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Study on Deformation and Shape Coexistence for ^{188}Pb *

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Abstract The deformation and shape coexistence in ^{188}Pb have been investigated in terms of the Projected Shell Model. Comparing the experimental data with the calculated results, it is shown that three shape configurations of sphere ($Z=82$ shell closure), oblate (two particle-two hole in proton $h_{9/2}$ orbital) and prolate (multi-particle-hole) coexist each other in the low-lying excited states and the prolate band exhibits a mixture between two kinds of multi-particle-hole configurations, which means that the neutron $i_{13/2}$ alignment happens gradually in this case. The mixing is discussed and the mixing coefficients are given. The oblate band structure is predicted and the 2^+ prolate state is estimated to be in the energy range of 804—880keV.

Key words deformation, shape coexistence, low-lying excited state, projected shell model

The shape coexistence has been observed for the nuclei in the vicinity of closed shells and ascribed to the particle-hole excitation at the intruder states in the core[1]. The occurrence of intruder state in closed shell causing shape coexistence at low spins is well established experimentally. A particular interest has been paid to the neutron-deficient even-even lead isotopes due to the variety of shapes[2]. They are spherical in their ground states with the $Z=82$ closed shell. However deformed configurations show up at excited states. The deformation and shape coexistence for ^{188}Pb have been observed in the low-lying excited states in three experiments: (1) A prolate rotational-like sequence in ^{188}Pb was observed by J. Heese et al. in a prompt γ-ray spectrum populated in the ^{156}Gd (^{36}Ar, 4n) reaction at VICKSI accelerator, Germany in 1993[3]. The prolate structure assignment was not made definitely because of its relatively high moment of inertia with respect to a regular band and the absence of the band head of a 0^+ excited state. (2) The oblate 0^+ state in ^{188}Pb was identified by N. Bijnens et al. at (571 ± 31)keV based on the observation of a fine structure in the α-decay of ^{192}Po with a half-life of (33.2 ± 1.4)ms, produced in the ^{160}Dy (^{36}Ar, 4n) reaction at ATLAS accelerator U. S. A. in 1996[4]. (3) Furthermore, the oblate state was confirmed being at (568 ± 4) keV by G. A. Allatt et al., through α-particle-conversion electron coincidences in the same reaction at JYFL, Finland in 1998. And the prolate 0^+ state was also observed at (767 ± 12)keV within the same spectrum[5]. Collecting these experimental data, it is suggested that the ^{188}Pb may be a candidate nucleus for three shape coexistence.

There are two problems which challenge to our knowledge: one is the reason why the three different shape configurations could coexist each other in such nucleus; the other one is how to understand the structure of the prolate band. This letter reports the study on deformation and shape coexistence in ^{188}Pb through comparing the experimental data with the calculated results.

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The calculation was carried out in a framework of shell model, exploring in a fully quantum mechanical way with the angular momentum projection method, so called Projected Shell Model (PSM)[6]. The PSM uses the Nilsson + BCS representation as the deformed quasi-particle basis which relate to the excitation of pairs of particlehole at the intruder states. The set of multi-quasi-particle states generated by the deformed (Nilsson + BCS) mean field concerning with even-even nuclei shell model configuration space is

$$|\Phi_k\rangle = \{ |0\rangle,\ a^+_{\nu1}a^+_{\nu2}|0\rangle,\ a^+_{\pi1}a^+_{\pi2}|0\rangle,\ a^+_{\nu1}a^+_{\nu2}a^+_{\pi1}a^+_{\pi2}|0\rangle \}\ , \qquad (1)$$

where a^+'s are the quasi-particle creation operators, ν's(π's) denote the neutron(proton) Nilsson quantum number which run over the properly selected (low-lying) orbits, that will be explained later. $|0\rangle$ is the Nilsson + BCS vacuum or 0-quasiparticle, i.e. the Nilsson single particle sates.

