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Z_n Belavin 模型反射方程的多参数解

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摘要 利用 $A_{n-1}^{(1)}$ 面模型反射方程的对角解,得到了 Z_n Belavin 模型反射方程的含有 $n+1$ 个参数的解. 当 $n=2$ 时,其结果与侯伯宇等人给出的8顶角反射方程的解是一致的.

关键词 $A_{n-1}^{(1)}$ 面模型 Z_n Belavin 模型 反射方程 多参数解

1 引言

在二维可解晶格模型中,非周期边界条件是人们颇感兴趣的一件事情. 自从 Sklyanin[1]提出反射方程来解决非周期边界条件问题以来,人们已经获得了许多精确可解晶格模型反射方程的解[1—10]. 在此之中,侯伯宇教授等人[5]给出了一种含有3个任意参数的8顶角模型反射方程的解,而对于含有3个以上任意参数的 Z_n Belavin 模型反射方程的解却未曾见到报道过($n\geqslant 3$).

利用 M.T. Batchelor 等人[8]所给的 $A_{n-1}^{(1)}$ 面模型反射方程的对角解,通过 Jimbo 等人[11]的面顶角对应的 intertwiner,得到了 Z_n Belavin 模型[12,13]反射方程的含有 $n+1$ 个任意参数的解. 在 $n=2$ 时,所得的结果与侯伯宇教授等人[5]所得的结果在形式上是一致的.

2 $A_{n-1}^{(1)}$ 面模型反射方程及其对角解

首先介绍 Z_n Belavin 模型的 R 矩阵[12,13]. Z_n Belavin 模型的 R 矩阵为

$$R_{lm}(z)=\frac{1}{n}\sum_{\alpha\in Z_n^2}W_\alpha(z)I_\alpha^{(l)}(I_\alpha^{-1})^{(m)}, \tag{1}$$

其中 $\alpha=(\alpha_1,\ \alpha_2)$, $\alpha_1,\ \alpha_2\in Z_n$, $l,\ m\in Z$, I_α 是 $n\times n$ 矩阵, $I_\alpha^{(l)}=I\otimes\cdots\otimes I\otimes I_\alpha\otimes I\otimes\cdots\otimes I$, I_α 在第 l 空间, I 是 $n\times n$ 单位矩阵, $I_\alpha=g^{\alpha_2}h^{\alpha_1}$, $h_{ij}=\delta_{i+1,\ j}$, $g_{ij}=w^i\delta_{ij}$, $w=\mathrm{e}^{\frac{2\pi\sqrt{-1}}{n}}$, g, h 均为 $n\times n$ 矩阵, $i,j\in Z$,

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$$W_\alpha(z)=\frac{\sigma_\alpha\left(z+\frac{w}{n}\right)}{\sigma_\alpha\left(\frac{w}{n}\right)},$$

$$\sigma_\alpha(z)\equiv\theta\begin{bmatrix}\frac{1}{2}+\frac{\alpha_1}{n}\\ \frac{1}{2}+\frac{\alpha_2}{n}\end{bmatrix}(z,\ \tau), \tag{2}$$

$$\theta\begin{bmatrix}a\\ b\end{bmatrix}(z,\ \tau)\equiv\sum_{m\in Z}\mathrm{e}^{\pi\sqrt{-1}(m+a)^2\tau+2\pi\sqrt{-1}(m+a)(z+b)},$$

w 是交叉参数．R 矩阵满足 Yang–Baxter方程 (YBE)

$$R_{ij}(z_i-z_j)R_{ik}(z_i-z_k)R_{jk}(z_j-z_k)=\\ R_{jk}(z_j-z_k)R_{ik}(z_i-z_k)R_{ij}(z_i-z_j). \tag{3}$$

R 矩阵也满足如下的幺正关系和幺正交叉关系，

$$R_{12}(z_1-z_2)R_{21}(z_2-z_1)=\rho(z_1-z_2)\cdot id, \tag{4}$$

$$R_{12}^{t_1}(z_1-z_2)R_{21}^{t_1}(z_2-z_1-nw)=\tilde{\rho}(z_1-z_2)\cdot id. \tag{5}$$

其中

$$\rho(z)=\frac{\sigma_0(z+w)\sigma_0(-z+w)}{\sigma_0^2(w)}, \tag{6}$$

$$\tilde{\rho}(z)=\frac{\sigma_0(z)\sigma_0(-z-nw)}{\sigma_0^2(w)}, \tag{7}$$

t_i 表示第 i 空间的转置．顶角型的反射方程[1]为

$$R_{12}(z_1-z_2)K_1(z_1)R_{21}(z_1+z_2)K_2(z_2)=\\ K_2(z_2)R_{12}(z_1+z_2)K_1(z_1)R_{21}(z_1-z_2). \tag{8}$$

