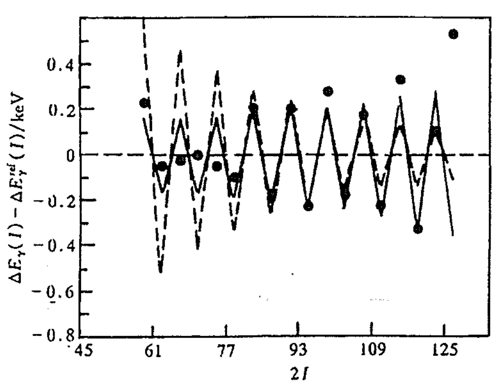

Fig. 4

$\Delta I = 2$ staggering as a function of the total angular momentum I for ^{149}Gd(b1).

Parameters used are: $\gamma = -10°$, $\lambda = -0.92\kappa$, $\Delta = 0.045\kappa$, $J_{10} = J_{20} = 700/\kappa$, $J_{30} = 220/\kappa$, $b = -1.0 \times 10^{-5}$, $B_2 = 0$. Dashed line: $B_1 = A_{30}/(80 \cdot f^2)$; solid line: $B_1 = A_{30}/(100 - 0.8(I - 22.5))$; points: experimental data.

$- E_\gamma(I-2)$ from which a smooth reference energy $\Delta E_\gamma^{ref}(I)$, expressed in the same way as that in Ref. [1], has been subtracted.

The calculated $\Delta I = 2$ energy staggering in ^{153}Dy(b1) is qualitatively in agreement with the observed data. However, there is an obvious disagreement between them: In the region $I < 91/2$, the observed values of $\Delta I = 2$ staggering nearly equal to zero. Apparently, the calculated values are too large. On the other hand, in region $I > 91/2$, the calculated values are less then the experimental data and decrease with increasing I. It is noticed that the amplitudes of $\Delta I = 2$ staggering are sensitively dependent on the perturbation intensity [8]. Therefore, it shows that the perturbation intensity is too strong in region $I <$ 91/2, but it is too weak in region $I > 91/2$ and with increasing I the perturbation intensity needs to increase. In order to examine this view the calculated $\Delta I = 2$ staggering for ^{149}Gd(b1) is given in Fig. 4, in which the dashed line denotes the case with $C_1 = 80$ while the solid line shows that of C_1 linearly decreasing with increasing I. (Suppose $C_1 = 100$ at $I = 45/2$ and $C_1 = 64$ at $I = 135/2$.) From this figure it is seen that in the case of $C_1 = 80$, the calculated $\Delta I = 2$ staggering is qualitatively in agreement with the observed data, but the amplitude of $\Delta I = 2$ staggering decrease with increasing I. However, the solid line shows that the amplitude of $\Delta I = 2$ staggering gradually increases with increasing I, which is consistent with the behavior of the observed data.

4. CONCLUSIONS

The properties of superdeformed nuclear states in ^{153}Dy(b1) and ^{149}Gd(b1) have been investigated using perturbed particle-rotor model. From the comparison between the calculated results and observed data the following conclusions can be obtained:

(1) The particle-rotor model can be used to investigate the properties of the odd A superdeformed nuclear states in the $A \sim 150$ mass region.

(2) The calculated γ-transition energies, dynamic moments of inertia and $\Delta I = 2$ energy staggering are in agreement with the experimental data on the whole. It shows that the perturbed particle-rotor model can be used to investigate the behavior of $\Delta I = 2$ staggering.

(3) The phenomenon of $\Delta I = 2$ staggering itself does not require C_4-symmetry in the Hamiltonian, but the appropriate distortion of the rotor will lead to $\Delta I = 2$ staggering.

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