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含 Chern-Simons 项的标量电动力学的 BFV 量子化

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摘要 应用 BFV 路径积分量子化方案,给出含 Chern-Simons 项的标量电动力学的量子化,得到了量子系统守恒的能量、动量和角动量,指出在量子水平上系统具有分数自旋性质.

关键词 BFV 量子化 Chern-Simons 理论 量子守恒量

1 引言

对奇异拉氏量描述的系统,Dirac 正则量子化方法用于非 Abel 规范理论时,遇到了困难;另外,正则量子化方法是算符形式的,在具体运用时就要考虑算符的次序问题. 而路径积分量子化中出现的是 c 数,不是 q 数,易于计算. 目前,约束系统用路径积分量子化较方便.

Faddev 和 Popov (F-P) 应用路径积分量子化方法,成功地实现了杨 -Mills 理论的量子化. 但 F-P 量子化方法人为的丢掉无穷大积分,是不严格的. Faddeev 在 Dirac 约束理论基础上,给出了含第一类约束系统的路径积分量子化,Senjanovic 解决了同时含第一类约束和第二类约束系统的路径积分量子化[1],称为 F-S (Faddeev-Senjanovic) 路径积分量子化. 不论是 F-P 还是 F-S 路径积分量子化,用于非 Abel 规范场时,要引进鬼场保证么正性.

Becchi, Rouet 和 Stora 发现有效拉氏量有一种新的不变性,即 BRS 不变性. Batalin, Fradkin 和Vilkovsky 在 BRS 对称变换基础上,利用约束 Hamilton 系统的经典结构,即存在结构函数,建立了协变的 BFV 量子化理论[2,3]. BFV 量子化方案是有严格论证的量子化方法;给出了确定的积分测度的表述;可以较方便地与其他量子化形式转换,在 BFV 方法中,对一些重要物理系统,选取不同形式的规范固定费米子 ψ,就可以得到 F-S 或 F-P 量子化的结果. BFV 量子化方法提供了一个诱人的描述约束 Hamilton 系统的方法,它不约化相空间,而是通过增添 Grassmann 变量扩展相空间,扩展相空间中的 BFV 泛函积分相应于

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一个没有约束的 Hamilton 系统.

BFV 量子化方法为 Hamilton 形式的 BRST(Becchi-Rouet-Stora-Tyutin)量子化方法. 建立在 Lagrangian 形式的 BRST 量子化方法，是BV(Batalin-Vilkovsky)量子化[4]. BRST 对称和鬼场在规范理论的协变量子化中有重要意义. 因此，需要一个理论在开始就自动包含鬼场和 BRST 对称性，BV 量子化方法具有上述特点. 但 BV 量子化方法中引入反括号–反场；解主(BV)方程相当麻烦，得到的解也很复杂，没有一个简单的方法来解复杂系统的主(BV)方程；反括号使计算变得不必要复杂；不能自动给出泛函积分测度. 这些困难联系着么正性，重整化，量子规范不变性和反常.

文 [5]中研究了含 Chern-Simons 项的量子标量电动力学，讨论了系统的质量激发谱. 这里应用 BFV 路径积分量子化方案[2,3]，讨论了含 Chern-Simons 项的标量电动力学的量子化，得到和 F-S 量子化方法一致的结果. 本文进一步导出了量子情形下系统守恒的能量、动量和角动量. 文 [6]和 [7]中用经典 Noether 定理计算了系统的角动量，说明了它的分数自旋性质，这里指出在量子水平上系统的分数自旋性质仍保持.

