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Antibunching Effect of k-Boson q-Coherent States

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Abstract The antibunching effect for the eigenstates of the q-deformed annihilation operator a_q^k ($k \geqslant 3$) is investigated. Using the numerical method, we have studied the influences of the q-parameter deformation on the effect in the case of $k = 3$. The results show that the eigenstates of a_q^k exhibit antibunching effect when $x = |z|^2$, i.e. the intensity of the oscillator in q-deformed coherent state, is in values of certain intervals, and the effect is evidently influenced by the values of parameter q. When the intensity of the q-deformed light field is changed stronger gradually, the fluctuation of the photon number in the light field is changed between the classical (or quantum) and the quantum (or classical) properties alternatively.

Key words q-deformation, eigenstates of operator a_q^k, antibunching effect

1 Introduction

In the past few years, much work has been devoted to quantum group versions of usual Lie (super) algebras, i.e. quantum groups, and their applications to many domains in physics and mathematical physics[1—13]. These algebrs may be viewed as deformations of classical Lie algebras, depending, in general, on one or more parameters. The representation theory of quantum algebras with a single deformation parameter q, has led to the development of q-deformed oscillator algebras[1,2]. Their annihilation and creation operators satisfy the quantum Heisenberg-Weyl algebra (q-HWA). On the other hand, the connecting of coherent states and quantum groups to get the Glauber-type q-coherent states (q-CSs) of the q-HWA have been well studied by many authors[2,6—13]. In Refs. [9,10], the even and odd q-CSs representations were constructed, and their optical statistics properties were studied[8—10]. The even and odd q-CSs are the eigenstates of the square (a_q^2) of the q-annihilation operator. In 1993, the eigenstates of the operator a_q^k were investigated by Kuang et al[12]. The states are the k-boson q-CSs. More recently, the quantum statistical properties (Nth-power squeezing and antibunching effect) of the eigenstates were investigated by Wang et al[13]. Based on these work, in this paper we investigate the antibunching effect of the states, and use numerical method to study the influences of the q-deformation on the case of $k = 3$.

2 The k-boson q-Coherent States

As is well known, the q-HWA[2] is generated by q-creation operator a_q^+, the q-annihilation operator a_q and a q-number operator N_q. These operators satisfy the following commutation relations[3]:

$$a_q a_q^+ - q a_q^+ a_q = q^{-N_q}, \tag{1}$$

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$$[N_q, a_q^+] =$$

where q is a deformed parameter.

$q < \infty$ then corresponds to the replacement $q \leftrightarrow q^{-1}$ throughout. The

on q-Fock space $\{|n\rangle_q,\ n = 0,1,2,\cdots\}$:

$$a_q\ |\ n\rangle_q = \sqrt{[n]_q}\ |\ n-1\rangle_q, \quad (3)$$

$$a_q^+\ |\ n\rangle_q = \sqrt{[n+1]_q}\ |\ n+1\rangle_q, \quad (4)$$

$$N_q\ |\ n\rangle_q = n\ |\ n\rangle_q. \quad (5)$$

The q-number $[n]_q$ is defined as $[n]_q = (q^n - q^{-n})/(q - q^{-1})$, and $|n\rangle_q$ is defined as

$$|\ n\rangle_q = \frac{(a_q^+)^n}{\sqrt{[n]_q!}}\ |\ 0\rangle_q,$$

where the q-factorial $[n]_q! = [n]_q[n-1]_q\cdots[1]_q$ and defining $[0]_q! = 1$. The q-Fock space construct a complete Hilbert space, thus the unity is written as

$$I = \sum_{n=0}^{\infty} |\ n\rangle_{qq}\langle n\ |.$$

The k-component (k is an integer and $k \geqslant 3$) q-CSs were given by[12]

$$|\ z,k,i\rangle_q = N_{qi}(z,k)\sum_{n=0}^{\infty}\frac{z^{kn+i}}{\sqrt{[kn+i]_q!}}\ |\ kn+i\rangle_q,\ (i = 0,1,2,\cdots,k-1), \quad (8)$$

where $N_{qi}(z,k)$ are the normalization constants and z is a complex number. It is easy to prove that the k states of Eq.(8) are all the eigenstates of the operator $a_q^k\ (k \geqslant 3)$ with the same eigenvalue z^k. Taking account of the normalizing conditions, the normalization factors can be calculated as follows:

