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正则 Ward 恒等式和 Abel 规范理论中动力学质量的产生*

隆正文[1;1)] 李子平[2;2)]

1(贵州大学物理系 贵阳 550025)

2(北京工业大学应用物理系 北京 100022)

摘要 文中基于约束 Hamilton 系统理论用 Faddeev-Senjanovic 路径积分量子化方法,重新讨论了 Cornwall-Norton 和 Jackiw-Johnson 模型的量子化,导出了这两个系统的正则 Ward 恒等式,利用导出的正则 Ward 恒等式,得到了包括费米子和束缚态的质量谱. 所得的结果与其他方法导出的结果相同.

关键词 正则 Ward 恒等式 约束 动力学对称破缺 Abel 规范理论

1 引言

动力学自发破缺机制[1,2]是在超导迈斯纳效应理论的启发下,提出的一种不同于 Higgs 机制产生破缺的可能机制,在动力学自发破缺机制中不需要引入 Higgs 标量场,拉氏函数只包含所要研究的场,自发破缺解存在于自己所满足的(非微扰的)方程之中. 它包含的基本场及参数更少. Nambu 和 Jona-Lasinio 的模型[3]就是一个 $SU(2)_L \times SU(2)_R$ 对称破缺的例子. 把强子作为有规范作用结合起来的夸克复合态来描述的量子色动力学,人们也试图用这种观点解释 $SU(2)_L \times SU(2)_R$ 对称性的破缺. 因而一直引起人们的研究兴趣[4—10]. 它的缺点是计算困难,因为动力学自发破缺是一个非微扰效应,不能作微扰展开. 最近,开展了包含复合场的 Ward-Takahashi(W-T)恒等式的研究. 并用于研究手征对称破缺和束缚态的性质[5]. 在一些模型的研究中,这种方法能够得到包括费米子和束缚态的质量谱. 但在存在明显破缺的情况下,用这种方法能更方便地讨论相结构、质量谱、PCAC 等[11],文献[4]曾将它推广到规范对称动力学破缺的情况,讨论规范玻色子获得质量的机制[12],当规范对称动力学破缺时,矢量介子获得的质量与 Schwinger 机制[13]结果是一致的. 但他们的讨论是基于位形空间的路径积分的 Ward 恒等式,位形空间路径积分是相空间路径积分的一种特殊情况,相空间路径积分比位形空间更基本,对于约束 Hamilton 系统含复杂约束的情形,要作出对动量的路径积分是十分困难的,甚至是不可能的,因此,基于相空间路径积分来研究就具有更普遍的意义. 这里用约束系统理论[13—18]来进一步讨论文献[4]中的模型. 采用 Faddeev-Senjanovic(F-S)相空间路径积分量子化方法[19,20],研究了有约束的动力学系统 Cornwall-Norton 和 Jackiw-Johnson 模型的路径积分量子化,并导出了这两个系统的正则 Ward 恒等式,利用导出的正则 Ward 恒等式,得到了与文献[4]中相同的有关费米子和束缚态质量谱的结果.

2 Cornwall-Norton 模型

Cornwall-Norton 模型由如下拉氏密度描述[1]

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1) E-mail: longshc@hotmail.com

2) E-mail: zpli@solaris.bjpu.edu.cn

$$\mathscr{L} = \bar{\psi}(\mathrm{i}\gamma \cdot \partial - m_0)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + g\bar{\psi}\gamma^\mu\psi A_\mu + g'\bar{\psi}\gamma^\mu\tau_2\psi B_\mu \quad (1)$$

其中 $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B'_\mu$, ψ 代表两个场 ψ_1 和 ψ_2, τ_2 为 Pauli 矩阵, A_μ, B_μ 为 Abel 规范场. 拉氏量(1)式是奇异的,设 $\bar{\psi}$, ψ, A_μ, B_μ 的正则共轭动量分别为 π, $\bar{\pi}$, π_A^μ, π_B^μ, 则

$$\pi = \frac{\partial \mathscr{L}}{\partial \dot{\bar{\psi}}} = 0, \bar{\pi} = \frac{\partial \mathscr{L}}{\partial \dot{\psi}} = -\bar{\psi}\mathrm{i}\gamma^0,$$
$$\pi_A^\mu = \frac{\partial \mathscr{L}}{\partial \dot{A}_\mu} = -F^{0\mu}, \pi_B^\mu = \frac{\partial \mathscr{L}}{\partial \dot{B}_\mu} = -G^{0\mu}. \quad (2)$$

