$a_2(1320)\to\pi^+\pi^+\pi^-$过程的三体振幅分析*

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摘要 对$a_2(1320)\to\pi^+\pi^+\pi^-$过程的衰变振幅进行张量分析，展示了如何得到三体衰变振幅.

关键词 张量分析 三体振幅 共振结构 背景项

1 引言

在高能粒子物理实验中，需要给出反应过程微分截面的解析形式，以便进行数据拟合.通常采用张量分析方法[1—7]. 这种方法与早期的螺旋度法、分波法[8—12]相比，能进一步给出振幅对能量的依赖关系. 但文献中的方法都是把反应过程看成级联的两体衰变. 本文以$a_2(1320)\to\pi^+\pi^+\pi^-$的振幅分析为例，说明怎样构造直接的三体衰变振幅. 第二节构造了末态为$\pi^+\pi^+\pi^-$的衰变过程一般的三体振幅，第三节考虑宇称守恒对振幅的限制，第四节讨论$a_2(1320)\to\pi^+\pi^+\pi^-$过程的共振结构.

2 三体振幅的构造

记$a_2(1320)$、π^+、π^+、π^-的四动量分别为p、p_1、p_2、p_3. 这一衰变过程的螺旋振幅可以形式的写为

$$F_\lambda = e_{\mu\nu}(p,\ 2\lambda)\Gamma^{\mu\nu},\qquad \lambda = 0,\ \pm 1,\ \pm 2. \tag{1}$$

其中λ是$a_2(1320)$的螺旋度，$e_{\mu\nu}(p,\ 2\lambda)$是母粒子$a_2(1320)$的极化张量，$\Gamma^{\mu\nu}$是外腿为$a_2(1320)$、π^+、π^+、π^-的四腿等效顶角.

$e_{\mu\nu}(p,2\lambda)$满足Rarita-Schwinger条件[13]:

$$e_{\mu\nu}(p,\ 2\lambda) = e_{\nu\mu}(p,\ 2\lambda), \tag{2a}$$

$$g^{\mu\nu}e_{\mu\nu}(p,\ 2\lambda) = 0, \tag{2b}$$

$$p^\mu e_{\mu\nu}(p,\ 2\lambda) = 0. \tag{2c}$$

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下面分析顶角$\Gamma^{\mu\nu}$的指标，$\Gamma^{\mu\nu}$只可能由

$$p_1^\mu,\quad p_2^\mu,\quad p_3^\mu,\quad g^{\mu\nu},\quad \varepsilon^{\mu\nu\alpha\beta}$$

构成；这里$g^{\mu\nu}$是度规，取作 diag{1, − 1, − 1, − 1}；$\varepsilon^{\mu\nu\alpha\beta}$是全反对称张量．由于$e_{\mu\nu}(p, 2\lambda)$无迹，可排除$g^{\mu\nu}$；并且全反对称张量只能以

$$Q^\mu \overset{\text{def}}{=} p_1^\alpha p_2^\beta p_3^\gamma \varepsilon^{\alpha\beta\gamma\mu} \tag{3}$$

的形式出现．考虑到μ、ν指标的对称性，所有可能的组合为

张量：

$$\begin{gathered} p_1^\mu p_2^\nu + p_1^\nu p_2^\mu,\quad p_1^\mu p_3^\nu + p_1^\nu p_3^\mu,\quad p_2^\mu p_3^\nu + p_2^\nu p_3^\mu, \\ p_1^\mu p_1^\nu,\ p_2^\mu p_2^\nu,\quad p_3^\mu p_3^\nu,\quad Q^\mu Q^\nu; \end{gathered} \tag{4a}$$

赝张量：

$$Q^\mu p_1^\nu + Q^\nu p_1^\mu,\quad Q^\mu p_2^\nu + Q^\nu p_2^\mu,\quad Q^\mu p_3^\nu + Q^\nu p_3^\mu. \tag{4b}$$

