

## Remarks about the Constituent Quark Model<sup>\*</sup>

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**Abstract** The problems related to the modeling of quark confinement and Goldstone boson-quark coupling in the prevailing constituent quark models are discussed based on the lattice QCD result and the chiral symmetry spontaneous breaking theory.

**Key words** quark model, confinement potential, scalar mesons

Constituent quark model (CQM) is the most successful one in describing hadron spectroscopy and hadron interactions. However for a long time the QCD basis of the CQM has been appeared to be the worst. N. Isgur discussed this problem in his last years<sup>[1]</sup>. The progress of nonperturbative QCD calculations gradually provides the QCD foundation of the CQM<sup>[2,3]</sup>. The quark confinement is due to color flux tube or color string formation and the large gluonic excitation energy ( $\sim 1\text{GeV}$ ) suppresses the explicit excitation of the gluon degree of freedom in the low energy QCD physics such as hadron spectroscopy and interactions. Chiral symmetry and its spontaneous breaking is another feature of low energy QCD physics. The nontrivial QCD vacuum dresses the light current quark to be constituent quark with much larger dynamic mass  $m(q^2)$  and in turn suppresses the explicit constituent  $q\bar{q}$  excitation. On the other hand, it leads to the appearance of Goldstone boson. These features have been shown in Ichie et al.'s Lattice QCD result and their schematic diagrams which I modified a little bit and reproduced in figures 1—4. Based on

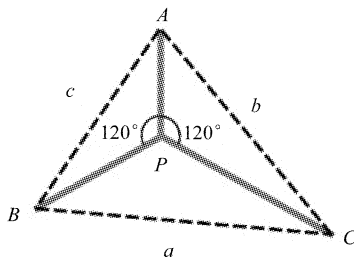


Fig.1. The flux-tube configuration of the 3Q system.

these we can reexamine the assumptions of the CQM.

Ichie et al.<sup>[3]</sup> found that the energy of the ground state gluonic configuration in a three quark system can be fitted with a potential

$$V_{3q} = -A_{3q} \sum_{i < j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3q} L_{\min} + C_{3q} \quad (1)$$

within 1% -level deviation.  $L_{\min}$  is the minimal value of the total gluon flux-tube length,  $\mathbf{r}_i$  is the position of quark  $i$  (see Fig.1). The first term in Eq.(1) is the color Coulomb interaction and the second term is similar to a linear confinement potential.

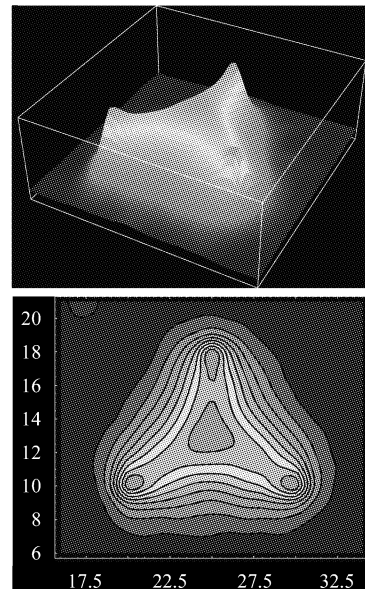


Fig.2. The lattice QCD result for the flux-tube profile in the spatially-fixed 3Q system.

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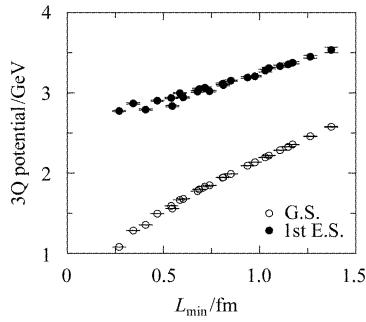


Fig.3. The lattice QCD results of the ground-state 3Q potential  $V_{3Q}^{g.s.}$  (open circles) and the 1st excited-state 3Q potential  $V_{3Q}^{e.s.}$  (filled circles) as the function of  $L_{min}$ .