2　Chern-Simons 理论的 BFV 路径积分量子化

含 Chern-Simons 项标量电动力学的拉氏量密度为[5]

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{k}{4}\epsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda + (\mathrm{D}_\mu\varphi)^*(\mathrm{D}_\mu\varphi) + m^2\varphi^*\varphi \ , \tag{1}$$

其中 $\mathrm{D}_\mu = \partial_\mu - \mathrm{i}A_\mu$，　$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

各场量相应的正则共轭动量分别为

$$\pi^i = \frac{\partial\mathscr{L}}{\partial\dot{A}_i} = F^{i0} + \frac{k}{2}\epsilon^{ij}A_j \ ,$$

$$\pi^0 = \frac{\partial\mathscr{L}}{\partial\dot{A}_0} = 0 \ , \tag{2}$$

$$\pi_\varphi = \frac{\partial\mathscr{L}}{\partial\dot{\varphi}} = (\mathrm{D}_0\varphi)^* \ ,$$

$$\pi_\varphi^* = \frac{\partial\mathscr{L}}{\partial\dot{\varphi}^*} = \mathrm{D}_0\varphi \ . \tag{3}$$

系统有初级约束

$$\phi^1 = \pi^0 \approx 0 \ , \tag{4}$$

系统的正则 Hamilton 密度 $\mathscr{H}_c$

$$\mathscr{H}_c = \pi^\mu\dot{A}_\mu + \pi_\varphi\dot{\varphi} + \pi^*_\varphi\dot{\varphi}^* - \mathscr{L} = \mathscr{H}_0 + A_0[\mathrm{i}(\pi_\varphi\varphi - \varphi^*\pi^*_\varphi) - (\partial_i\pi^i + \frac{k}{4}\epsilon^{ij}F_{ij})] \ , \tag{5}$$

其中

$$\mathscr{H}_0 = \pi_\varphi\pi^*_\varphi - (\mathrm{D}_i\varphi)^*(\mathrm{D}^i\varphi) - m^2\varphi^*\varphi - \frac{1}{2}\pi^i\pi_i + \frac{k}{2}\epsilon^{ij}\pi_i A_j - \frac{k^2}{8}A^iA_i + \frac{1}{4}F_{ij}F^{ij} \ . \tag{6}$$

初级约束的自洽性条件给出次级约束

$$\phi^2 = \mathrm{i}(\pi_\varphi \varphi - \varphi^* \pi_\varphi^*) - (\partial_i \pi^i + \frac{k}{4} \epsilon^{ij} F_{ij}) \approx 0. \tag{7}$$

次级约束的自洽性条件不给出新的次级约束．系统的全部约束

$$\phi^1 = \pi^0 \approx 0 , \tag{8}$$

$$\phi^2 = \mathrm{i}(\pi_\varphi \varphi - \varphi^* \pi_\varphi^*) - (\partial_i \pi^i + \frac{k}{4} \epsilon^{ij} F_{ij}) \approx 0 \tag{9}$$

为第一类约束．从(5)式可以看出，A_0为约束ϕ^2的拉氏乘子．系统的相空间为 $Z_A(A_i, \varphi, \varphi^*; \pi^i, \pi_\varphi, \pi_\varphi^*)$．把拉氏乘子$\lambda = A_0$看作动力学变量后的相空间记为 $Z_\Delta(Z_A, \lambda = A_0, \pi = \pi^0)$，这样就相当于系统只有一个约束$\phi = \phi^2$，而$\pi^0$是由于把拉氏乘子$\lambda = A_0$看作动力学变量而引入的约束$\pi = \pi^0 \approx 0$．将系统的约束重新记为 G_a:

$$G_1 = \pi = \pi^0 \approx 0 , \tag{10}$$

$$G_2 = \phi = \mathrm{i}(\pi_\varphi \varphi - \varphi^* \pi_\varphi^*) - (\partial_i \pi^i + \frac{k}{4} \epsilon^{ij} F_{ij}) \approx 0 . \tag{11}$$

对每一个约束 G_a，引入费米鬼场η^a及其正则共轭量 $\mathscr{P}_a$，按约束分类 $G_a(G_1, G_2) = (\pi, \phi)$，鬼场分为两部分

$$\begin{aligned} \eta &= (-\mathrm{i}\mathscr{P}, C) , \\ \mathscr{P} &= (\mathrm{i}\bar{C} , \bar{\mathscr{P}}) . \end{aligned} \tag{12}$$