由(2c)式及能动量守恒$p_1 + p_2 + p_3 = p$知(4)式中含p_3^μ的项可以表示为其它项的线性组合，独立的项为

张量：

$$p_1^\mu p_2^\nu + p_1^\nu p_2^\mu,\quad p_1^\mu p_1^\nu,\quad p_2^\mu p_2^\nu,\quad Q^\mu Q^\nu; \tag{5a}$$

赝张量：

$$Q^\mu p_1^\nu + Q^\nu p_1^\mu,\ Q^\mu p_2^\nu + Q^\nu p_2^\mu. \tag{5b}$$

对$a_2(1320)\to\pi^+\pi^+\pi^-$过程，1、2 两个$\pi^+$粒子是全同玻色子，顶角$\Gamma^{\mu\nu}$应当具有 1、2 粒子的交换对称性．将(5)式的结果对 1、2 粒子作对称化得

张量：

$$\begin{gathered} p_1^\mu p_2^\nu + p_1^\nu p_2^\mu,\quad p_1^\mu p_1^\nu + p_2^\mu p_2^\nu,\quad Q^\mu Q^\nu, \\ (p_1 - p_2)\cdot p_3(p_1^\mu p_1^\nu - p_2^\mu p_2^\nu); \end{gathered} \tag{6a}$$

赝张量：

$$\begin{gathered} Q^\mu(p_1^\nu - p_2^\nu) + Q^\nu(p_1^\mu - p_2^\mu), \\ (p_1 - p_2)\cdot p_3(Q^\mu p_3^\nu + Q^\nu p_3^\mu). \end{gathered} \tag{6b}$$

一般的顶角形式为

$$\begin{aligned} \Gamma^{\mu\nu} = {} & c_1[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu] + c_2[p_1^\mu p_1^\nu + p_2^\mu p_2^\nu] + c_3 Q^\mu Q + \\ & c_4[(p_1 - p_2)\cdot p_3(p_1^\mu p_1^\nu - p_2^\mu p_2^\nu)] + \\ & c_5[Q^\mu(p_1^\nu - p_2^\nu) + Q^\nu(p_1^\mu - p_2^\mu)] + \\ & c_6[(p_1 - p_2)\cdot p_3(Q^\mu p_3^\nu + Q^\nu p_3^\mu)]. \end{aligned} \tag{7}$$

3　空间反射对称性

记η、η_1、η_2、η_3为$a_2(1320)$、π^+、π^+、π^-的内禀宇称，J、J_1、J_2、J_3分别是相应粒子的自旋，λ、λ_1、λ_2、λ_3是粒子的螺旋度；初态为$|\boldsymbol{p}, J, \lambda\rangle$，末态为$|\boldsymbol{p}_1, J_1, \lambda_1; \boldsymbol{p}_2, J_2, \lambda_2; \boldsymbol{p}_3, J_3, \lambda_3\rangle$，$P$是宇称算符．

采用文献[5]的约定，有

$$
\begin{aligned}
&P|\boldsymbol{p},\ J,\ \lambda\rangle \overset{\text{def}}{=} \eta \mathrm{e}^{-\mathrm{i}\pi J}|-\boldsymbol{p},\ J,\ -\lambda\rangle,\\
&P|\boldsymbol{p}_1,\ J_1,\ \lambda_1;\ \boldsymbol{p}_2,\ J_2,\ \lambda_2;\ \boldsymbol{p}_3,\ J_3,\ \lambda_3\rangle \equiv\\
&\eta_1\eta_2\eta_3\mathrm{e}^{-\mathrm{i}\pi(J_1+J_2+J_3)}|-\boldsymbol{p}_1,\ J_1,\ -\lambda_1;\ -\boldsymbol{p}_2,\ J_2,\ -\lambda_2;\ -\boldsymbol{p}_3,\ J_3,\ -\lambda_3\rangle.
\end{aligned}
\tag{8}
$$