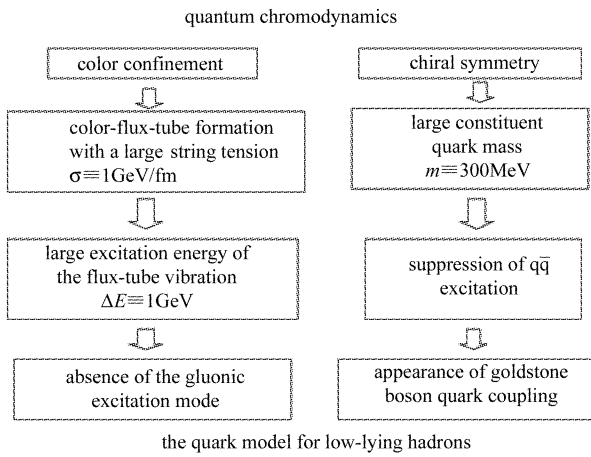


Fig.4. Connection from QCD to the quark model for low-lying hadrons.

Most of constituent quark models use a quadratic or linear potential to model the quark confinement,

$$V_{conf}(\mathbf{r}_{ij}) = -a\lambda_i \cdot \lambda_j r_{ij}^n, \quad (2)$$

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j, n = 1, 2,$$

here  $\lambda_i^a$  ( $a = 1 \cdots 8$ ) is the color  $SU(3)$  group generator. For a single hadron,  $q\bar{q}$  mesons or  $q^3$  baryons, such a modeling can be achieved by adjusting the strength constant  $a$  of the confinement potential. The color factor  $\lambda_i \cdot \lambda_j$  gives rise a strength ratio 1/2 for baryon and meson which is almost the ratio for the minimum length of the flux tube to the circumference of the triangle in Fig.1.

However to extend the confinement potential with parameter fixed by hadron spectroscopy to multi-quark system, such as the baryon-baryon (BB) interaction, is questionable. Up to now there is still no lattice QCD result of the color flux tube or string structure for BB system. But from a general  $SU(3)$  color group consideration, there might be the following color

structures: Fig.5(a) corresponds to two isolated color singlet baryons; Fig.5(b) is a simple rearrangement of the color flux tube but still two isolated color singlet baryons; Fig.5(c) is the hidden color channel and Fig.5(d) is a genuine six quark state. When two baryons are separated far away, one expects that the two isolated color singlet baryons represent the real situation. In this case there will be no BB interaction at all. This might be the mechanism which transforms the long range quark confinement interaction to the observed short range hadron interaction. When two baryons are close together, the color structure shown in Fig.5(c) and 5(d) will appear. This will induce effective BB interaction. On the other hand, it is well known that the quadratic confinement will not induce any effective BB interaction and the linear confinement almost does not induce effective BB interaction if only color singlet baryon channels are included. The linear or quadratic confinement does induce effective BB interaction if the hidden color channels are included. But this leads to the unphysical color van der Waals force. Therefore the linear or quadratic confinement potential is pathological to model the quark confinement for the BB system. A similar conclusion is also true for other multi-quark systems.

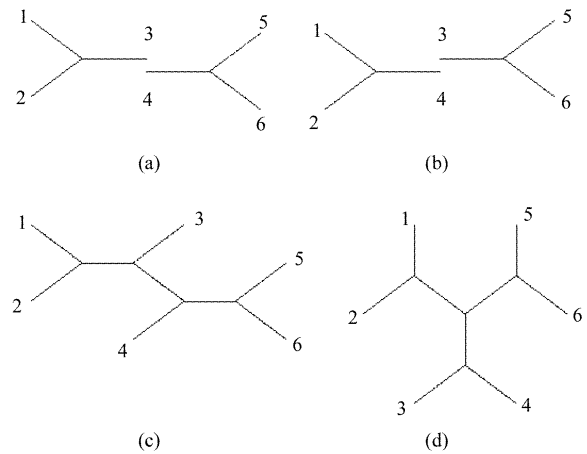


Fig.5. Color structures of six quark system.