不为零的鬼场的 Poisson 括号

$$\begin{aligned} \{\mathscr{P}(x), \ \bar{C}(y)\} &= -\delta(x-y), \\ \{\bar{\mathscr{P}}(x), \ C(y)\} &= -\delta(x-y) \end{aligned} \tag{13}$$

扩展相空间为 $Z(Z_\Delta, \eta, \mathscr{P})$．在扩展相空间中，存在 BRS 规范变换生成元．对 Abel 理论，BRS 变换生成元$\Omega = \eta^a G_a$[3]，即

$$\Omega = \int \mathrm{d}^2 x (C\phi - \mathrm{i}\mathscr{P}\pi) = \int \mathrm{d}^2 x \{C[\mathrm{i}(\pi_\varphi \varphi - \varphi^* \pi_\varphi^*) - (\partial_i \pi^i + \frac{k}{4} \epsilon^{ij} F_{ij})] - \mathrm{i}\mathscr{P}\pi^0] . \tag{14}$$

BRS 变换为

$$\begin{aligned}
&\delta A_i = \{A_i, \Omega\} = \partial_i C, && \delta A_0 = \{A_0, \Omega\} = -\mathrm{i}\mathscr{P}, \\
&\delta \pi^i = \{\pi^i, \Omega\} = 0, && \delta \pi^0 = \{\pi^0, \Omega\} = 0, \\
&\delta\varphi = \{\varphi, \Omega\} = \mathrm{i}C\varphi, && \delta\varphi^* = \{\varphi^*, \Omega\} = -\mathrm{i}C\varphi^*, \\
&\delta\pi_\varphi = \{\pi_\varphi, \Omega\} = -\mathrm{i}C\pi_\varphi , && \delta\pi_\varphi^* = \{\pi_\varphi^*, \Omega\} = \mathrm{i}C\pi_\varphi^* , \\
&\delta C = \{C, \Omega\} = 0, && \delta \mathscr{P} = \{\mathscr{P}, \Omega\} = 0, \\
&\delta \bar{\mathscr{P}} = \{\bar{\mathscr{P}}, \Omega\} = \mathrm{i}(\pi_\varphi \varphi - \varphi^* \pi_\varphi^*) - (\partial_i \pi^i + \frac{k}{4} \epsilon^{ij} F_{ij}), && \delta \bar{C} = \{\bar{C}, \Omega\} = \mathrm{i}\pi^0 .
\end{aligned} \tag{15}$$

Hamilton 量 H_0是物理系统可观测量，其密度为 $\mathscr{H}_0$．约束 G_a和 Hamilton 量密度 $\mathscr{H}_0$之

间的 Poisson 括号

$$\{\mathscr{H}_0(x), G_a(x')\} = V_a^b G_b = 0 \quad , \tag{16}$$

$$\{G_a(x), G_b(x')\} = C_{ab}^{\,c} G_c = 0 \quad . \tag{17}$$

扩展相空间 BRS 变换不变函数 H 为[3]

$$H = \int d^2x\,(\mathscr{H}_0 + \eta^a V_a^b \mathscr{P}_b + \cdots) = \int d^2x \mathscr{H}_0 \ . \tag{18}$$

BFV 量子化有效 Hamilton 量为[3]

$$H_{\rm eff} = H - \{\psi, \Omega\} = \int d^2x \mathscr{H}_{\rm eff} \ , \tag{19}$$