取极化矢量为

$$
(e^{\mu}(p,0)) = \begin{bmatrix} \dfrac{|\boldsymbol{p}|}{W} \\ \dfrac{E}{W}\sin\theta\cos\phi \\ \dfrac{E}{W}\sin\theta\sin\phi \\ \dfrac{E}{W}\cos\theta \end{bmatrix},
$$

$$
(e^{\mu}(p,\pm 1)) = \begin{bmatrix} 0 \\ \mp\cos\theta\cos\phi + \mathrm{i}\sin\phi \\ \mp\cos\theta\sin\phi - \mathrm{i}\cos\phi \\ \pm\sin\phi \end{bmatrix};
\tag{9}
$$

其中

$$
\begin{aligned}
&(p^{\mu}) = (E,\ \boldsymbol{p}),\\
&\boldsymbol{p} = (|\boldsymbol{p}|\sin\theta\cos\phi,\ |\boldsymbol{p}|\sin\theta\sin\phi,\ |\boldsymbol{p}|\cos\theta),\\
&W = \sqrt{p^{\mu}p_{\mu}}\ .
\end{aligned}
\tag{10}
$$

任意阶的极化张量按下式递归定义：

$$
e^{\mu_1\mu_2\cdots\mu_j}(p,\ j,\ \lambda) \overset{\text{def}}{=} \sum_{\lambda_1,\lambda_2}(1,\lambda_1;\ j-1,\ \lambda_2|j,\ \lambda)e^{\mu_j}(p,\ \lambda_1)e^{\mu_1\mu_2\cdots\mu_{j-1}}(p,\ j-1,\lambda_2).
\tag{11}
$$

定义空间反射矩阵

$$
(P^{\mu}_{\nu}) = \mathrm{diag}\{1,\ 1,\ 1,\ 1\},
\tag{12}
$$

并记

$$
\begin{aligned}
&\bar{p}^{\mu} \equiv P^{\mu}_{\nu}p^{\nu} = (E,\ -\boldsymbol{p}),\\
&\bar{p}^{\mu}_{i} \equiv P^{\mu}_{\nu}p^{\nu}_{i} = (E_i,\ -\boldsymbol{p}_i),\quad i = 1,\ 2,\ 3;\\
&\bar{e}^{\mu_1\mu_2\cdots\mu_j} \equiv P^{\mu_1}_{\nu_1}P^{\mu_2}_{\nu_2}p^{\mu j}_{\nu j}e^{\nu_1\nu_2\cdots\nu_j}\ .
\end{aligned}
\tag{13}
$$

由(9)式可以看出

$$
e^{\mu}(\bar{p},\ \lambda) = \bar{e}^{\mu}(p,\ -\lambda).
\tag{14}
$$

利用(11)式的定义可得

$$
e^{\mu_1\mu_2\cdots\mu_j}(\bar{p},\ j,\ \lambda) = \bar{e}^{\mu_1\mu_2,\cdots\mu_j}(p,\ j,\ -\lambda).
\tag{15}
$$

由宇称守恒得螺旋振幅为

$$\begin{aligned}&\langle \boldsymbol{p},\ J,\ \lambda|M|\boldsymbol{p}_1,\ J_1,\ \lambda_1;\ \boldsymbol{p}_2,\ J_2,\ \lambda_2;\ \boldsymbol{p}_3,\ J_3,\ \lambda_3\rangle =\\ &\langle \boldsymbol{p},\ J,\ \lambda|P^+MP|\bar{\boldsymbol{p}}_1,\ J_1,\ \lambda_1;\ \boldsymbol{p}_2,\ J_2,\ \lambda_2;\ \boldsymbol{p}_3,\ J_3,\ \lambda_3\rangle =\\ &\eta^*\eta_1\eta_2\eta_3 e^{i\pi(J-J_1-J_2-J_3)}\langle -\boldsymbol{p},\ J,\ -\lambda|M|-\boldsymbol{p}_1,J_1,-\lambda_1;\ -\boldsymbol{p}_2,J_2,-\lambda_2;-\boldsymbol{p}_3,J_3,-\lambda_3\rangle.\end{aligned} \tag{16}$$

