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Description of Superdeformed Bands in ^{148}Gd in q-deformed Moment of Inertia*

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Abstract The $E2$ transition γ-ray energies and the dynamical moment of inertia of the six superdeformed (SD) bands in ^{148}Gd have been investigated by use of the rigid-rotor with the q-deformed moment of inertia. The calculated results coincide with experimental data. It indicates that the approach of rigid-rotor with the q-deformed moment of inertia may be powerful in description for both the yrast SD band and the excited bands simultaneously.

Key words superdeformed band, q-deformed moment of inertia, $E2$ transition γ-ray energy, dynamical moment of inertia

Soon after the discovery of the first discrete superdeformed (SD) rotational band in the nucleus ^{152}Dy[1] in 1986, the yrast SD band in ^{148}Gd was observed[2]. Afterwards, all six SD excited bands have been confirmed[3]. With six known SD bands, the nucleus ^{148}Gd appears an excellent case to extract more physics in the second minimum of the potential energy surface. Many microscopic investigations on the SD states in ^{148}Gd have been accomplished and the quasiparticle configurations have then been assigned[4,5-7]. However, in both non-relativistic approach[6,7,8] and the cranked relativistic mean field[7] theory, the dynamical moment of inertia of SD states have not yet been described well. On phenomenological side, only the yrast band in ^{148}Gd has been discussed[8]. Therefore it is interesting to describe the six bands together in a phenomenological model. Recently this work has been done with a four-parameter formula in the spirit of *sdg* IBM[9]. In this paper, a three-parameter formula, based on the rigid-rotor with the q-deformed moment of inertia (MoI), is proposed to describe the all six SD bands.

Unlike normal deformed nuclei, for SD nuclei three simple consequences may be identified: (1) the nuclear deformation can vary during the rotation, a phenomenon known as the stretching effect[10]; (2) the Coriolis anti-pairing effect[11], which is caused by the weakening pairing correlation across many orbitals due to the Coriolis force; and (3) rotation alignment[12], which emphasizes an alignment along the rotational axis of a pair in an orbital particularly susceptible to the Coriolis effect. Generally, all of these effects may be expected for SD nuclei, but in special cases one or two of them may be considered dominantly.

Being less clear situation of the SD nuclei on both microscopic and macroscopic view, we may consider, on phenomenological side, that the SD bands are dominated by the nuclear deformation, the stretching effect, and many-body statistics effect, and all these effects lead to a variation in MoI. As an assumption, the MoI of the SD nuclei in I state may be formulated by

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$$J = J_0(1 + B[I]_q[I+1]_q), \tag{1}$$

and in rotational framework, the rotational energy in I state can be expressed as

$$E(I) = \frac{\hbar^2}{2J(I)} I(I+1) + E_0, \tag{2}$$

where J_0 is the MoI of band-head; B is a real parameter; and $[X]_q$ is q-number defined as[13-15]

$$[X]_q = \frac{q^X - q^{-X}}{q - q^{-1}}. \tag{3}$$

In the case of q being a phase $q = e^{i\tau}$ (with a real τ), Eq. (2) can be written in details as

$$E(I) = \frac{\hbar^2}{2J_0} I(I+1) \Big/ \left[1 + B\frac{\sin(\tau I)\sin(\tau I + \tau)}{\sin\tau \sin\tau}\right] + E_0. \tag{4}$$

The behavior of Eq. (4) is clear: the parameter τ or q-number $[X]_q$ is related to the many-body statistics effect; if $\tau = 0$ or q-number $[X]_q = X$, Eq. (4) is just same as the modified rotational term in Ref. [16], where $J_0 B$ is regarded as Arima coefficient related to the stretching effect[10]; if $B = 0$ Eq. (4) becomes the rigid-rotor formula. For a sphere shape contributing nothing to the rotational band, J_0 is connected with the shape deformation and the orientation of rotational axis.