其中 $K_1(z)=K(z)\otimes I$, $K_2(z)=I\otimes K(z)$, 作用在 $C^n\otimes C^n$空间上．$K(z)$ 如满足反射方程 (8)，则被称为反射方程的一个解．

Jimbo 等人[11]给出了 Z_n模型与 $A_{n-1}^{(1)}$面模型面顶角对应的 intertwiner，它是一个列矢量 $\phi_{a,\ a+\hat{\mu}}(z)$, 其分量为

$$\phi_{a,\ a+\hat{\mu}}^{(j)}(z)=\theta^{(j)}(z-nw\bar{a}_\mu,\ n\tau),$$

$$\theta^{(j)}(z,\ n\tau)=\theta\begin{bmatrix}\frac{1}{2}-\frac{j}{n}\\ \frac{1}{2}\end{bmatrix}(z,\ n\tau). \tag{9}$$

$a=(a_0,\ a_1,\ \cdots,\ a_{n-1})\in Z^n$, $\hat{\mu}=(0,\ \cdots,\ 0,\ 1,\ 0,\ \cdots,\ 0)$(第$\mu$个位置为1，其余位置为零)，$\bar{a}_\mu=a_\mu-\frac{1}{n}\sum_\nu a_\nu+\delta_\mu,\ \mu,\ \nu\in Z_n,\ \delta_\mu$ 是一些一般的复数．对于其它情况有

$$\phi^{(j)}_{a,\ b}(z)=0\qquad(b\neq a+\hat{\nu}).\tag{10}$$

利用 intertwiner，面顶角对应可以写为

$$R_{12}(z_1-z_2)\phi_{a,\ b}(z_1)\otimes\phi_{b,\ c}(z_2)=$$
$$\sum_d W\begin{bmatrix}a & d\\ b & c\end{bmatrix}(z_1-z_2)\phi_{d,\ c}(z_1)\otimes\phi_{a,\ d}(z_2)\tag{11}$$

$W\begin{bmatrix}a & d\\ b & c\end{bmatrix}(z)$为 $A^{(1)}_{n-1}$面模型的 Boltzmann 权．其定义为

$$W\begin{bmatrix}a & d\\ b & c\end{bmatrix}(z)=\frac{\sigma_0(z+w)}{\sigma_0(w)}\qquad(b=d=a+\hat{\mu},\ c=a+2\hat{\mu}),$$
$$W\begin{bmatrix}a & d\\ b & c\end{bmatrix}(z)=\frac{\sigma_0(-z+a_{\mu\nu}w)}{\sigma_0(a_{\mu\nu}w)}\qquad(b=d=a+\hat{\mu},\ c=b+\hat{\nu}),\tag{12}$$
$$W\begin{bmatrix}a & d\\ b & c\end{bmatrix}(z)=\frac{\sigma_0(z)\sigma_0((a_{\mu\nu}+1)w)}{\sigma_0(w)\sigma_0(a_{\mu\nu}w)}\quad(b=a+\hat{\mu},\ d=a+\hat{\nu},\ c=b+\hat{\nu}),$$
$$W\begin{bmatrix}a & d\\ b & c\end{bmatrix}(z)=0\qquad(\text{其它情况}),$$

其中 $a_{\mu\nu}=\bar{a}_\mu-\bar{a}_\nu$．还可以构造满足下列关系的行矢量$\tilde{\phi}$和$\bar{\phi}$，

$$\sum_k\tilde{\phi}^{(k)}_{a-\hat{\mu},\ a}(z)\phi^{(k)}_{a-\hat{\nu},\ a}(z)=\delta_{\mu\nu},$$
$$\sum_k\bar{\phi}^{(k)}_{a,\ a+\hat{\mu}}(z)\phi^{(k)}_{a,\ a+\hat{\nu}}(z)=\delta_{\mu\nu}.\ (k\in Z_n)\tag{13}$$

对于其它情况，$\tilde{\phi}$ 和$\bar{\phi}$ 定义为

$$\tilde{\phi}_{b,\ a}(z)=0\qquad(b\neq a-\hat{\mu}),$$
$$\bar{\phi}_{a,\ c}(z)=0\qquad(c\neq a+\hat{\mu}).\tag{14}$$

上述关系(13)又可写成

$$\sum_\mu\phi_{a-\hat{\mu},\ a}(z)\tilde{\phi}_{a-\hat{\mu},\ a}(z)=I,$$
$$\sum_\mu\phi_{a,\ a+\hat{\mu}}(z)\bar{\phi}_{a,\ a+\hat{\mu}}(z)=I.\tag{15}$$