又

$$\begin{aligned}&\langle \boldsymbol{p},\ J,\ \lambda|M|\boldsymbol{p}_1,\ J_1,\ \lambda_1;\ \boldsymbol{p}_2,\ J_2,\ \lambda_2;\ \boldsymbol{p}_3,\ J_3,\ \lambda_3\rangle\equiv\\ &e^*(p,J,\lambda)\Gamma(p,\ p_1,\ p_2,\ p_3,\ g^{\mu\nu},\ \varepsilon^{\alpha\beta\gamma\delta})e(p_1,\ J_1,\ \lambda_1)e(p_2,\ J_2,\ \lambda_2)e(p_3,\ J_3,\ \lambda_3),\end{aligned} \tag{17}$$

上式省略了指标；Γ 是 p^μ、p_1^μ、p_2^μ、p_3^μ、$g^{\mu\nu}$、$\varepsilon^{\alpha\beta\gamma\delta}$ 的缩并(只对整数自旋适用)．所以宇称守恒相当于要求

$$\begin{aligned}&e^*(\bar{p},\ J,\ -\lambda)\Gamma(\bar{p},\ \bar{p}_1,\ \bar{p}_2,\ \bar{p}_3,\ g^{\mu\nu},\ \varepsilon^{\alpha\beta\gamma\delta})e(\bar{p}_1,\ J_1,\ -\lambda_1)\times\\ &\quad e(\bar{p}_2,\ J_2,\ -\lambda_2)e(\bar{p}_3,\ J_3,\ -\lambda_3)=\\ &\eta\eta_1^*\eta_2^*\eta_3^*(-1)^{J-J_1-J_2-J_3}e^*(p,J,\lambda)\Gamma(p,p_1,p_2,p_3,g^{\mu\nu},\varepsilon^{\alpha\beta\gamma\delta})e(p_1,J_1,\lambda_1)\times\\ &\quad e(p_2,J_2,\lambda_2)e(p_3,J_3,\lambda_3),\end{aligned} \tag{18}$$

其中用了 $\eta^*\eta=1$，$\eta_i^*\eta_i=1\,(i=1,2,3)$．

本例中 $J=2$，$\eta=+1$，$J_i=0$，$\eta_i=-1(i=1,\ 2,\ 3)$[14]；从而

$$e(p_1,J_1,\lambda_1)=e(p_2,J_2,\lambda_2)=e(p_3,J_3,\lambda_3)=1. \tag{19}$$

代入(18)式，

$$\begin{aligned}&e^*_{\mu\nu}(p,J,\lambda)\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},\ \varepsilon^{\alpha\beta\gamma\delta})=\\ &-e^*_{\mu\nu}(\bar{p},J,\ -\lambda)\Gamma^{\mu\nu}(\bar{p},\bar{p}_1,\ \bar{p}_2,\bar{p}_3,\ g^{\sigma\tau},\varepsilon^{\alpha\beta\gamma\delta}),\end{aligned} \tag{20}$$

这里 $\Gamma^{\mu\nu}$ 是对称无迹的张量．再利用(15)式以及

$$\bar{g}^{\mu\nu}\overset{\mathrm{def}}{=}P^\mu_\alpha P^\nu_\beta g^{\alpha\beta}\equiv g^{\mu\nu},$$

$$\bar{\varepsilon}^{\alpha\beta\gamma\delta}\overset{\mathrm{def}}{=}P^\alpha_{\alpha'}P^\beta_{\beta'}P^\gamma_{\gamma'}P^\delta_{\delta'}\varepsilon^{\alpha'\beta'\gamma'\delta'}\equiv-\varepsilon^{\alpha\beta\gamma\delta}; \tag{21}$$