Moreover, for multi-quark systems, there are multi-gluon exchange interactions. One example is given in Fig.6<sup>[2]</sup>. Such a three gluon interaction will have a color factor  $f_{abc}\lambda_i^a\lambda_j^b\lambda_k^c$ , which will not contribute to the internal energy of a color singlet baryon, because  $\epsilon_{ijk}f_{abc}\lambda_i^a\lambda_j^b\lambda_k^c\epsilon_{lmn} = 0$ . However it does contribute to the BB interaction. Any model with only two body quark interactions is certainly not able to model such kind gluon interactions. Baryon spectroscopy study restricted in  $q^3$  configuration can not obtain any infor-

mation about such kind multi-gluon interaction. To fix the model parameters of a two body Hamiltonian through baryon spectroscopy and then directly extend it to multi-quark system, such a quite successful method used in atomic, molecular, and nuclear structure studies will certainly missed part of the quark-gluon interactions.

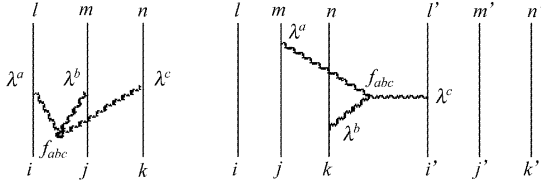


Fig.6. Three gluon exchange interactions.

QCD Lagrangian has chiral symmetry for the massless quark. The current u, d quark mass is small (few MeV) in comparison with the chiral symmetry spontaneous breaking scale ( $\chi \sim 1\text{GeV}$ ). Even the current s quark mass ( $\sim 150\text{MeV}$ ) is also small in comparison with  $\chi$ . Therefore QCD has good  $SU_L(2) \times SU_R(2)$  and approximate  $SU_L(3) \times SU_R(3)$  chiral symmetry. Various QCD models want to keep this chiral symmetry in their model Lagrangians. However some chiral quark model Lagrangians in fact do not satisfy the chiral symmetry. For example, the Glozman-Riska chiral quark model<sup>[4]</sup> employs the following quark-Goldstone meson coupling Lagrangian,

$$\mathcal{L}_{\text{int}}^{\text{ch}} = -g_{\text{ch}} \bar{\psi} i \gamma_5 \sum_{a=0}^8 \lambda_a \phi_a \psi. \quad (3)$$

It is easy to show that this Lagrangian is not a chiral invariant one, only after including the scalar nonets then one can have a  $SU(3)$  chiral symmetry Lagrangian.

For low energy QCD physics the chiral symmetry is spontaneously broken due to  $q\bar{q}$  condensation. Therefore to study low energy QCD physics, one must use a chiral symmetry spontaneous breaking Lagrangian rather than a chiral symmetric one. Weinberg, Coleman et al.<sup>[5]</sup> have given a general procedure to derive the Goldstone boson-Fermion coupling Lagrangian. Manohar and Georgi<sup>[6]</sup> gave an explicit Goldstone boson-constituent quark coupling Lagrangian. Such a chiral symmetry spontaneous breaking Goldstone boson-nucleon coupling Lagrangian has been used by Machleidt<sup>[7]</sup> in the calculation of NN interaction and a perfect fit to the scattering data below  $300\text{MeV}$  has been obtained. Machleidt's Lagrangian is as follows,

$$\begin{aligned} \mathcal{L}_{\pi N} &= -\bar{\psi}_N \gamma^\mu (V_\mu + g_A \gamma_5 A_\mu) \psi_N, \\ V_\mu &= i/2 (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\ A_\mu &= i/2 (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \\ \xi &= \exp(i\boldsymbol{\tau} \cdot \boldsymbol{\pi}/2f_\pi), \end{aligned} \quad (4)$$

To the third order, the above Lagrangian includes the following terms,

$$\begin{aligned} \mathcal{L}_{\pi N} &= -\bar{\psi}_N \gamma^\mu (\gamma_5 \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi}/2f_\pi + \boldsymbol{\tau} \cdot \boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi}/4f_\pi^2 + \\ &\quad \gamma_5 (\boldsymbol{\tau} \cdot \boldsymbol{\pi} \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} - \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi} \boldsymbol{\pi}^2)/12f_\pi^3 + \dots) \psi_N, \end{aligned} \quad (5)$$

here the  $f_\pi$  is the  $\pi$  decay constant. In this Lagrangian, there are only  $\pi$  fields and no low mass ( $\sim 500\text{--}600\text{MeV}$ ) flavor singlet scalar  $\sigma$  field. The  $\sigma$  exchange effect of the usual Yukawa meson exchange model is attributed to the nonlinear  $\pi$  field contribution.