利用$\tilde{\phi}$，$\bar{\phi}$，ϕ 之间的关系，面顶角又可写成以下其它几种形式，

$$1\otimes\bar{\phi}_{a,\ d}(z_2)R_{12}(z_1-z_2)\phi_{a,\ b}(z_1)\otimes 1=\sum_c W\begin{bmatrix}a & d\\ b & c\end{bmatrix}(z_1-z_2)\phi_{d,\ c}(z_1)\otimes\bar{\phi}_{b,\ c}(z_2),\tag{16}$$

$$\tilde{\phi}_{c,\,b}(z_1)\otimes 1R_{12}(z_1-z_2)1\otimes\phi_{a,\,b}(z_2)=\sum_d W\begin{bmatrix} d & c \\ a & b \end{bmatrix}(z_1-z_2)\tilde{\phi}_{d,\,a}(z_1)\otimes\phi_{d,\,c}(z_2), \tag{17}$$

$$\bar{\phi}_{d,\,c}(z_1)\otimes\bar{\phi}_{a,\,d}(z_2)R_{12}(z_1-z_2)=\sum_b W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z_1-z_2)\bar{\phi}_{a,\,b}(z_1)\otimes\bar{\phi}_{b,\,c}(z_2), \tag{18}$$

$$\tilde{\phi}_{d,\,c}(z_1)\otimes\tilde{\phi}_{a,\,d}(z_2)R_{12}(z_1-z_2)=\sum_b W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z_1-z_2)\tilde{\phi}_{a,\,b}(z_1)\otimes\tilde{\phi}_{b,\,c}(z_2). \tag{19}$$

将反射方程(8)等式两边同时左乘$\tilde{\phi}_{d,\,c}(z_1)\otimes\tilde{\phi}_{e,\,d}(z_2)$和右乘$\phi_{b,\,c}(-z_1)\otimes\phi_{a,\,b}(-z_2)$，利用面顶角对应关系(19)，(17)和(11)，得到$A_{n-1}^{(1)}$面模型的反射方程(参见附录1)

$$\sum_{fs} W\begin{bmatrix} e & d \\ s & c \end{bmatrix}(z_1-z_2)K(s_e^f)(z_1)W\begin{bmatrix} f & s \\ b & c \end{bmatrix}(z_1+z_2)K(b_f^a)(z_2)=$$

$$\sum_{fs} K(d_e^f)(z_2)W\begin{bmatrix} f & d \\ s & c \end{bmatrix}(z_1+z_2)K(s_f^a)(z_1)W\begin{bmatrix} a & s \\ b & c \end{bmatrix}(z_1-z_2). \tag{20}$$

M. T. Batchelor 等人[8]给出了上述面型反射方程(20)的一种解

$$K(a+\hat{\mu}_a^{\,b})(z)=\frac{\sigma_0(\bar{a}_\mu w-\xi+z)}{\sigma_0(\bar{a}_\mu w-\xi-z)}f_a(z)\,\delta_{a,\,b}. \tag{21}$$

ξ为任意参数，$f_a(z)$为任一解析函数．

3 顶角型反射方程的解

对于面型反射方程的对角解 $K(a_b^c)(z)=K(a_b^b)\,\delta_{b,c}$，将面型反射方程(20)等式两边同时左乘$\phi_{d,c}(z_1)\otimes\phi_{e,d}(z_2)$和右乘$\bar{\phi}_{b,c}(-z_1)\otimes\bar{\phi}_{a,b}(-z_2)$并对$b,c,d$求和，利用面顶角对应关系(18)，(16)和(11)式，可得到顶角型反射方程(参见附录2)

$$\begin{aligned} &R_{12}(z_1-z_2)K_1(a,\ z_1)R_{21}(z_1+z_2)K_2(a,\ z_2)= \\ &K_2(a,\ z_2)R_{12}(z_1+z_2)K_1(a,\ z_1)R_{21}(z_1-z_2), \end{aligned} \tag{22}$$

其中

$$K(a,\ z)=\sum_\mu \phi_{a,\,a+\hat{\mu}}(z)K(a+\hat{\mu}_a^{\,a})(z)\bar{\phi}_{a,\,a+\hat{\mu}}(-z). \tag{23}$$

可以证明，任何一个$n\times n$矩阵$A=(a_{ij})$都可以用$I_\alpha=g^{\alpha_2}h^{\alpha_1}$展开．令

$$A=\sum_{\alpha\in Z_n^2} C_\alpha I_\alpha \tag{24}$$

则有

$$C_\alpha=\frac{1}{n}\,\mathrm{tr}AI_\alpha^{-1}=\sum_i a_{i,\,i+\alpha_1}w^{-\alpha_2 i}. \tag{25}$$