The Hamiltonian operator used here is one as the usual form

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi\sum_{\mu}\hat{Q}^+_{\mu}\hat{Q}_{\mu} - G_{\mathrm{M}}\hat{P}^+\hat{P} - G_{\mathrm{Q}}\sum_{\mu}\hat{P}^+_{\mu}\hat{P}_{\mu}, \qquad (2)$$

where $\hat{H}_0$ is the spherical single-particle Hamiltonian which contains a proper spin-orbital term, whose strength is represented by the Nilsson parameters of κ and μ. The second term in the Harmiltonian is the $Q\cdot Q$ interaction responsible for the deformation. The last two terms are the monopole and quadrapole pairing interactions, respectively.

Introducing the angular momentum projection operator $\hat{P}^I_{MK}$, an (un-normalized) eigenstate of the angular momentum I is generated. The eigenvalue equation of the Hamiltonian for a given spin I then takes the form:

$$\sum_{k'}\{H^I_{kk'} - E^I N_{kk'}\}f^I_{k'} = 0\ , \qquad \sum_{kk'}f_k N_{kk'} f_{k'} = 1\ , \qquad (3)$$

where the Hamiltonian and norm matrix elements are defined by

$$H^I_{kk'} = \langle\Phi_k|\hat{H}\hat{P}^I_{kk'}|\Phi_{k'}\rangle\ , \qquad N^I_{kk'} = \langle\Phi_k|\hat{P}^I_{kk'}|\Phi_{k'}\rangle\ , \qquad (4)$$

respectively. The PSM basis taken in the present calculation consists of 4 types of projected quasi-particle states:

$$\{\hat{P}^I_{MK}|0\rangle,\ \hat{P}^I_{MK}a^+_{\nu1}a^+_{\nu2}|0\rangle,\ \hat{P}^I_{MK}a^+_{\pi1}a^+_{\pi2}|0\rangle,\ \hat{P}^I_{MK}a^+_{\nu1}a^+_{\nu2}a^+_{\pi1}a^+_{\pi2}|0\rangle\}\ , \qquad (5)$$

each projected state in (5) may be considered to represent a rotational band. The first one, $\hat{P}^I_{MK}|0\rangle$ represents the ground-state band or 'g-band'. The remainders represent various bands built upon the multi-quasi-particle states. We will discuss later how the oblate and prolate bands are involved in those states.

There are at least two advantages in this approach over the usual shell model. Firstly, the vector addition of angular momenta, which is the most awkward procedure in the usual shell model, is not required as this is carried out automatically by the projection operator irrespective of the number of quasi-particle involved. Secondly (and more importantly), the projected states constructed by taking only a few quasi-particle states span already a reasonably good shell model basis due to the hierarchy of the quasi-particle states, that makes physics transparent and interpretation of numerical results easily.

We believe that a relative large single-particle basis is important to reproduce a good rotating frame (projected quasi-particle vacuum state). The active neutron (proton) single particle basis extend over all the major shell. In the calculation, $N_{\mathrm{n}} = 4,5,6$ ($N_{\mathrm{p}} = 3,4,5$). The Nilsson parameters κ and μ relevant to those shells were taken as the same values as those in reference[7]. The monopole pairing strength G_{M} is taken to be

$$G_{\mathrm{M}} = C_{\mathrm{i}}\left(20.12 \mp 13.13\,\frac{N-Z}{A}\right)A^{-1} \quad (\text{'}-\text{' for neutron; '}+\text{' for proton, i = p, n}) \qquad (6)$$

The quadrupole pairing strength G_{Q} is assumed to be proportional to G_{M}. and the proportionality constant, which is usually taken in the range of 0.14—0.18, is being fixed to 0.16. The $Q\cdot Q$ interaction strength χ is adjusted by the self-consistent relation, so as to make the input quadrupole deformation ε_2 and the one resulting from the HFB procedure in coincidence with each other, and with the monopole pairing force constant C_i, which is adjusted to give the pairing energy gap as the known experimental value, $\Delta p = 1.385\text{MeV}$, $\Delta n = 1.1125\text{MeV}$ for ^{188}Pb. The hexa-deformed parameter ε_4 was fixed to be 0.007 for different ε_2 cases. The calculation started with the total potential energy as a function of deformation. The result was plotted in Fig. 1, which exhibits three different shape configurations: sphere (0^+ ground state), oblate (0_2^+ excite state) and prolate (0_3^+ excite state) at $\varepsilon_2 = 0$, -0.17 and 0.25, respectively.