$$N_{qi}(z,k) = A_i^{-1/2}(|\ z\ |^2,\ k),\ (i = 0,1,2,\cdots,k-1), \quad (9)$$

$$A_i(x,k) = \sum_{n=0}^{\infty}\frac{x^{kn+i}}{[kn+i]_q!},\ (i = 0,1,2,\cdots,k-1), \quad (10)$$

where we have let $|z|^2 = x$ corresponding to the intensity of the oscillator in q-deformed coherent state, which reflects the intensity of q-light field. Therefore, the general expression for the k-boson q-coherent states are expressed as

$$|\ z,k,i\rangle_q = A_i^{-1/2}(|\ z\ |^2,k)\sum_{n=0}^{\infty}\frac{z^{kn+i}}{\sqrt{[kn+i]_q!}}\ |\ kn+i\rangle_q,\ (i = 0,1,2,\cdots,k-1), \quad (11)$$

where $A_i(x,k)(i = 0,1,2,\cdots,k-1)$ are given by Eq. (10).

3 The Antibunching Effect of the k-Boson q-Coherent States

3.1 The Antibunching Effect of the Eigenstates of the Operator a_q^k

It is well known that, if the normalized second-order correlation function of a light field[14] $g^{(2)}(0) < 1$, one says the light field exhibits antibunching effect. It is a nonclassical property of the light field. In a similar way, we may introduce the second-order q-correlation function for the q-deformed light field

$$g_q^{(2)}(0) = \frac{{}_q\langle\ |\ a_q^{+2}a_q^2\ |\ \rangle_q}{|\ {}_q\langle\ |\ a_q^+a_q\ |\ \rangle_q\ |^2}. \quad (12)$$

Substituting Eq. (11) into (12), and taking account of Eq. (3), it is straightforward to evaluate the second-order q-correlation function for the k eigenstates of a_q^k, respectively

$$g_{q,0}^{(2)}(0) = \frac{{}_q\langle z,k,0 \mid a_q^{+2}a_q^2 \mid z,k,0\rangle_q}{|{}_q\langle z,k,0 \mid a_q^{+}a_q \mid z,k,0\rangle_q|^2} = \frac{A_0A_{k-2}}{A_{k-1}^2}, \tag{13}$$

$$g_{q,1}^{(2)}(0) = \frac{{}_q\langle z,k,1 \mid a_q^{+2}a_q^2 \mid z,k,1\rangle_q}{|{}_q\langle z,k,1 \mid a_q^{+}a_q \mid z,k,1\rangle_q|^2} = \frac{A_1A_{k-1}}{A_0^2}, \tag{14}$$

$$g_{q,j}^{(2)}(0) = \frac{{}_q\langle z,k,j \mid a_q^{+2}a_q^2 \mid z,k,j\rangle_q}{|{}_q\langle z,k,j \mid a_q^{+}a_q \mid z,k,j\rangle_q|^2} = \frac{A_{j-2}A_j}{A_{j-1}^2}, \quad (j = 2,3,\cdots,k-1). \tag{15}$$

From Eq. (10) we know that the above second-order q-correlation functions are the functions of x, which reflects the intensity of q-deformed light field. According to the definition, we can say that an eigenstate exhibits antibunching effect as long as the second-order q-correlation function are less than 1 when x equals a certain value. We will prove that all of the eigenstates of a_q^k exhibit the antibunching effect.

According to Eqs. (10) and (13), we have

$$g_{q,0}^{(2)}(0) = \frac{\sum_{m=0}^{\infty}\left(\sum_{n=0}^{m}\frac{1}{[kn]_q![km-kn+k-2]_q!}\right)x^{km}}{x^k\sum_{m=0}^{\infty}\left(\sum_{n=0}^{m}\frac{1}{[kn+k-1]_q![km-kn+k-1]_q!}\right)x^{km}} = \frac{f_{q,0}(x)}{x^k\varphi_{q,0}(x)}.$$