在下面将$n\times n$矩阵$K(a,z)$按I_α展开，令

$$K(a,\ z) = \sum_{\alpha \in Z_n^2} D_\alpha I_\alpha, \tag{26}$$

则由(25)式，可以得到

$$D_\alpha = \frac{1}{n} \sum_\mu \sum_i \phi^{(i)}_{a,\ a+\hat{\mu}}(z) K(a + \hat{\mu}^a_a)(z) \bar{\phi}_{a,\ a+\hat{\mu}}(-z) w^{-\alpha_2 i} =$$
$$\frac{1}{n} \sum_\mu \frac{\sigma_0(\bar{a}_\mu w - \xi + z)}{\sigma_0(\bar{a}_\mu w - \xi - z)} f_a(z) E(\mu,\ \alpha,\ z),$$

其中

$$E(\mu,\ \alpha,\ z) = \sum_i \phi^{(i)}_{a,\ a+\hat{\mu}}(z) \bar{\phi}^{(i+\alpha_1)}_{a,\ a+\hat{\mu}}(-z) w^{-\alpha_2 i}.$$

利用

$$\theta^{(i)}(z,\ n\tau) = (-1)^{\alpha_2} w^{\alpha_1\alpha_2} w^{\alpha_2 i} \mathrm{e}^{2\pi\sqrt{-1}\frac{\alpha_1}{n}\left(\frac{\alpha_1}{2}\tau + z + \frac{1}{2}\right)} \times \theta^{(i+\alpha_1)}(z + \alpha_1\tau + \alpha_2,\ \tau), \tag{27}$$

得

$$E(\mu,\ \alpha,\ z) = \sum_i (-1)^{\alpha_2} w^{\alpha_1\alpha_2} \mathrm{e}^{2\pi\sqrt{-1}\frac{\alpha_1}{n}\left(\frac{\alpha_1}{2}\tau + z - nw\bar{a}_\mu + \frac{1}{2}\right)} \times$$
$$\phi^{(i+\alpha_1)}_{a,\ a+\hat{\mu}}(z + \alpha_1\tau + \alpha_2) \bar{\phi}^{(i+\alpha_1)}_{a,\ a+\hat{\mu}}(-z). \tag{28}$$

如有一矩阵 $B = (B_{ij}) = (\theta^{(i)}(nz_j))$，则可以证明

$$\det B = C(\tau)\sigma_0\left(\sum_i z_i - \frac{n-1}{2}\right) \prod_{j<k} \sigma_0(z_j - z_k). \tag{29}$$

利用上述公式(28)，得

$$\sum_i \phi^{(i+\alpha_1)}_{a,\ a+\hat{\mu}}(z + \alpha_1\tau + \alpha_2) \bar{\phi}^{(i+\alpha_1)}_{a,\ a+\hat{\mu}}(-z) =$$
$$\frac{\sigma_0\left(-z - w\delta - \frac{n-1}{2} + \frac{2z}{n} + \frac{1}{n}(\alpha_1\tau + \alpha_2)\right)}{\sigma_0\left(-z - w\delta - \frac{n-1}{2}\right)} \times$$
$$\prod_{j \neq \mu} \frac{\sigma_0\left(w(\bar{a}_j - \bar{a}_\mu) + \frac{1}{n}(2z + \alpha_1\tau + \alpha_2)\right)}{\sigma_0(w(\bar{a}_j - \bar{a}_\mu))}. \tag{30}$$

$\left(\delta = \sum_\mu \delta_\mu\right)$. 再利用

$$\sigma_0\left(z + \frac{1}{n}(\alpha_1\tau + \alpha_2)\right) = \mathrm{e}^{-2\pi\sqrt{-1}\frac{\alpha_1}{n}\left(\frac{\alpha_1}{2n}\tau + z + \frac{1}{2} + \frac{\alpha_2}{n}\right)} \sigma_{(\alpha_1,\ \alpha_2)}(z), \tag{31}$$

得

$$E(\mu,\ \alpha,\ z) = (-1)^{\alpha_2} \frac{\sigma_{(\alpha_1,\ \alpha_2)}\left(-z - w\delta - \frac{n-1}{2} + \frac{2z}{n}\right)}{\sigma_0\left(-z - w\delta - \frac{n-1}{2}\right)} \times$$

$$\prod_{j \neq \mu} \frac{\sigma_{(\alpha_1,\ \alpha_2)}\left(w(\bar{a}_j - \bar{a}_\mu) + \frac{2z}{n}\right)}{\sigma_0(w(\bar{a}_j - \bar{a}_\mu))}, \tag{32}$$