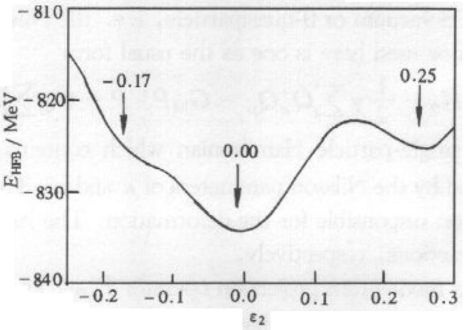

Fig. 1 $E_{\rm HFB}$ as a function of ε_2 for ^{188}Pb

We simply took this information and calculated the collective band structure at those deformations. The level scheme was obtained and summarized in Table. 1 The results were compared with experimental data in Fin1.

Table 1. Summery of the three shape characters in ^{188}Pb

Shape	ε_2	Configuration Space	Character	Comments
I	0.00	$\hat{P}^I_{MK}\|0\rangle$	Spherical ground state	
II	-0.17	$\hat{P}^I_{MK}\|0\rangle$ $\hat{P}^I_{MK}a^+_{\pi1}a^+_{\pi2}\|0\rangle$ $(\pi h_{9/2})$ [*]	$\pi h_{9/2}$ $2p-2h$ excited oblate band	$0^+ = 578\text{keV}$ oblate band, predicted
III	0.25	$\hat{P}^I_{MK}\|0\rangle$ $\hat{P}^I_{MK}a^+_{\pi1}a^+_{\pi2}\|0\rangle$ $(\pi h_{9/2}, \pi f_{7/2})$ [*]	$\pi h_{9/2}$, $\pi f_{7/2}$ excited multi-particle-hole prolate band (Fig. 2 band B)	Two kinds of multi-particle-hole mixing prolate band (Fig. 2 band A) $2^+ = 804$—880keV, pointed
		$\hat{P}^I_{MK}\|0\rangle$ $\hat{P}^I_{MK}a^+_{\pi1}a^+_{\pi2}\|0\rangle$, $\hat{P}^I_{MK}a^+_{\nu1}a^+_{\nu2}\|0\rangle$ $\hat{P}^I_{MK}a^+_{\pi1}a^+_{\pi2}a^+_{\nu1}a^+_{\nu2}\|0\rangle$ $(\pi h_{9/2}$、$\pi f_{7/2}$、$\nu i_{13/2})$ [*]	$\pi h_{9/2}$、$\pi f_{7/2}$、$\nu i_{13/2}$ excited multiparticle-hole prolate band (Fig. 2 band C)	
Note	* : The projected multi-quasi-particle states are constructed only with those intruder states branching from the orbital in brackets.			

The experimental band III can not be considered as a regular rotational band, but a mixture of band B and C.

$$E_{\rm A} = a^2 E_{\rm B} + b^2 E_{\rm C} \quad \text{and} \quad a^2 + b^2 = 1, \tag{7}$$

the 0^+ is the pure unalignment state, and the 12^+ is the complete alignment state due to the

$\nu i_{13/2}$ pair broken. Fitting the E_A to the experimental one, taking $a^2 = 100\%$ for $E_A(0^+)$ and $b^2 = 100\%$ for $E_A(12^+)$, the mixing coefficients b^2 for different levels were deduced with equation (7) and listed in the brackets of band A in the Fig.2. In conclusion, the ^{188}Pb low-lying level scheme has been studied by PSM calculation and explained as three shape coexistence: sphere (g-state), oblate(o-band) and prolate (p-band). Although the potential well where the deformed bands locate are not so deep, even flat in oblate case, the three shapes are associated with different particle configurations. It is because the nucleus will keep certain rigidity among those configurations that the three shapes will coexist. From the mixing coefficients obtained it is turned out that the influence of alignment from the $\nu i_{13/2}$ pair broken on the band structure is not a steep but gradual process, and it is pointed out that the 2^+ will be in the range of 804—880keV.