Generally the parameter τ is small in practical calculations ($\tau \sim 10^{-2}$), and Eq. (1) can be expanded as the series in power of $I(I+1)$[17]

$$J(I) = J_0\{1 + B[I(I+1) - \tau^2 I^2(I+1)^2/6\cdots\cdots]\}. \tag{5}$$

Extending the idea suggested in Ref. [11], it is obvious that when $\tau = 0$, the MoI has a driving (positive B) or restraining (negative B) effect which leads to the dynamical MoI changing monotonously. Because of B and $B\tau^2$ with opposite symbols, both driving effect and restraining effect (anti-pairing effect and pairing effect) are taken into account, and the competition between them determines the changing characteristic of the dynamical MoI. If the driving effect is stronger, $J^{(2)}$ increase with $\hbar\omega$. If the restraining effect is stronger, $J^{(2)}$ decrease with $\hbar\omega$. When these two effects are in balance with each other, an extreme value appears. Then a turnover emerges. From the above discussion, we have found that the SD bands may be governed by the rigid-rotor with the q-deformed MoI.

With Eq. (4) the transition energies $E_\gamma(I) = E(I+2) - E(I)$ of the SD bands are obtained. After a nonlinear least fitting to the experimentally observed E_γ energies, the spin of band-head I_0 is consequently determined. The fitting parameters J_0, B and τ are listed in Table 1 and the calculated results compared with the experimental data of six SD bands in ^{148}Gd are listed in Table 2, where σ is the root-mean-square (rms) deviation definde as

$$\sigma = \{\frac{1}{N}\sum_i [E_{exp}(i) - E_{cal}(i)]^2\}^{1/2} \tag{6}$$

In details of calculation, the agreement between the calculated and observed transition energies depends sensitively on the prescribed level spins. When a correct I_0 value is assigned, the calculated energies coincide with the experimental data incredibly well. However, if I_0 is shifted away from the assigned one, even merely by ± 1, the rms will increase rapidly. Therefore, the spin value of the band-head I_0, hence all the spin values of the yrast SD bands, can be determined unambiguously

Table 1. The fitting parameters, the assigned spin and the fitting quality rms. The spin I_0 determined by $E(I_0+2)\to E(I_0)$.

	σ/keV	$(\hbar^2/2J_0)$/keV	$B(10^{-5})$	$\tau(10^{-2})$	I_0
^{148}Gd (1)	0.417	4.808	−4.07	1.476	32
^{148}Gd (2)	1.131	6.409	−2.50	4.971	29
^{148}Gd (3)	0.660	5.878	0.414	2.895	35
^{148}Gd (4)	0.546	3.459	−5.25	1.347	50
^{148}Gd (5)	0.359	4.430	−3.51	1.273	43
^{148}Gd (6)	0.339	4.285	−3.94	1.350	42

Table 2. Calculated $E2$ γ-ray energies of six SD bands in ^{148}Gd and comparison with experiment. The experimental data a and b are taken from Refs. [6] and [3] respectively.

Spin	Band 1 Exp^a	Cal	Spin	Band 2 Exp^a	Cal	Spin	Band 3 Exp^b	Cal
32	700.2	699.7	29	790.2	790.5	35	853.8	853.1
34	748.6	748.1	31	839.3	840.9	37	900.9	899.7
36	797.7	797.5	33	890.1	891.0	39	945.4	946.3
38	847.8	848.0	35	941.3	940.6	41	992.4	993.0
40	899.1	899.4	37	990.8	989.7	43	1038.7	1039.8
42	951.5	951.9	39	1040.0	1038.5	45	1086.5	1086.6
44	1004.8	1005.3	41	1088.9	1087.2	47	1133.8	1133.6
46	1059.5	1059.8	43	1136.0	1135.8	49	1180.4	1180.7
48	1114.9	1115.1	45	1183.1	1184.6	51	1228.7	1227.9
50	1171.4	1171.3	47	1233.3	1233.7	53	1275.5	1275.2
52	1228.4	1228.3	49	1281.7	1283.4	55	1323.2	1322.6
54	1286.4	1286.0	51	1333.1	1333.7	57	1370.7	1370.2
56	1344.4	1344.3	53	1385.7	1385.1	59	1417.1	1417.9
58	1404.0	1403.1	55	1438.0	1437.3	61	1465.6	1465.7
60	1462.7	1462.3	57	1492.0	1490.6	63	1513.0	1513.6
62	1521.0	1521.7	59	1544.0	1545.0	65	1562.0	1561.6
64	1581.2	1581.3						
66	1640.4	1640.7						
68	1700.3	1700.1						