初级约束为

$$\Phi_1 = \pi \approx 0,\ \Phi_2 = \bar{\pi} + \bar{\psi}\mathrm{i}\gamma^0 \approx 0,$$
$$\Phi_3 = \pi_A^0 \approx 0,\ \Phi_4 = \pi_B^0 \approx 0. \quad (3)$$

正则哈密顿量密度为

$$\mathscr{H}_c = \dot{\psi}\bar{\pi} + \dot{\bar{\psi}}\pi + \dot{A}_\mu\pi_A^\mu + \dot{B}_\mu\pi_B^\mu - \mathscr{L} = \frac{1}{2}\pi_{Ai}^2 + \frac{1}{2}\pi_{Bi}^2 - A_0\partial_i\pi_A^i - B_0\partial_i\pi_B^i - \bar{\psi}(\mathrm{i}\gamma^i \cdot \partial_i - m_0)\psi + \frac{1}{4}F_{ij}F^{ij} + \frac{1}{4}G_{ij}G^{ij} - g\bar{\psi}\gamma^\mu\psi A_\mu - g'\bar{\psi}\gamma^\mu\tau_2\psi B_\mu. \quad (4)$$

总哈密顿量为

$$H_T = H_c + \lambda^i\Phi_i \mathrm{d}^3x, \quad (5)$$

$\lambda^i(x)$为约束乘子. 初级约束的自洽性条件

$$\{\Phi_1, H_T\} = \{\pi, H_0\} = (\mathrm{i}\gamma^i\partial_i - m_0)\psi + g\gamma^\mu\psi A_\mu + g'\gamma^\mu\tau_2\psi B_\mu - \lambda_2\mathrm{i}\gamma^0 \approx 0,$$
$$\{\Phi_2, H_T\} = \{\bar{\pi} + \bar{\psi}\mathrm{i}\gamma^0, H_T\} = (\mathrm{i}\gamma^i\partial_i - m_0)\bar{\psi} - g\bar{\psi}\gamma^\mu A_\mu - g'\bar{\psi}\gamma^\mu\tau_2 B_\mu + \lambda_1\mathrm{i}\gamma^0 \approx 0. \quad (6)$$

它们不给出约束,而是给出乘子满足的关系式. 次级约束为

$$\chi_1 = \{\Phi_3, H_T\} = \{\dot{\pi}_A^0, H_T\} = \partial_i\pi_A^i + g\bar{\psi}\gamma^0\psi \approx 0,$$
$$\chi_2 = \{\Phi_4, H_T\} = \{\pi_B^0, H_T\} = \partial_i\pi_B^i + g'\bar{\psi}\gamma^0\tau_2\psi \approx 0. \quad (7)$$

此外再无约束了,由 Dirac 约束分类的定义,通过计算约束函数的 Poisson 括号,不难验证 $\Phi_3 = \pi_A^0$, $\Phi_4 = \pi_B^0$ 为第一类约束; Φ_1, Φ_2, χ_1, χ_2 为第二类约束,为了得到第一类约束的最大数目,可将其组合成二个第一类约束

$$\Phi_5 = \partial_i\pi_A^i + \mathrm{i}g(\bar{\pi}\psi + \bar{\psi}\pi),$$
$$\Phi_6 = \partial_i\pi_B^i + \mathrm{i}g'\tau_2(\bar{\pi}\psi + \bar{\psi}\pi). \quad (8)$$

这就给出了第一类约束的最大数目(Φ_3, Φ_4, Φ_5, Φ_6). Φ_1, Φ_2 为第二类约束. 按 Faddeev-Senjanovic (F-S)量子化方法[18,19],相应于每一个第一类约束应取一规范条件,其规范条件可取为[15,16]

$$\Omega_1 = \partial_i\pi_A^i + \partial_i\partial_i A_0 \approx 0, \Omega_2 = \partial_i A^i \approx 0,$$
$$\Omega_3 = \partial_i\pi_B^i + \partial_i\partial_i B_0 \approx 0, \Omega_4 = \partial_i B^i \approx 0. \quad (9)$$

其中 $\Omega_1 \approx 0$ 和 $\Omega_3 \approx 0$ 分别可由库仑规范 $\Omega_2 \approx 0$ 和 $\Omega_4 \approx 0$ 的自洽性、相容性条件而得.