相应于规范固定费米子 $\psi = \int d^3x(i\bar{C}_a\chi^a + \bar{\mathscr{P}}_a\lambda^a)$，有

$$\psi = \int d^2x(i\bar{C}\chi + \bar{\mathscr{P}}\lambda) \ . \tag{20}$$

在(20)式中χ选为

$$\chi = \partial_i A^i \ , \tag{21}$$

直接计算 $\{\psi, \Omega\}$，得

$$\{\psi, \Omega\} = \int d^2x(-\lambda\phi - \pi\chi + i\bar{C}\{\chi, \phi\}C - i\bar{\mathscr{P}}\mathscr{P}) =$$

$$\int d^2x[-A_0[i(\pi_\varphi\varphi - \varphi^*\pi_\varphi^*) - (\partial_i\pi^i + \frac{k}{4}\epsilon^{ij}F_{ij})] - \pi^0\partial_i A^i - i\bar{C}\partial_i\partial^i C - i\bar{\mathscr{P}}\mathscr{P}) \ . \tag{22}$$

有效作用量 $S_{\rm eff}$ 为

$$S_{\rm eff} = \int d^3x(\pi^k\dot{A}_k + \pi_\varphi\dot{\varphi} + \pi_\varphi^*\dot{\varphi}^* + \pi\dot{\lambda} + \dot{C}\bar{\mathscr{P}} + \dot{\bar{\mathscr{P}}}\bar{C} - \mathscr{H}_{\rm eff}) \ , \tag{23}$$

由(18,19)式和(22,23)式，得

$$S_{\rm eff} = \int d^3x\Big\{(\pi^\mu\dot{A}_\mu + \pi_\varphi\dot{\varphi} + \pi_\varphi^*\dot{\varphi}^* + \dot{C}\bar{\mathscr{P}} - \dot{\bar{C}}\mathscr{P}) -$$

$$\left[\pi_\varphi\pi_\varphi^* - (D_i\varphi)^*(D^i\varphi) - m^2\varphi^*\varphi - \frac{1}{2}\pi^i\pi_i + \frac{k}{2}\epsilon^{ij}\pi_i A_j - \frac{k^2}{8}A^iA_i + \frac{1}{4}F_{ij}F^{ij})\right] +$$

$$\{-A_0[i(\pi_\varphi\varphi - \varphi^*\pi_\varphi^*) - (\partial_i\pi^i + \frac{k}{4}\epsilon^{ij}F_{ij})] - \pi^0\partial_i A^i - i\bar{C}\partial_i\partial^i C - i\bar{\mathscr{P}}\mathscr{P}\}\Big\} \ . \tag{24}$$

含 Chern-Simons 项的标量电动力学的 BFV 路径积分量子化跃迁振幅为[3]

$$Z[0] = \int \mathscr{D}A_\mu\mathscr{D}\pi^\mu\mathscr{D}\varphi\mathscr{D}\pi_\varphi\mathscr{D}\varphi^*\mathscr{D}\pi_\varphi^*\mathscr{D}C\mathscr{D}\bar{\mathscr{P}}\mathscr{D}\bar{C}\mathscr{D}\mathscr{P}\exp iS_{\rm eff} \ . \tag{25}$$

(24)式也可以写为

$$S_{\rm eff} = \int d^3x(\pi^\mu\dot{A}_\mu + \pi_\varphi\dot{\varphi} + \pi_\varphi^*\dot{\varphi}^* + \dot{C}\bar{\mathscr{P}} - \dot{\bar{C}}\mathscr{P} -$$

$$i\bar{\mathscr{P}}\mathscr{P} - i\bar{C}\partial_i\partial^i C - \mathscr{H}_0 - A_0\phi - \pi^0\partial_i A^i) \ . \tag{26}$$