因此

$$K(a,\ z) = \frac{1}{n} \sum_{\alpha \in Z_n^2} \sum_{\mu} \frac{\sigma_0(\bar{a}_\mu w - \xi + z)}{\sigma_0(\bar{a}_\mu w - \xi - z)} f_a(z)(-1)^{\alpha_2} \times$$

$$\frac{\sigma_{(\alpha_1,\ \alpha_2)}\left(-z - w\delta - \frac{n-1}{2} + \frac{2z}{n}\right)}{\sigma_0\left(-z - w\delta - \frac{n-1}{2}\right)} \prod_{j \neq \mu} \frac{\sigma_{(\alpha_1,\ \alpha_2)}\left(w(\bar{a}_j - \bar{a}_\mu) + \frac{2z}{n}\right)}{\sigma_0(w(\bar{a}_j - \bar{a}_\mu))} \times I_\alpha. \tag{33}$$

4 $n = 2$ 时 $K(a, z)$ 的具体形式

当 $n = 2$ 时，

$$K(a,\ z) = \sum_{\alpha \in Z_2^2} D_\alpha I_\alpha, \tag{34}$$

$$D_\alpha = \frac{1}{2}(-1)^{\alpha_2} \frac{\sigma_{(\alpha_1,\ \alpha_2)}\left(-w\delta - \frac{1}{2}\right)}{\sigma_0\left(-w\delta - z - \frac{1}{2}\right)\sigma_0(a_{10}w)} \left[\frac{\sigma_0(\bar{a}_0 w - \xi + z)}{\sigma_0(\bar{a}_0 w - \xi - z)} \sigma_{(\alpha_1,\ \alpha_2)}(z + wa_{10}) - \right.$$

$$\left. \frac{\sigma_0(\bar{a}_1 w - \xi + z)}{\sigma_0(\bar{a}_1 w - \xi - z)} \sigma_{(\alpha_1,\ \alpha_2)}(z - wa_{10})\right] f_a(z). \tag{35}$$

令

$$F_{(\alpha_1,\ \alpha_2)}(z) = \sigma_0(\bar{a}_0 w - \xi + z)\sigma_0(\bar{a}_1 w - \xi - z)\sigma_{(\alpha_1,\ \alpha_2)}(z + wa_{10}) -$$
$$\sigma_0(\bar{a}_1 w - \xi + z)\sigma_0(\bar{a}_0 w - \xi - z)\sigma_{(\alpha_1,\ \alpha_2)}(z - wa_{10}), \tag{36}$$

则有

$$F_{(\alpha_1,\ \alpha_2)}(z + \tau) = e^{-2\pi\sqrt{-1}\left(3z + \frac{3}{2}\tau + \frac{1}{2} + \frac{\alpha_2}{2}\right)} F_{(\alpha_1,\ \alpha_2)}(z),$$

$$F_{(\alpha_1,\ \alpha_2)}(z + 1) = e^{2\pi\sqrt{-1}\left(\frac{1}{2} + \frac{\alpha_1}{2}\right)} F_{(\alpha_1,\ \alpha_2)}(z). \tag{37}$$

即 $F_{(\alpha_1,\ \alpha_2)}(z)$ 具有三个零点，且零点之和为

$$\sum_{i=1}^{3} z_i = \frac{3}{2} - \left(\frac{3\tau}{2} + \frac{1}{2} + \frac{\alpha_2}{2}\right) - \left(\frac{1}{2} - \frac{\alpha_1}{2}\right)\tau = -\frac{\alpha_1}{2}\tau - \frac{\alpha_2}{2} \quad (\mathrm{mod}\Lambda\tau). \tag{38}$$

可以找到 $F_{(\alpha_1, \alpha_2)}(z)$具有以下的三个零点，

$$\begin{aligned}
&\text{I: } \alpha_1 = \alpha_2 = 0 \text{ 时，零点分别为} \frac{1}{2}, \ \frac{\tau}{2}, \ \frac{1}{2} + \frac{\tau}{2}, \\
&\text{II: } \alpha_1 = 0, \alpha_2 = 1 \text{ 时，零点分别为 } 0, \ \frac{\tau}{2}, \ \frac{1}{2} + \frac{\tau}{2}, \\
&\text{III: } \alpha_1 = 1, \alpha_2 = 0 \text{ 时，零点分别为 } 0, \ \frac{1}{2}, \ \frac{1}{2} + \frac{\tau}{2}, \\
&\text{IV: } \alpha_1 = \alpha_2 = 1 \text{ 时，零点分别为 } 0, \ \frac{1}{2}, \ \frac{\tau}{2}.
\end{aligned} \tag{39}$$