对复合场也引入外源,则相空间 Green 函数的生成泛函为[15,16]

$$Z[J] = \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}\pi_A^\mu\mathrm{D}B_\mu\mathrm{D}\pi_B^\mu \times \delta(\Phi_3)\delta(\Phi_4)\delta(\Phi_5)\delta(\Phi_6)\delta(\Omega_1)\delta(\Omega_2) \cdot \delta(\Omega_3)\delta(\Omega_4)\delta(\Phi_1)\delta(\Phi_2) \cdot \exp\{\mathrm{i}\,\mathrm{d}^4x[\mathscr{L}^p + J_\mu B_\mu + I_\mu A_\mu + \bar{\eta}\psi + \bar{\psi}\eta + \bar{\psi}\tau_a\psi K_a]\}, \quad (10)$$

其中 $\mathscr{L}^p = \dot{\psi}\bar{\pi} + \dot{\bar{\psi}}\pi + \dot{A}_\mu\pi_A^\mu + \dot{B}_\mu\pi_B^\mu - \mathscr{H}_c$,不难验证以约束函数 Poisson 括号为元素构成的行列式 $[\det|\{\Phi_i, \Phi_j\}|]^{\frac{1}{2}}(i,j=1,2)$与 $\det|\{\Omega_k, \Phi_l\}|(k=1,2,3,4,;\ l=3,4,5,6)$均与场量无关,在生成泛函(10)式中已略去这两个行列式. 利用 δ 函数的性质,并对乘子场也引入外源,则上式写为[13—15]

$$Z[J] = \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}\pi_A^\mu\mathrm{D}B_\mu\mathrm{D}\pi_B^\mu\mathrm{D}\mu_k\mathrm{D}\omega_l \cdot \exp\{\mathrm{i}\,\mathrm{d}^4x[\mathscr{L}_{\text{eff}}^p + J_\mu B_\mu + I_\mu A_\mu + \bar{\eta}\psi + \bar{\psi}\eta + U_k\mu_k + V_l\omega_l + \bar{\psi}\tau_a\psi K_a]\} \equiv \mathrm{e}^{\mathrm{i}W[J]} \quad (11)$$

J 代表所有外源(J_μ, I_μ, U_k, V_l, $\bar{\eta}$, η, K_a), $\mathscr{L}_{\text{eff}}^p = \mathscr{L}^p + \mathscr{L}_m = \mathscr{L}^p + \mu_k\Phi_k + \omega_l\Omega_l$, μ_k, ω_l 为乘子场.

在下列变换下

$$\delta\psi(x) = \mathrm{i}[\alpha(x) + \tau_2\beta(x)]\psi(x),$$
$$\delta\bar{\pi}(x) = -[\alpha(x) + \tau_2\beta(x)]\bar{\psi}(x)\gamma^0,\ \delta\pi(x) = 0,$$
$$\delta A_\mu(x) = \frac{1}{g}\partial_\mu\alpha(x),\ \delta\pi_A^\mu(x) = 0,$$
$$\delta B_\mu(x) = \frac{1}{g'}\partial_\mu\beta(x),\ \delta\pi_B^\mu(x) = 0. \quad (12)$$

正则 Lagrange 量密度 $\mathscr{L}^p$ 不变,其中 $\alpha(x)$, $\beta(x)$是无穷小函数,此变换的 Jacobi 行列式为 1. 而

$$\delta(\mu_k\Phi_k + \omega_l\Omega_l) = \omega_1\nabla^2\partial_0\frac{1}{g}\alpha(x) + \omega_2\frac{1}{g}\nabla^2\alpha(x) + \omega_3\nabla^2\partial_0\frac{1}{g'}\beta(x) + \omega_4\frac{1}{g'}\nabla^2\beta(x), \quad (13)$$

定义

$$\frac{\delta W[J]}{\delta J_\mu(x)} = B_c^\mu(x),\ \frac{\delta W[J]}{\delta I_\mu(x)} = A_c^\mu(x),$$

$$\frac{\delta W[J]}{\delta \bar{\eta}(x)} = \psi_c(x),\ \frac{\delta W[J]}{\delta \eta(x)} = -\bar{\psi}_c(x), \tag{14}$$

$$\frac{1}{\mathrm{i}}\frac{\delta}{\delta\eta(x)}\tau_a\frac{\delta}{\delta\bar{\eta}(x)}W[J] = G_a(x),$$

$$\frac{\delta W[J]}{\delta U_k(x)} = \mu_k(x),\ \frac{\delta W[J]}{\delta V_l(x)} = \omega_l(x).$$