把(26)式代入(25)中,对鬼场 $C, \bar{\mathscr{P}}, \bar{C}, \mathscr{P}$,拉氏乘子 A_0及其动量π_0积分,得

$$Z[0] = \int \mathscr{D} A_k \mathscr{D}\pi^k \mathscr{D}\varphi \mathscr{D}\pi_\varphi \mathscr{D}\varphi^* \mathscr{D}\pi_\varphi^* \delta(\phi)\delta(\partial_i A^i)\det\left|\{\phi, \partial_i A^i\}\right| \times$$

$$\exp i\left\{\int d^3x(\pi^k \dot{A}_k + \pi_\varphi\dot{\varphi} + \pi_\varphi^* \dot{\varphi}^* - \mathscr{H}_0)\right\} =$$

$$\int \mathscr{D} A_k \mathscr{D}\pi^k \mathscr{D}\varphi \mathscr{D}\pi_\varphi \mathscr{D}\varphi^* \mathscr{D}\pi_\varphi^* \delta(\phi)\delta(\partial_i A^i)\exp i\left\{\int d^3x(\pi^k \dot{A}_k + \pi_\varphi\dot{\varphi} + \pi_\varphi^* \dot{\varphi}^* - \mathscr{H}_0)\right\} . \quad (27)$$

这和用 Faddeev-Senjanovic 路径积分量子化计算的结果相同. 其中在上述积分过程中,利用了如下公式[8]

$$\int \mathscr{D}\bar{\mathscr{P}}\mathscr{D}\mathscr{P}\mathscr{D}\bar{C}\mathscr{D}C\exp\left[i\int_{t_1}^{t_2} dt(-\mathscr{P}\dot{\bar{C}} + \dot{C}\bar{\mathscr{P}} - i\bar{\mathscr{P}}\mathscr{P})\right] =$$

$$\int \mathscr{D}\bar{C}\mathscr{D}C\exp\left[\int_{t_1}^{t_2} dt\,\dot{\bar{C}}\dot{C}\right] = -(t_2 - t_1) . \quad (28)$$

上述量子系统具有时空平移和空间转动不变性,且相应场量变换的 Jacobi 行列式为1,在约束超曲面上量子水平下的能量、动量和角动量守恒. 守恒荷分别为[9]:

守恒能量

$$E = \int d^2x\left[-\frac{1}{2}\pi^i\pi_i + \frac{k}{2}\epsilon^{ij}\pi_i A_j - \frac{k^2}{8}A^iA_i + \frac{1}{4}F_{ij}F^{ij} + \pi_\varphi\pi_\varphi^* - (D_i\varphi)^*(D^i\varphi) - m^2\varphi^*\varphi\right] , \quad (29)$$

守恒动量

$$P = -\int d^2x(\pi_k\nabla A^k + \pi_\varphi\nabla\varphi + \pi_\varphi^*\nabla\varphi^*) , \quad (30)$$

守恒角动量

$$Q^{12} = \int d^2x\left[\pi^i\left(S_{ij}^{12}A^j + x^1\frac{\partial A^i}{\partial x^2} - x^2\frac{\partial A^i}{\partial x^1}\right) + \pi_\varphi\left(x^1\frac{\partial\varphi}{\partial x^2} - x^2\frac{\partial\varphi}{\partial x^1}\right) + \pi_\varphi^*\left(x^1\frac{\partial\varphi^*}{\partial x^2} - x^2\frac{\partial\varphi^*}{\partial x^1}\right)\right] . \quad (31)$$

上述结果与经典 Neother 定理导致的结果是一致的. 量子水平的守恒角动量(31)式包含场的轨道角动量和矢量场的自旋角动量,由于存在涡旋,其边界项给出"分数自旋"项[6],文[6]和[7]中(无 Maxmell 项)在经典水平下讨论了此问题,由上述结果可知,在量子水平上系统具有分数自旋性质.

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Quantization of a Scalar QED With Chern-Simons Term in the Batalin-Fradkin-Vilkovisky Scheme

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Abstract　In this paper we use the Batalin-Fradkin-Vilkovisky (BFV) formalism to study the quantization of a scalar QED with Chern-Simons term. The conserved laws of energy, momentum and angular momentum at the quantal level of the system are obtained. The property of fractional spin of the system at the quantal level is pointed out.

Key words BFV approach, Chern-Simons theory, quantal conserved charges

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