当$\alpha_1 = \alpha_2 = 0$ 时，令

$$G_{(0,0)} = \frac{F_{(0,0)}(z)}{\sigma_{(0,1)}(z)\sigma_{(1,0)}(z)\sigma_{(1,1)}(z)}, \tag{40}$$

可知 $G_{(0,0)}$是复平面上的双周期全纯函数．利用刘维定理，得 $G_{(0,0)}$为复平面上的常数．则有

$$F_{(0,0)}(z) = G_{(0,0)}\sigma_{(0,1)}(z)\sigma_{(1,0)}(z)\sigma_{(1,1)}(z). \tag{41}$$

令 $z = \bar{a}_0 w - \xi$，可得

$$G_{(0,0)} = \frac{\sigma_0(2\bar{a}_0 w - 2\xi)\sigma_0(a_{10}w)\sigma_0(\bar{a}_1 w - \xi)}{\sigma_{(0,1)}(\bar{a}_0 w - \xi)\sigma_{(1,0)}(\bar{a}_0 w - \xi)\sigma_{(1,1)}(\bar{a}_0 w - \xi)}. \tag{42}$$

同理，有

$$F_{(0,1)}(z) = G_{(0,1)}\sigma_0(z)\sigma_{(1,0)}(z)\sigma_{(1,1)}(z), \tag{43}$$

$$F_{(1,0)}(z) = G_{(1,0)}\sigma_0(z)\sigma_{(0,1)}(z)\sigma_{(1,1)}(z), \tag{44}$$

$$F_{(1,1)}(z) = G_{(1,1)}\sigma_0(z)\sigma_{(0,1)}(z)\sigma_{(1,0)}(z). \tag{45}$$

其中

$$G_{(0,1)} = \frac{\sigma_0(2\bar{a}_0 w - \xi)\sigma_0(a_{10}w)\sigma_{(0,1)}(\bar{a}_1 w - \xi)}{\sigma_0(\bar{a}_0 w - \xi)\sigma_{(1,0)}(\bar{a}_0 w - \xi)\sigma_{(1,1)}(\bar{a}_0 w - \xi)}, \tag{46}$$

$$G_{(1,0)} = \frac{\sigma_0(2\bar{a}_0 w - \xi)\sigma_0(a_{10}w)\sigma_{(1,0)}(\bar{a}_1 w - \xi)}{\sigma_0(\bar{a}_0 w - \xi)\sigma_{(0,1)}(\bar{a}_0 w - \xi)\sigma_{(1,1)}(\bar{a}_0 w - \xi)}, \tag{47}$$

$$G_{(1,1)} = \frac{\sigma_0(2\bar{a}_0 w - \xi)\sigma_0(a_{10}w)\sigma_{(1,1)}(\bar{a}_1 w - \xi)}{\sigma_0(\bar{a}_0 w - \xi)\sigma_{(1,0)}(\bar{a}_0 w - \xi)\sigma_{(0,1)}(\bar{a}_0 w - \xi)}. \tag{48}$$

利用以上结果，有

$$K(a, z) = g(a, z)\left[I + C_1\frac{\sigma_0(z)}{\sigma_{(0,1)}(z)} I_{(0,1)} + C_2\frac{\sigma_0(z)}{\sigma_{(1,0)}(z)} I_{(1,0)} + C_3\frac{\sigma_0(z)}{\sigma_{(1,1)}(z)} I_{(1,1)}\right]. \tag{49}$$

其中

$$g(a,\ z)=\frac{1}{2}\left[\frac{\sigma_0(2\bar{a}_0w-2\xi)\sigma_0(\bar{a}_1w-\xi)\sigma_0\left(-w\delta-\frac{1}{2}\right)}{\sigma_{(0,\ 1)}(\bar{a}_0w-\xi)\sigma_{(1,\ 0)}(\bar{a}_0w-\xi)\sigma_{(1,\ 1)}(\bar{a}_0w-\xi)}\times\right.$$

$$\left.\frac{\sigma_{(0,\ 1)}(z)\sigma_{(1,\ 0)}(z)\sigma_{(1,\ 1)}(z)}{\sigma_0\left(-z-w\delta-\frac{1}{2}\right)\sigma_0(\bar{a}_0w-\xi-z)\sigma_0(\bar{a}_1w-\xi-z)}\right]f_a(z),\tag{50}$$

$$C_1=-\frac{\sigma_{(0,\ 1)}(\bar{a}_0w-\xi)\sigma_{(0,\ 1)}(\bar{a}_1w-\xi)\sigma_{(0,\ 1)}\left(-w\delta-\frac{1}{2}\right)}{\sigma_0(\bar{a}_0w-\xi)\sigma_0(\bar{a}_1w-\xi)\sigma_0\left(-w\delta-\frac{1}{2}\right)},\tag{51}$$