Considering $k \geqslant 3$ and $[n]_q > n$, we have

$$\sum_{n=0}^{m}\frac{1}{[kn]_q![km-kn+k-2]_q!} > \sum_{n=0}^{m}\frac{1}{[kn+k-1]_q![km-kn+k-1]_q!},$$

and hence $f_{q,0}(x) > \varphi_{q,0}(x)$ for $x > 0$, so that $g_{q,0}^{(2)} > 1$ when $0 < x \leqslant 1$. However, when $x > 1$, there surely exist values of x, i.e. $x^k > f_{q,0}(x)/\varphi_{q,0}(x)$, for which the following relation holds:

$$g_0^{(2)} = \frac{f(x)}{x^k\varphi(x)} < 1. \tag{17}$$

From Eqs. (10) and (14), we obtain

$$g_{q,1}^{(2)}(0) = \frac{x^k\sum_{m=0}^{\infty}\left(\sum_{n=0}^{m}\frac{1}{[kn+1]_q![km-kn+k-1]_q!}\right)x^{km}}{\sum_{m=0}^{\infty}\left(\sum_{n=0}^{m}\frac{1}{[kn]_q![km-kn]_q!}\right)x^{km}} = \frac{x^kf_{q,1}(x)}{\varphi_{q,1}(x)}.$$

Obviously,

$$\sum_{n=0}^{m}\frac{1}{[kn+1]_q![km-kn+k-1]_q!} < \sum_{n=0}^{m}\frac{1}{[kn]_q![km-kn]_q!},$$

so that $f_{q,1}(x) < \varphi_{q,1}(x)$. Therefore, $g_{q,1}^{(2)}(0) < x^k$, i.e. $g_{q,1}^{(2)}(0) < 1$, when $x \leqslant 1$.

Substituting Eq. (10) into (15), for the second-order q-correlation function of $|z,k,j\rangle_q$ $(j = 2,3,\cdots,k-1)$, we have

$$g_{q,j}^{(2)}(0) = \frac{\sum_{m=0}^{\infty}\left(\sum_{n=0}^{m}\frac{1}{[kn+j-2]_q![km-kn+j]_q!}\right)x^{km}}{\sum_{m=0}^{\infty}\left(\sum_{n=0}^{m}\frac{1}{[kn+j-1]_q![km-kn+j-1]_q!}\right)x^{km}}, (j = 2,3,\cdots,k-1).$$

If $x < 1$, then

$$g_{q,j}^{(2)}(0) < \frac{\sum_{m=0}^{\infty} \frac{(m+1)}{[j-2]_q![j]_q!} x^{km}}{\sum_{m=0}^{\infty} \frac{(m+1)}{([km+j-1]_q!)^2}} < \frac{\frac{1}{[j-2]_q![j]_q!} \sum_{m=0}^{\infty} (m+1) x^{km}}{\frac{1}{([j-1]_q!)^2}},$$

and

$$\sum_{m=0}^{\infty} (m+1) x^{km} = \frac{1}{(1-x^k)^2}.$$

Therefore, when $x < 1$, we have

$$g_{q,j}^{(2)}(0) < \frac{[j-1]_q}{[j]_q} \frac{1}{(1-x^k)^2}, (j = 2,3,\cdots,k-1). \tag{20}$$

As long as x is small enough, the right-hand side of the inequality (20) can equal 1 or less than 1. Practically, if $x^k \leqslant 1-([j-1]_q/[j]_q)^{1/2}$, then $[j-1]_q/\{[j]_q(1-x^k)^2\} \leqslant 1$. For example, when $k=3$, taking $q=1.0$, in the range of $0 < x \leqslant 0.664$; taking $q=0.5$, in the range of $0 < x \leqslant 0.716$; and taking $q=0.01$, in the range of $0 < x \leqslant 0.965$ (Obviously, if $q \to 0$, the range is $0 < x < 1$), the second-order q-correlation functions less than 1, i.e. $g_{q,j}^{(2)}(0) < 1 (j=2, 3,\cdots,k-1)$. As a result, in the range of $0 < x < 1$, there surely exist values of x for which the following relation holds:

$$g_{q,j}^{(2)}(0) < 1, \quad (j = 2,3,\cdots,k-1). \tag{21}$$

Therefore, the relevant result in Ref. [13] is not precise, because that states $|z,k,j\rangle_q (j = 2,3,\cdots,k-1)$ exhibit antibunching effect not only when $x \to 0$. In fact, the states $|z,k,j\rangle_q (j = 2,3,\cdots,k-1)$ can show antibunching effect in the range of $x > 1$. It will be proved by numerical study in the following subsection.