由此可得

$$\frac{\delta W[J]}{\delta K_a(x)} = G_a(x) + \bar{\psi}_c(x)\tau_a\psi_c(x). \tag{15}$$

由 Legendre 变换,引入正规顶角生成泛函

$$\Gamma[\phi] = W[J] - \int \mathrm{d}^4x\{\bar{\psi}_c(x)\eta(x) + \bar{\eta}(x)\psi_c(x) + J_\mu(x)B_c^\mu(x) + I_\mu(x)A_c^\mu(x) + U_k(x)\mu_k(x) + V_l(x)\omega_l(x) + [G_a(x) + \bar{\psi}_c(x)\tau_a\psi_c(x)]K_a(x)\}, \tag{16}$$

那么,有

$$\frac{\delta\Gamma[\phi]}{\delta\psi_c(x)} = \bar{\eta}(x) + \bar{\psi}_c(x)\tau_a K_a(x),$$

$$\frac{\delta\Gamma[\phi]}{\delta\bar{\psi}_c(x)} = -\eta(x) - \tau_a\psi_c(x)K_a(x),$$

$$\frac{\delta\Gamma[\phi]}{\delta G_a(x)} = -K_a(x),\ \frac{\delta\Gamma[\phi]}{\delta B_c^\mu(x)} = -J_\mu(x), \tag{17}$$

$$\frac{\delta\Gamma[\phi]}{\delta\mu_k(x)} = -U_k(x),\ \frac{\delta\Gamma[\phi]}{\delta A_c^\mu(x)} = -I_\mu(x),$$

$$\frac{\delta\Gamma[\phi]}{\delta\omega_l(x)} = -V_l(x).$$

在(12)式的变换下,由正则形式的恒等式[13—15],得

$$-\partial_0\nabla^2\frac{1}{g'}\omega_3 + \nabla^2\frac{1}{g'}\omega_4 + \frac{\delta\Gamma[\phi]}{\delta\psi_c(x)}\mathrm{i}\tau_2\psi_c(x) + \bar{\psi}_c(x)\mathrm{i}\tau_2\frac{\delta\Gamma[\phi]}{\delta\psi_c(x)} + \frac{1}{g'}\partial^\mu\frac{\delta\Gamma[\phi]}{\delta B_c^\mu} - 2\epsilon_{2ab}\sigma_c^a(x)\frac{\delta\Gamma[\phi]}{\delta\sigma_c^b(x)} = 0. \tag{18}$$

这里引入 $\sigma^a(x) = aG^a(x)$来描述束缚态 $G^a(x)$,由于考虑了系统在相空间中存在约束,此时正则形式的恒等式与文献[4]中不同,出现附加的乘子场项,但如果对乘子场取 Gauss 平均,就不会有乘子的附加项[16]. 由(18)式对场量求泛函微商导致的诸 Green 函数的关系,与位形空间生成泛函求出的诸 Green 函数的关系相同. 这表明 Faddeev-Popov 方法对该模型适用,这里采用相空间的正则 Ward 恒等式,导出了从位形空间中的 Ward 恒等式给出的结果,其显著优点是勿需作出相空间对动量的路径积分,即可化到位形空间的生成泛函来讨论[13—16],由(18)式可得到一些关于二点顶角的 Ward 恒等式. 由于两点顶角和质量谱有关,这样就能得到费米子和规范场的质量谱. 对(18)式的 $\psi_c(x)$和 $\bar{\psi}_c(z)$求泛函微商,有

$$\bar{\psi}_c(x)\mathrm{i}\tau_2\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\bar{\psi}_c(x)} - \delta(x-z)\mathrm{i}\tau_2\frac{\delta^2\Gamma[\phi]}{\delta\psi_c(y)\delta\bar{\psi}_c(x)} - \delta(x-z)\frac{\delta^2\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(x)}\mathrm{i}\tau_2 + \frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\bar{\psi}_c(x)}\mathrm{i}\tau_2\psi_c(x) + \frac{1}{g'}\partial_\mu\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta B_c^\mu(x)} - 2\epsilon_{2ab}\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\sigma_c^a(x)}\sigma_c^b(x) = 0. \tag{19}$$