Spin	Band 4 Exp^a	Cal	Spin	Band 5 Exp^a	Cal	Spin	Band 6 Exp^b	Cal
50	899.0	898.6	43	891.1	890.7	42	849.44	848.81
52	946.7	947.2	45	939.6	939.6	44	897.40	897.00
54	997.0	996.9	47	988.6	989.6	46	945.86	946.12

(Continued)

Spin	Band 4 Exp[a]	Cal	Spin	Band 5 Exp[a]	Cal	Spin	Band 6 Exp[b]	Cal
56	1047.8	1047.7	49	1041.0	1045.5	48	996.08	996.28
58	1099.0	1099.6	51	1092.7	1092.4	50	1046.82	1047.41
60	1152.4	1152.5	53	1145.3	1145.2	52	1099.39	1099.48
62	1206.0	1206.4	55	1199.0	1199.0	54	1152.20	1152.45
64	1260.7	1261.0	57	1253.4	1253.6	56	1206.76	1206.26
66	1317.3	1316.3	59	1309.2	1309.0	58	1261.00	1260.86
68	1372.7	1372.2	61	1365.1	1365.2	60	1316.57	1316.17
70	1429.0	1428.4	63	1422.0	1422.2	62	1372.10	1372.10
72	1485.0	1484.7	65	1480.0	1479.8	64	1428.55	1428.54
74	1540.0	1541.0	67	1538.0	1537.9	66	1485.15	1485.39
						68	1542.40	1542.53

As I_0 and the fitting parameters determined, the dynamical MoI $J^{(2)} = 4\hbar^2 / [E_\gamma(I+2) - E_\gamma(I)]$ and the rotational frequency $\hbar\omega = [E_\gamma(I+2) - E_\gamma(I)]/4$ can be also obtained. The calculated results and comparison with experimental data are illustrated in Figs. 1 - 6. It is shown from the figures that the smooth decrease of the dynamical MoI with rotational frequency in bands 1, 4, 5 and 6 is reproduced well. Furthermore, the platform or even turnover in band 1 is appeared.

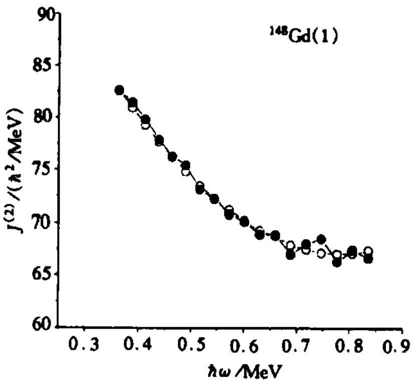

Fig. 1. The calculated results of the dynamical moments of inertia with the rotational frequency of the SD band in ^{148}Gd (1).
The experimental data denoted by —•—, the calculated results, by —∘—.

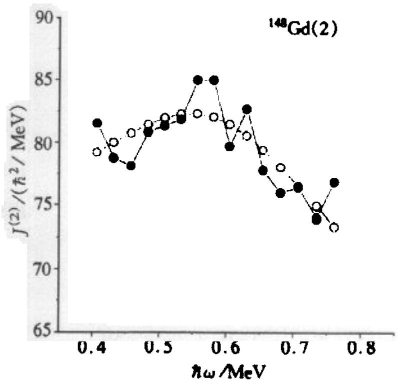

Fig. 2. Same as Fig. 1., but for ^{148}Gd (2).