In the  $SU(3)$  case, the chiral symmetry spontaneous breaking Goldstone boson-constituent quark coupling Lagrangian to third order is

$$\begin{aligned} \mathcal{L}_{\text{int}} &= -\bar{\psi} \gamma^\mu (\gamma_5 \lambda_a \partial_\mu \phi_a/2f_\pi + \lambda_a f_{abc} \phi_b \partial_\mu \phi_c/4f_\pi^2 + \\ &\quad \gamma_5 \lambda_a f_{abc} f_{cde} \phi_b \phi_c \partial_\mu \phi_d/12f_\pi^3 + \dots), \end{aligned} \quad (6)$$

here  $\lambda_a$  is the  $SU(3)$  flavor group generator,  $f_{abc}$  is the  $SU(3)$  group structure constant. Again it only includes the octet Goldstone boson fields and no low mass flavor singlet scalar  $\sigma$  meson. One can not derive a universal effective  $\sigma$  coupling to u, d and s quark through the nonlinear terms of this Lagrangian.

Zhang et al. proposed a  $SU(3)$  chiral quark model<sup>[8]</sup> to study the BB interaction and dibaryon states. Their Lagrangian includes both the scalar and pseudo scalar meson nonets, which satisfies the chiral symmetry but is not the right chiral symmetry spontaneous breaking version. Such a model Lagrangian is a direct extension of the Yukawa meson baryon coupling down to the constituent quark meson coupling. If it is used to fit the NN scattering data, a reasonable fit is expected, where the flavor singlet scalar  $\sigma$  meson models the correlated S-wave  $\pi\pi$  exchange. However if this model is applied to the high strangeness BB channels, the fictitious  $\sigma$  meson-s quark coupling will give rise to the spurious attraction. Therefore the binding energies of the high strangeness dibaryons are quite possible over estimated. The Nijmegen group has found that the flavor singlet scalar meson over estimated the attraction between the strange baryons<sup>[9]</sup>. Fujiwara also found that his quark model, which uses the Nijmegen model F, might over estimated the attraction between  $\Lambda$ s in comparison with the result obtained from the di- $\Lambda$  hypernuclei<sup>[10]</sup>.

The Glozman-Riska model Lagrangian, except the flavor singlet pseudo scalar meson, can be viewed as a linear approximation of the  $SU(3)$  chiral symmetry sponaneous breaking one. Such a model Lagrangian can not fit the BB interaction because the nonlinear terms are missing. If the  $\sigma$  meson is recalled to provide the intermediate range attraction as done by Stancu et al.<sup>[11]</sup>, it might be possible to fit the NN scattering. If it is extened to the high strangeness BB channels it will have the same problem as discussed above for Zhang et al.'s model. Isgur<sup>[1]</sup> had pointed out that this model ruins the symmetry between meson and baryon internal structures and this symmetry seems to be confirmed by the lattice QCD calculation. Ichie et al.'s result, Eq.(1), has a Coulomb term which should be due to massless gluon exchange rather than a Yukawa term due to meson exchange, this might be a signal that to neglect the gluon exchange totally might be question-

able.

The Manohar-Georgi model<sup>[6]</sup> has the right chiral symmetry spontaneous breaking version. One point we like to mention is that after the current quark has been dressed to be the constituent quark by the nonperturbative QCD vacuum, it is no longer the point particle. The vector and axial vector vertexes both are dressed by the nonperturbative QCD vacuum<sup>[13,14]</sup>. It is interesting to check if such a modified Manohar-Georgi model can describe the baryon internal structure, e. g., the spin flavor structure of nucleon, the right  $G_A/G_V$  without the phenomenological  $g_a$  used in Manohar-Georgi model and the BB interactions without the phenomenological contact terms used in the chiral perturbation approach.

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## 对于组分夸克模型的几点评注\*

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**摘要** 根据现有的格点 QCD 结果和手征对称自发破缺理论研讨了组分夸克模型中夸克囚禁和夸克介子耦合方案.指出了两体囚禁势不能直接推广应用于多夸克系统,  $\sigma$  介子作为双  $\pi$  交换的等效描述只能用于 u, d 夸克, 不能用于 s 夸克.

**关键词** 夸克模型 囚禁势 标量介子

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