在没有外源时,即 $\psi_c(x) = \bar{\psi}_c(x) = 0$,并作 Fourier 变换得

$$\Gamma^{(2)}_{\psi,\bar{\psi}}\mathrm{i}\tau_2 - \mathrm{i}\tau_2\Gamma^{(2)}_{\psi,\bar{\psi}}(p+k) = 2\epsilon_{2ab}\Gamma^{(3)}_{\psi,\bar{\psi};\sigma_a}(p+k,-p;-k)\sigma_c^b - \frac{\mathrm{i}}{g'}k_\mu\Gamma^{(3)}_{\psi,\bar{\psi};B_\mu}(P+k,-p;-k). \tag{20}$$

让 $k_\mu\to 0$,让 $p\to 0$,由于只有凝结$\langle\bar{\psi}\tau_3\psi\rangle\neq 0$, Fermi 子质量能表示为

$$m_f = m_f^0 + \tau_3\delta m_f. \tag{21}$$

使用此性质,有

$$\delta m_f = -\tau_1 z_\psi\Gamma^{(3)}_{\psi,\bar{\psi};\sigma_1}(0,0;0)\sigma_c^3. \tag{22}$$

为了得到束缚态的质量谱,对(18)式的 $\sigma_s(y)$, $\sigma_t(z)$求导,类似于文献[4]的讨论,可得束缚态的质量谱为

$$m^2_{\sigma_1} = 0, \tag{23a}$$

$$m^2_{\sigma_3} = -z^{-1}_{\sigma_1}\Gamma^{(3)}_{\sigma_1,\sigma_2,\sigma_3}(0,0,0)\sigma_3. \tag{23b}$$

σ_1 对应于无质量的 Goldstone 玻色子,在(23b)中,用了关系

$$z_{\sigma_1} = z_{\sigma_3}. \tag{24}$$

此式由 $O(2)$对称性确定. 由于 σ_1 是无质量的 Goldstone 玻色子,能从自能中得到波函数的重正化常数 z_{σ_1} 为

$$z_{\sigma_1} = \frac{\mathrm{d}}{\mathrm{d}p^2}\Gamma^{(2)}_{\sigma_1}(p)\bigg|_{p^2=0}. \tag{25}$$

类似地对(18)式的 $B_c^\nu(y)$求导,做同样的讨论,有

$$\frac{\mathrm{i}}{g'}p_\mu\Gamma^{(2)}_{B_\nu,B_\mu}(p) = -2\sigma_c^3\Gamma^{(2)}_{B_\nu,\sigma_1}(p). \tag{26}$$

可以看出，如果没有费米子对凝聚，即$\langle\bar{\psi}\tau_3\psi\rangle = 0$则对称性保留，即

$$p_\mu\Gamma^{(2)}_{B_\nu,B_\mu}(p) = 0, \tag{27}$$

这表明规范场 B_μ 仅具有横分量.

若有费米子对凝聚，即$\langle\bar{\psi}\tau_3\psi\rangle \neq 0$，则

$$p_\mu\Gamma^{(2)}_{B_\nu,B_\mu}(p) \neq 0 \tag{28}$$

这表明规范对称性是动力学破缺的.(26)式两边同乘 $p_\nu^{-1} = \frac{p_\nu}{p^2}$，让 $p_\mu\to 0$，有

$$\frac{p_\mu p_\nu}{p^2}\Gamma^{(2)}_{B_\nu,B_\mu}(p) = \mathrm{i}2\frac{p_\nu}{p^2}\Gamma^{(2)}_{B_\nu,\sigma_1}(p)\sigma_c^3. \tag{29}$$

应用关系式

$$\lim_{p\to 0}\Gamma^{(2)}_{B_\nu,B_\mu} = -z_B\delta_{\mu,\nu}m_B^2 \tag{30}$$

得到规范玻色子 B_μ 的质量

$$m_B^2 = -\lim_{p\to 0}z_B^{-1}g'2\frac{p_\nu}{p^2}\Gamma^{(2)}_{B_\nu,\sigma_1}(p)\sigma_c^{(3)}. \tag{31}$$

这样就得到了与文献[4]相同的结果，显然，它是不同于 Higgs 机制的.