3.2 The Numerical Study for the Antibunching Effect of the *k*-Boson *q*-CSs

For the sake of simplicity, the second-order q-correlation functions $g_{q,i}^{(2)}(0) (i = 0,1,2,\cdots,k-1)$ will be only studied by numerical method in the case of $k=3$. By the same way, the other case in $k > 3$ can be investigated. From Eqs. (13)—(15), tahing $k=3$, we obtain

$$g_{q,0}^{(2)}(0) = A_0 A_1 / A_2^2, \quad g_{q,1}^{(2)}(0) = A_1 A_2 / A_0^2, \quad g_{q,3}^{(2)}(0) = A_0 A_2 / A_1^2. \tag{22}$$

We numerically calculate the second-order q-correlation functions Eqs. (22). The relation of $g_{q,j}^{(2)}(0) (i = 0,1,2)$ varying with x, or the intensity of q-deformed light field (q-LF), for $q = 1.0$, 0.5 and 0.1 are demonstrated in Figs. 1—3. The intervals that the eigenstates ($|z, 3, i\rangle_q$, $i = 0,1,2$) of a_q^3 can exhibit antibunching effect are listed in Table 1. It is demonstrated in Figs. 1—3 and Table 1 that these intervals are evidently influenced by the values of parameter q, where the case of $q = 1.0$ expresses the non-deformed ones. Comparing the second to the fourth column in Table 1, the state $|z,3,0\rangle_q$ shows antibunching effect, i.e. $g_{q,0}^{(2)}(0) < 1$, in the range of $x > 1$, the smaller values of q, the larger the intervals exhibiting the effect, and the intervals shift toward the positive x direction excepting nearby $q = 0.5$, which may be caused by the characteristic of q-variable $[n]_q$ itself. The first interval that the state $|z,3,1\rangle_q$ shows antibunching effect is the range from $x = 0$ to a certain value of $x > 1$, and the first interval that the state $|z,3,2\rangle_q$ exhibits antibunching effect is the range from $x > 0$ to a certain value of $x > 1$. It can be seen from Table 1 that the number of intervals in which the eigenstates $|z,3,i\rangle_q (i = 0,1,2)$ show antibunching effect may be more than one, and the second intervals as well as the latter are larger than the first ones.

Table 1. The intervals of the states $|z,3,i\rangle_q\ (i=0,1,2)$ showing antibunching effect.

	$q=1.0$	$q=0.5$	$q=0.1$
$\vert z,3,0\rangle_q$, $g^{(2)}_{q,0}(0)<1$	$3.02\leqslant x\leqslant 6.65$, $10.29\leqslant x\leqslant 13.38$	$2.40\leqslant x\leqslant 6.44$, $17.84\leqslant x\leqslant 51.10$, $142.52\leqslant x\leqslant 408.75$, $1140.14\leqslant x\leqslant$?	$4.84\leqslant x\leqslant 211.52$
$\vert z,3,1\rangle_q$, $g^{(2)}_{q,1}(0)<1$	$0\leqslant x\leqslant 1.84$, $5.45\leqslant x\leqslant 9.06$	$0\leqslant x\leqslant 1.61$, $4.57\leqslant x\leqslant 12.80$, $35.64\leqslant x\leqslant 102.19$, $285.04\leqslant x\leqslant 817.50$	$0\leqslant x\leqslant 2.17$, $48.24\leqslant x\leqslant$?
$\vert z,3,2\rangle_q$, $g^{(2)}_{q,2}(0)<1$	$0< x\leqslant 4.23$, $7.86\leqslant x\leqslant 11.45$	$0< x\leqslant 3.37$, $8.95\leqslant x\leqslant 25.56$, $71.27\leqslant x\leqslant 204.37$, $570.07\leqslant x\leqslant$?	$0< x\leqslant 21.15$, $482.36\leqslant x\leqslant$?

Note: where the question mark "?" expresses a greater number.

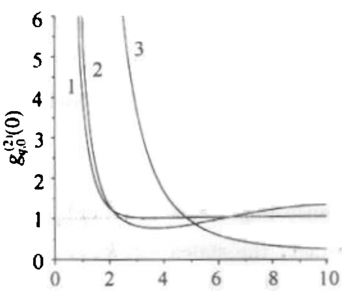

Fig.1. The function $g^{(2)}_{q,0}(0)$ vs the intensity of q-LF $x=|z|^2$ in different q.
Line 1, 2, and 3 correspond to $q=1.0$, 0.5 and 0.1, espectively.