$$C_2=\frac{\sigma_{(1,\ 0)}(\bar{a}_0w-\xi)\sigma_{(1,\ 0)}(\bar{a}_1w-\xi)\sigma_{(1,\ 0)}\left(-w\delta-\frac{1}{2}\right)}{\sigma_0(\bar{a}_0w-\xi)\sigma_0(\bar{a}_1w-\xi)\sigma_0\left(-w\delta-\frac{1}{2}\right)},\tag{52}$$

$$C_3=-\frac{\sigma_{(1,\ 1)}(\bar{a}_0w-\xi)\sigma_{(1,\ 1)}(\bar{a}_1w-\xi)\sigma_{(1,\ 1)}\left(-w\delta-\frac{1}{2}\right)}{\sigma_0(\bar{a}_0w-\xi)\sigma_0(\bar{a}_1w-\xi)\sigma_0\left(-w\delta-\frac{1}{2}\right)},\tag{53}$$

I_α是 I 和 3 个泡利矩阵σ_1, σ_2, σ_3, 即 $I_{(0,\ 0)}=I$, $\sigma_1=I_{(1,\ 0)}$, $\mathrm{i}\sigma_2=I_{(1,\ 1)}$, $\sigma_3=I_{(0,\ 1)}$. 可以证明C_1, C_2, C_3是相互独立的. 这同侯伯宇教授等人[5]给出的形式是一致的.

5 结论

可以看到, 在 $K(a,\ z)$的表示式(33)中, 含有 $n+1$ 个彼此独立的参数$\bar{a}_0w$, $\bar{a}_1w$, $\cdots$, $\bar{a}_{n-1}w$, ξ. $K(a, z)$的存在只要求$\bar{a}_\mu$ 中的δ_μ是一些一般的复数, 因此$\bar{a}_\mu$ 是可任意选取的, 又ξ为一任意数, 故 $K(a, z)$中含有 $n+1$ 个任意参数. 得到了含有 $n+1$ 个参数的与 Z_n Belavin R矩阵相关的反射方程的解.

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附录 1

给反射方程(8)等式两边同时左乘$\bar{\phi}_{d,c}(z_1)\otimes\bar{\phi}_{e,d}(z_2)$和右乘$\phi_{b,c}(-z_1)\otimes\phi_{a,b}(-z_2)$，则

$$LHS=\bar{\phi}_{d,c}(z_1)\otimes\bar{\phi}_{e,d}(z_2)R_{12}(z_1-z_2)K_1(z_1)R_{21}(z_1+z_2)\times K_2(z_2)\phi_{b,c}(-z_1)\otimes\phi_{a,b}(-z_2).$$

利用面顶角对应关系(19)，得

$$LHS=\sum_s W\begin{bmatrix} e & d \\ s & c \end{bmatrix}(z_1-z_2)\bar{\phi}_{e,s}(z_1)\otimes\bar{\phi}_{s,c}(z_2)K_1(z_1)\times R_{21}(z_1+z_2)K_2(z_2)\phi_{b,c}(-z_1)\otimes\phi_{a,b}(-z_2).$$

利用面顶角对应关系(17)，得

$$LHS=\sum_{sf}\left(W\begin{bmatrix} e & d \\ s & c \end{bmatrix}(z_1-z_2)(\bar{\phi}_{e,s}(z_1)K(z_1)\phi_{f,s}(-z_1))\otimes 1\right)\times$$
$$\left(W\begin{bmatrix} f & s \\ b & c \end{bmatrix}(z_1+z_2)1\otimes(\bar{\phi}_{f,b}(z_2)K(z_2)\phi_{a,b}(-z_2))\right). \tag{54}$$

同理，利用面顶角对应关系(11)和(17)，得

$$RHS=\sum_{sf}\left(1\otimes(\bar{\phi}_{e,d}(z_2)K(z_2)\phi_{f,d}(-z_2))W\begin{bmatrix} f & d \\ s & c \end{bmatrix}(z_1+z_2)\right)\times$$
$$\left((\bar{\phi}_{f,s}(z_1)K(z_1)\phi_{a,s}(-z_1))\otimes 1W\begin{bmatrix} a & s \\ b & c \end{bmatrix}(z_1-z_2)\right). \tag{55}$$

令

$$K(s_e^f)(z)=\bar{\phi}_{e,s}(z)K(z)\phi_{f,s}(-z), \tag{56}$$

则由 $LHS=RHS$ 得，

$$\sum_{sf}W\begin{bmatrix} e & d \\ c & s \end{bmatrix}(z_1-z_2)K(s_e^f)(z_1)W\begin{bmatrix} f & s \\ b & c \end{bmatrix}(z_1+z_2)K(b_f^a)(z_2)=$$
$$\sum_{sf}K(d_e^f)(z_2)W\begin{bmatrix} f & d \\ s & c \end{bmatrix}(z_1+z_2)K(s_f^a)(z_1)W\begin{bmatrix} a & s \\ b & c \end{bmatrix}(z_1-z_2).$$

即为(20)式.