3 Jackiw-Johnson 模型

Jackiw-Johnson 模型被如下拉氏密度描述[2]

$$\mathscr{L} = \bar{\psi}\mathrm{i}\gamma\cdot\partial\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + gJ_{5\mu}A_\mu, \quad J_{5\mu} = \mathrm{i}\bar{\psi}\gamma_\mu\gamma_5\psi,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{32}$$

设 $\bar{\psi},\psi,A_\mu$ 的正则共轭动量分别为 $\pi,\bar{\pi},\pi^\mu$，则

$$\pi = \frac{\partial\mathscr{L}}{\partial\dot{\bar{\psi}}} = 0,\ \bar{\pi} = \frac{\partial\mathscr{L}}{\partial\dot{\psi}} = -\bar{\psi}\mathrm{i}\gamma^0,\ \pi^\mu = -F^{0\mu}. \tag{33}$$

初级约束为 $\Phi_1 = \pi\approx 0, \Phi_2 = \bar{\pi} + \bar{\psi}\mathrm{i}\gamma^0\approx 0, \Phi_3 = \pi^0\approx 0$.

$$\mathscr{H}_c = \dot{\psi}\bar{\pi} + A_\mu\pi^\mu - \bar{\psi}\mathrm{i}\gamma\cdot\partial\psi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - gJ_{5\mu}A_\mu = \frac{1}{2}\pi_i^2 + \frac{1}{4}F_{ij}F^{ij} - \bar{\psi}\mathrm{i}\gamma^i\partial_i\psi - gJ_{5\mu}A_\mu - A_0\partial_i\pi_i, \tag{34}$$

$$H_T = H_c + \mathrm{d}^3x\lambda^i\Phi_i. \tag{35}$$

初级约束 Φ_1,Φ_2 的自洽性条件给出乘子 $\lambda^1(x)$，$\lambda^2(x)$满足的方程. 次级约束为

$$\chi = \{\Phi_3, H_T\} = \{\pi^0, H_T\} = \partial_i\pi^i + gJ_{50}\approx 0, \tag{36}$$

此外，再无约束了.

不难验证 $\Phi_3 = \pi^0$ 为第一类约束；$\Phi_1 = \pi, \Phi_2 = \bar{\pi} + \bar{\psi}\mathrm{i}\gamma^0, \chi = \partial_i\pi^i + gJ_{50}$为第二类约束，它们的线性组合 $\Phi_4 = \partial_i\pi^i - g(\bar{\pi}\gamma_5\psi + \bar{\psi}\gamma_5\pi)$为第一类约束. 因此，第一类约束为 Φ_3,Φ_4. 第二类约束为 Φ_1,Φ_2. 对第一类约束相应地选取规范条件[15,16]

$$\Omega_1 = \partial_i\pi^i + \partial_i\partial_i A_0\approx 0,\ \Omega_2 = \partial_i A^i\approx 0. \tag{37}$$

由 F-S 量子化方案，对复合场也引入外源，略去与场量无关的项，则 Green 函数的生成泛函为

$$Z[J] = \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}\pi^\mu\delta(\Omega_1)\delta(\Omega_2)\cdot\delta(\Omega_3)\delta(\Omega_4)\delta(\Phi_1)\delta(\Phi_2)\times\exp\{\mathrm{i}\,\mathrm{d}^4x[\mathscr{L}^p + J_\mu A_\mu + \bar{\eta}\psi + \bar{\psi}\eta + \bar{\psi}\psi K + \bar{\psi}\gamma_5\psi K_5]\}\equiv \mathrm{e}^{\mathrm{i}W[J]}, \tag{38}$$

其中 J 代表所有的外源. $\mathscr{L}^p = \dot{\psi}\bar{\pi} + \dot{\bar{\psi}}\pi + \dot{A}_\mu\pi^\mu - \mathscr{H}_c$. 利用 δ 函数的性质，则上式可写为

$$Z[J] = \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}\pi^\mu\mathrm{D}\mu_k\mathrm{D}\omega_l\times\exp\left\{\mathrm{i}\int\mathrm{d}^4x[\mathscr{L}^p_{\mathrm{eff}} + J_\mu A_\mu + \bar{\eta}\psi + \bar{\psi}\eta + \bar{\psi}\psi K + \bar{\psi}\gamma_5\psi K_5 + U_k\mu_k + V_l\omega_l]\right\}\equiv \mathrm{e}^{\mathrm{i}W[J]}, \tag{39}$$

其中 $\mathscr{L}^p_{\mathrm{eff}} = \mathscr{L}^p + \mathscr{L}_m = \mathscr{L}^p + \omega_l\Omega_l + \mu_k\Phi_k$，$\mu_k,\omega_l$ 为乘子场.