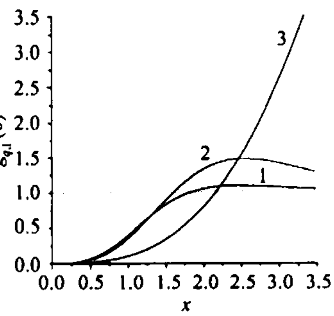

Fig.2. The function $g^{(2)}_{q,1}(0)$ vs the intensity of q-LF $x=|z|^2$ in different q.
The illustration is the same as Fig.1.

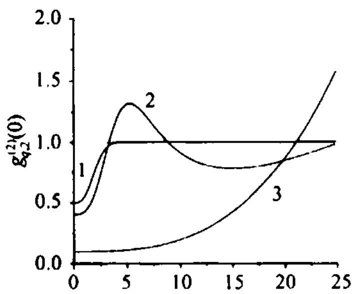

Fig.3. The function $g^{(2)}_{q,2}(0)$ vs the intensity of q-LF $x=|z|^2$ for different q.
The illustration is the same as Fig.1.

As is well known, if the second-order correlation function of a light field is greater than 1, the light field exhibits bunching effect, and the effect reflects a classical property of the field, i.e. the fluctuation of the photon number in the light field is greater than zero. If the second-order correlation function of a light field is less than 1, the light field exhibits antibunching effect, and this reflects a quantum property of the field. The second-order correlation function of a coherent state is equal to 1, and this is situated between the classical and the quantum properties, i.e. coherent state is a minimum-uncertainty state. The results of the numerical method in Table 1 show that the second-order correlation functions of q-deformed light field are situated between greater (or less) than 1 and less (or greater) than 1 alternately when the intensity of the q-light field becoms stronger gradually. It is indicated that the fluctuation of the photon number in the q-deformed light field is changed between the classical (or quantum) and the quantum (or classical) properties alternately when the intensity of the light field becoms stronger gradually.

4 Conclusion

To summarize, for particular values of x, which reflects the intensity of q-LF, the second-order q-correlation functions $g^{(2)}_{q,0}(0)$, $g^{(2)}_{q,1}(0)$ and $g^{(2)}_{q,j}(0)$ $(j=2,3,\cdots,k-1)$ can be less than 1.

Therefore all of the k-boson q-coherent states show the antibunching effect. For the case of $k=3$, the results of the numerical method show that the antibunching effect is evidently influenced by the values of parameter q. The state $|z,3,0\rangle_q$ exhibits antibunching effect in the intervals of $x>1$. The first intervals in which the states $|z,3,i\rangle_q (i=1,2)$ show the antibunching effect are the range from $x>0$ to a certain value of $x>1$, and the antibunching effect can be exhibited in several intervals. Furthermore, the fluctuation of the photon number in the light field expressed by the states is changed between the classical (or quantum) and the quantum (or classical) properties alternately when the intensity of the q-light field becoms stronger gradually.

The q-parameter deformation of quantum algebra has been used in description for some domain of physics and the results coinciding with experimental data[15]. It is interesting to note that when $q\to 1$, the results of this paper become the results in Ref. [16]. Therefore, the systems composed by the eigenstates of the q-deformed oscillator annihilation operator $a_q^k (k\geqslant 3)$ have more extensive physical connotation than that of the systems composed by the eigenstates of the conventional non-deformed ones. If the systems have been achieved in experiments, we may control the parameter q to control some quantum statistics properties of the light field. For this reason, it is worthy to make a through study to the systems since they have latent and important application prospects.

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k玻色子q相干态的反聚束效应

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摘要 研究了q变形湮没算符高次幂(a_q^k, $k\geqslant 3$)本征态的反聚束效应,并就$k=3$的情况用数值计算方法研究了q变形参数对该效应的影响. 结果表明,当q变形相干态中谐振子的强度$x=|z|^2$在某些区间内取值时,a_q^k的本征态将呈现反聚束效应,并且这一效应明显地受到q参数的影响. 当参数q取定时,随着q变形光场强度的变大,该光场的光子数涨落在经典(或量子)和量子(或经典)特性之间交替地变化.

关键词 q变形 a_q^k的本征态 反聚束效应