得

$$\begin{aligned}&e^*_{\mu\nu}(p,J,\lambda)\Gamma^{\mu\nu}(p,p_1,p_2,p_3,\ g^{\sigma\tau},\varepsilon^{\alpha\beta\gamma\delta})=\\ &-e^*_{\mu\nu}(p,J,\lambda)\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},\ -\varepsilon^{\alpha\beta\gamma\delta}),\\ &\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},\varepsilon^{\alpha\beta\gamma\delta})=-\Gamma^{\mu\nu}(p,p_1,p_2,p_3,g^{\sigma\tau},-\varepsilon^{\alpha\beta\gamma\delta}).\end{aligned} \tag{22}$$

即由 $a_2(1320)\to\pi^+\pi^+\pi^-$ 过程的宇称守恒知全反对称张量 $\varepsilon^{\alpha\beta\gamma\delta}$ 在顶角 $\Gamma^{\mu\nu}$ 中只能出现奇数次．于是顶角的一般形式为

$$\Gamma^{\mu\nu}=c\,[Q^\mu(p_1^\nu-p_2^\nu)+Q^\nu(p_1^\mu-p_2^\mu)]+d\,(p_1-p_2)\cdot p_3(Q^\mu p_3^\nu+Q^\nu p_3^\mu). \tag{23}$$

上式中的 c、d 是 Lorentz 标量．

4　振幅的共振结构和背景项

$a_2(1320)\to\pi^+\pi^+\pi^-$ 的主要共振衰变模式是 $a_2(1320)\to\pi^+\rho^0$，$\rho^0\to\pi^+\pi^-$．对这两体级联过程进行张量分析得[5]

$$F_{\lambda\,共振}=c_r e_{\mu\nu}(p,\,2\lambda)\,\Phi^{\mu\nu}=c_r e_{\mu\nu}(p,2\lambda)\,\Phi^{\{\mu,\,\nu\}}\ ,$$
$$\Phi^{\mu\nu}=(D_{23}Q^{\mu}p_1^{\nu}-D_{13}Q^{\mu}p_2^{\nu}),$$
$$\Phi^{\{\mu,\,\nu\}}=\frac{1}{2}\,(\Phi^{\mu\nu}+\Phi^{\nu\mu});\qquad(24)$$

其中 c_r是 Lorentz 标量，D_{13}、D_{23}为 Breit-Wigner因子，

$$D_{23}=\frac{1}{(p_2+p_3)^2-M_\rho^2+\mathrm{i}\,\Gamma_\rho M_\rho}\ ,$$
$$D_{13}=\frac{1}{(p_1+p_3)^2-M_\rho^2+\mathrm{i}\,\Gamma_\rho M_\rho}\ ;\qquad(25)$$

M_ρ、Γ_ρ 是ρ_0粒子的质量和宽度．(24)式满足 1、2 粒子的玻色对称．

按第 3 节的结果，$a_2(1320)\rightarrow\pi^+\pi^+\pi^-$的一般振幅可以写作

$$F_\lambda=e_{\mu\nu}(p,2\lambda)\,\Gamma^{\mu\nu}=ce_{\mu\nu}(p,2\lambda)[Q^{\mu}(p_1^{\nu}-p_2^{\nu})+Q^{\nu}(p_1^{\mu}-p_2^{\mu})]+ de_{\mu\nu}(p,2\lambda)(p_1-p_2)\cdot p_3(Q^{\mu}p_3^{\nu}+Q^{\nu}p_3^{\mu}).\qquad(26)$$

将上式形式的写成

$$F_\lambda=\begin{bmatrix}c\\ d\end{bmatrix},\qquad(27)$$

即把 F_λ 中正比于 c、d 的部分看成两个正交的矢量；满足宇称守恒和玻色对称性的 $a_2(1320)\rightarrow\pi^+\pi^+\pi^-$螺旋振幅是这两个矢量张成的二维空间的矢量．此时，共振项成为

$$F_{\lambda\,共振}=c_r\begin{bmatrix}\frac{1}{4}(D_{13}+D_{23})\\ \frac{D_{13}-D_{23}}{4(p_1-p_2)\cdot p_3}\end{bmatrix}.\qquad(28)$$