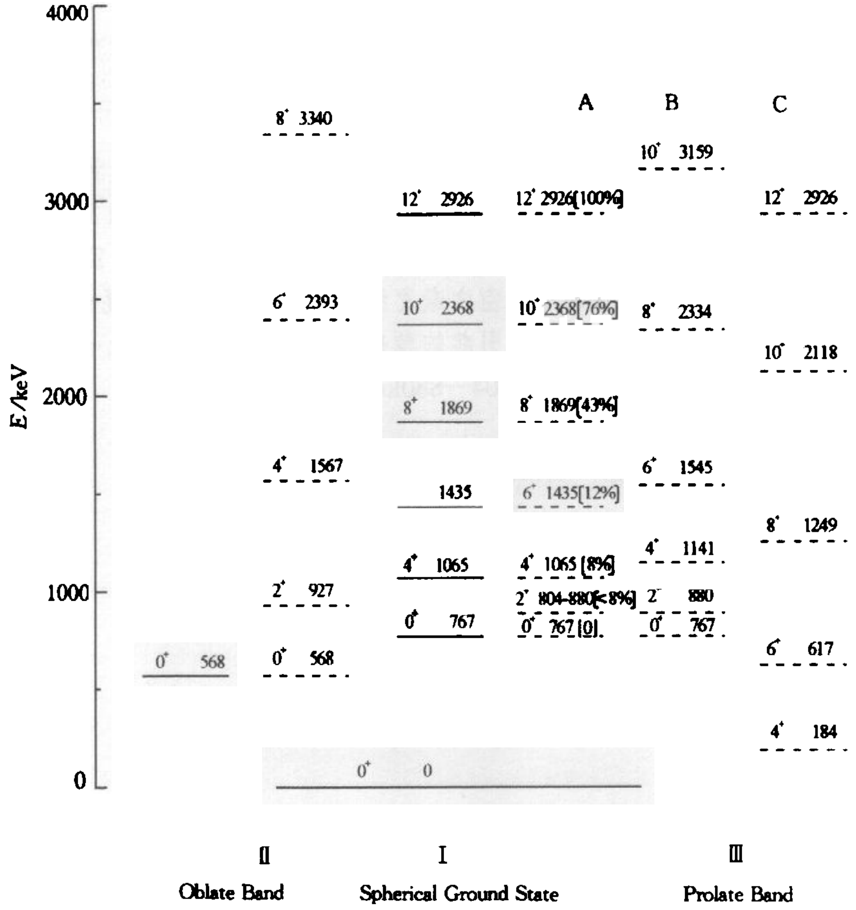

Fig.2 Level scheme for ^{188}Pb indicating their coexisting shape configurations (I , II , III).
The solid lines show the experimental data taken from Refs. [3,4,5]. The dash lines are the resuls from the PSM calculation. The influence of alignment on the prolate band is shown. B: the calculated values without alignment; C: the calculated values with alignment, A: the results obtained from a mixture of B and C with the alignment coefficients in brackets as described in the text.

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^{188}Pb形变和形状共存的研究

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摘要 用角动量投影壳模型研究了^{188}Pb核素的形状共存. 实验数据经过计算结果的分析,指出^{188}Pb的低能激发态存在着对应于不同内部粒子组态的球形基态、扁椭球和长椭球三种形状的共存;预言了质子$h_{9/2}$两粒子－两空穴扁椭球带;指出长椭球带是两种多粒子－空穴激发组态的混杂,并导出了混合系数. 由这些系数,可以说明中子$i_{13/2}$破对引起的顺排对长椭球带的影响是一个渐进过程;并指出了2^+态的能量范围在804—880keV.

关键词 形变 形状共存 低位激发态 投影壳模型

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