In conclusion, by a rigid-rotor with the q-deformed MoI all six SD bands in ^{148}Gd have been described quantitatively well. It manifests that the fixed deformation, stretching effect and many-body statistics effect are possible sources of the collective rotational bands. Especially, the parameter τ plays a crucial role in governing the turnover of dynamical MoI against the rotational frequency.

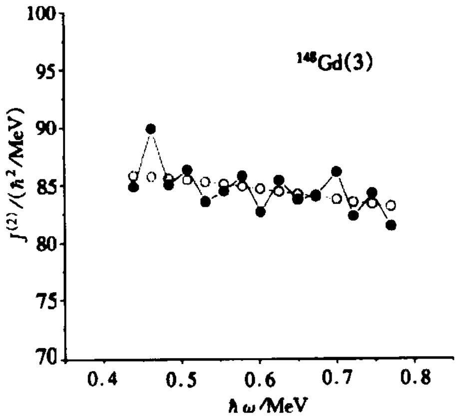

Fig. 3. Same as Fig. 1., but for ^{148}Gd (3).

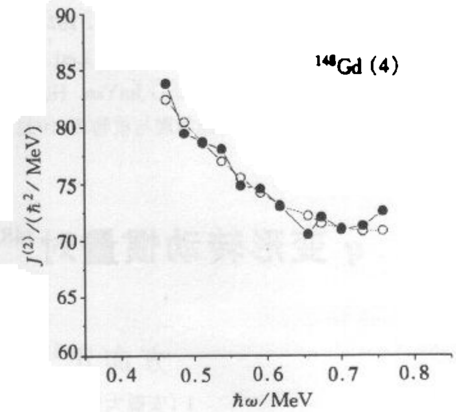

Fig. 4. Same as Fig. 1., but for ^{148}Gd (4).

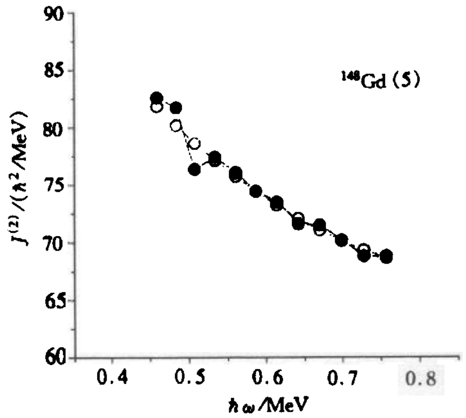

Fig. 5. Same as Fig. 1., but for ^{148}Gd (5).

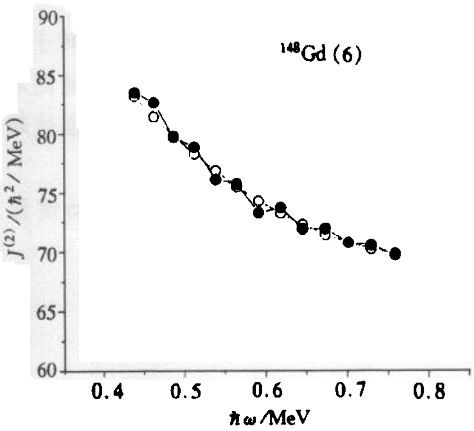

Fig. 6. Same as Fig. 1., but for ^{148}Gd (6).

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q 变形转动惯量对 ^{148}Gd 核超形变带的描述*

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摘要 利用 q 变形转动惯量的转子模型,系统地计算了 ^{148}Gd 核 6 个超形变带的 $E2\gamma$ 跃迁能谱以及相应的动力学转动惯量随转动频率的变化关系。计算结果表明,q 变形转动惯量的转子模型不仅能较精确地描述超形变晕带,而且可以描述超形变激发带。

关键词 超形变带 q 变形转动惯量 $E2\gamma$ 跃迁能量 动力学转动惯量

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