附录 2

给面型反射方程(20)式等式两边同时左乘$\phi_{d,c}(z_1)\otimes\phi_{e,d}(z_2)$和右乘$\bar{\phi}_{b,c}(-z_1)\otimes\bar{\phi}_{a,b}(-z_2)$并对 b,c,d 求和，得

$$LHS=\sum_{sf}\sum_{bcd}\phi_{d,c}(z_1)\otimes\phi_{e,d}(z_2)W\begin{bmatrix} e & d \\ c & s \end{bmatrix}(z_1-z_2)K(s_e^f)(z_1)\times$$

$$W\begin{bmatrix} f & s \\ b & c \end{bmatrix}(z_1+z_2)K(b_f^a)(z_2)\bar{\phi}_{b,\,c}(-z_1)\otimes\bar{\phi}_{a,\,b}(-z_2).$$

利用面顶角对应关系(11),得

$$LHS=\sum_{sf}\sum_{bc}R_{12}(z_1-z_2)(\phi_{e,\,s}(z_1)K(s_e^f)(z_1)\otimes 1)W\begin{bmatrix} f & s \\ b & c \end{bmatrix}(z_1+z_2)\times$$
$$\bar{\phi}_{b,\,c}(-z_1)\otimes\phi_{s,\,c}(z_2)(1\otimes K(b_f^a)(z_2)\bar{\phi}_{a,\,b}(-z_2)).$$

利用面顶角关系(16),得

$$LHS=\sum_f R_{12}(z_1-z_2)\left(\left(\sum_s\phi_{e,\,s}(z_1)K(s_e^f)(z_1)\bar{\phi}_{f,\,s}(-z_1)\right)\otimes 1\right)\times$$
$$R_{21}(z_1+z_2)\left(1\otimes\left(\sum_b\phi_{f,\,b}(z_2)K(b_f^a)(z_2)\bar{\phi}_{a,\,b}(-z_2)\right)\right).$$

代入(21)式,并令

$$K\left(\begin{smallmatrix} f \\ e \end{smallmatrix}\middle| z\right)=\sum_s\phi_{e,\,s}(z)K(s_e^f)(z)\bar{\phi}_{f,\,s}(-z), \tag{57}$$

则有:

$$LHS=R_{12}(z_1-z_2)K_1\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_1\right)R_{21}(z_1+z_2)K_2\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_2\right)\delta_{a,\,e}. \tag{58}$$

同理,利用面顶角对应关系(18)和(16),得

$$RHS=K_2\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_2\right)R_{12}(z_1+z_2)K_1\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_1\right)R_{21}(z_1-z_2)\delta_{a,\,e}. \tag{59}$$

由 $LHS=RHS$,得

$$R_{12}(z_1-z_2)K_1\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_1\right)R_{21}(z_1+z_2)K_2\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_2\right)=K_2\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_2\right)R_{12}(z_1+z_2)K_1\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z_1\right)R_{21}(z_1-z_2). \tag{60}$$

由(14)可知,$K\left(\begin{smallmatrix} a \\ a \end{smallmatrix}\middle| z\right)$仅当 $s=a+\hat{\mu}$ 时才不为零,令

$$K(a,\ z)=\sum_\mu\phi_{a,\,a+\hat{\mu}}(z)K(a+\hat{\mu}_a^a)(z)\bar{\phi}_{a,\,a+\hat{\mu}}(-z), \tag{61}$$

则(60)式变为

$$R_{12}(z_1-z_2)K_1(a,\ z_1)R_{21}(z_1+z_2)K_2(a,\ z_2)=K_2(a,\ z_2)R_{12}(z_1+z_2)K_1(a,\ z)R_{21}(z_1-z_2),$$

即为(22)式.

Multi-Parameter Solution to the Reflection Equation of Z_n Belavin Model

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Abstract By using the diagonal solution of reflection equation for $A_{n-1}^{(1)}$ IRF model, We obtain the solution with $n+1$ parametcrs to the reflection equation of Z_n Belavin model. The result we get when $n=2$ coincides formally with that given by Hou et al.

Key words $A_{n-1}^{(1)}$ IRF model, Z_n Belavin model, reflection equation, multi-parameter solution

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