F_λ 可展开为与 $F_{\lambda\,共振}$平行的项和与之垂直的项

$$F_\lambda=c_{/\!/}\begin{bmatrix}\frac{1}{4}(D_{13}+D_{23})\\ \frac{D_{13}-D_{23}}{4(p_1-p_2)\cdot p_3}\end{bmatrix}+c_\perp\begin{bmatrix}\frac{D_{13}-D_{23}}{4(p_1-p_2)\cdot p_3}\\ -\frac{1}{4}(D_{13}+D_{23})\end{bmatrix}=$$

$$c_r\begin{bmatrix}\frac{1}{4}(D_{13}+D_{23})\\ \frac{D_{13}-D_{23}}{4(p_1-p_2)\cdot p_3}\end{bmatrix}+\begin{bmatrix}(c_{/\!/}-c_r)\,\frac{D_{13}+D_{23}}{4}+c_\perp\frac{D_{13}-D_{23}}{4(p_1-p_2)\cdot p_3}\\ (c_{/\!/}-c_r)\,\frac{D_{13}-D_{23}}{4(p_1-p_2)\cdot p_3}-c_\perp\frac{D_{13}+D_{23}}{4}\end{bmatrix}$$

$$\equiv F_{\lambda\,共振}+F_{\lambda\,背景}.\qquad(29)$$

在相空间的ρ_0共振点附近，$F_{\lambda共振}$ 是 F_λ 的领头项，背景项对能量的依赖较小，可取为常数：

$$F_{\lambda背景}=\begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

$$F_\lambda=F_{\lambda共振}+F_{\lambda背景}=\begin{bmatrix} \frac{c_r}{4}(D_{13}+D_{23})+b_1 \\ \frac{c_r(D_{13}-D_{23})}{4(p_1-p_2)\cdot p_3}+b_2 \end{bmatrix}. \tag{30}$$

所以考虑了ρ_0共振的螺旋振幅是

$$F_\lambda=\left[\frac{c_r}{4}(D_{13}+D_{23})+b_1\right]e_{\mu\nu}(p,2\lambda)[Q^\mu(p_1^\nu-p_2^\nu)+Q^\nu(p_1^\mu-p_2^\mu)]+$$
$$\left[\frac{c_r}{4}(D_{13}-D_{23})+b_2(p_1-p_2)\cdot p_3\right]e_{\mu\nu}(p,2\lambda)(Q^\mu p_3^\nu+Q^\nu p_3^\mu). \tag{31}$$

在进行数据拟合时可把 c_r、b_1、b_2当作常数处理．

5　结论

本文通过对四腿顶角的协变性和对称性分析，得到了 $a_2(1320)\to\pi^+\pi^+\pi^-$衰变振幅的一般形式．所得的振幅比两体级联振幅的形式更广泛．在对衰变进行共振结构分析时，此结果可得到背景项．

以本文的例子来说，如果没有背景项，在ρ_0共振点附近时，两体级联衰变分析所给出的振幅是合理的；但在不是很靠近共振点的相空间区域，此振幅失去了合理性，背景项可能贡献较大．因此需要一个本文所给出的三体衰变振幅的一般形式作为背景项．

对初末态为任意自旋的情形，可以通过类似于本文的分析得出．

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Three Body Amplitude Analysis of the Process $a_2(1320) \to \pi^+\pi^+\pi^-$

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Abstract We analyze the decay amplitudes of the process $a_2(1320) \to \pi^+\pi^+\pi^-$ using the tensor analysis method and show how to obtain three body amplitudes.

Key words tensor analysis, three body amplitude, resonant structure, background term

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