在下列变换下

$$\begin{aligned}&\delta\psi(x) = \mathrm{i}\alpha(x)\gamma_5\psi(x),\ \delta\pi(x) = 0,\\ &\delta\bar{\pi}(x) = \bar{\psi}\alpha(x)\gamma_5\gamma^0;\\ &\delta A_\mu(x) = -\frac{\mathrm{i}}{g}\partial_\mu\alpha(x),\ \delta\pi_\mu(x) = 0.\end{aligned} \tag{40}$$

$\mathscr{L}^p$ 不变，且 $\delta(\omega_l\Omega_l + \mu_k\Phi_k) = -\frac{\mathrm{i}}{g}\nabla^2\partial_0\alpha(x) - \frac{\mathrm{i}}{g}\omega_2\nabla^2\alpha(x)$.

定义

$$\begin{aligned}&\frac{\delta W[J]}{\delta J_\mu(x)} = A_c^\mu(x),\ \frac{\delta W[J]}{\delta\bar{\eta}(x)} = \psi_c(x),\\ &\frac{\delta W[J]}{\delta\eta(x)} = -\bar{\psi}_c(x),\ \frac{\delta W[J]}{\delta U_k(x)} = \mu_k(x),\\ &\frac{\delta W[J]}{\delta V_l(x)} = \omega_l(x),\\ &-\frac{1}{\mathrm{i}}\frac{\delta}{\delta\eta(x)}\frac{\delta}{\delta\bar{\eta}(x)}W[J] = G(x),\\ &-\frac{1}{\mathrm{i}}\frac{\delta}{\delta\eta(x)}\gamma_5\frac{\delta}{\delta\bar{\eta}(x)}W[J] = G_5(x).\end{aligned} \tag{41}$$

由此得

$$\frac{\delta W[J]}{\delta K(x)} = G(x) + \bar{\psi}_c(x)\psi_c(x),$$

$$\frac{\delta W[J]}{\delta K_5(x)} = G_5(x) + \bar{\psi}_c(x)\gamma_5\psi_c(x). \qquad (42)$$

作 Legendre 变换,类似于第2部分的讨论,则得

$$\partial_0 \nabla^2 \omega_1(x) - \nabla^2 \omega_2(x) + \frac{\delta \Gamma[\phi]}{\delta \psi_c(x)} \frac{i}{2}\gamma_5 \psi_c(x) - \bar{\psi}_c(x)\frac{i}{2}\frac{\delta\Gamma[\phi]}{\delta\bar{\psi}_c(x)} - \frac{i}{2g}\partial^\mu \frac{\delta\Gamma[\phi]}{\delta A_c^\mu(x)} + \sigma_c(x)\frac{\delta\Gamma[\phi]}{\delta\lambda_c(x)} - \lambda_c(x)\frac{\delta\Gamma[\phi]}{\delta\sigma_c(x)} = 0, \qquad (43)$$

这里引进标量和赝标量场 $\sigma(x)$, $\lambda(x)$,即 $\sigma(x) = aG(x)$, $\lambda(x) = aG_5(x)$.

对(43)式关于 $\psi_c(y)$, $\bar{\psi}_c(z)$ 求泛函微商,可得

$$\delta(x-z)\frac{i}{2}\gamma_5 \frac{\delta^2\Gamma[\phi]}{\delta\psi_c(y)\delta\bar{\psi}_c(x)} - \delta(x-y)\frac{\delta^2\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(x)} \times \frac{i}{2}\gamma_5 - \bar{\psi}_c(x)\frac{i}{2}\gamma_5\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\bar{\psi}_c(x)} + \frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\bar{\psi}_c(x)}\frac{i}{2}\gamma_5\psi_c(x) - \frac{i}{2g}\partial_\mu\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta A_c^\mu(x)} - \frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\sigma_c(x)}\lambda_c(x) + \frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\lambda_c(x)}\sigma_c(x) = 0, \qquad (44)$$

此结果和文献[4]相同,在没有外源时,当选择如下破缺

$$\langle\bar{\psi}(x)\psi(x)\rangle \neq 0, \qquad (45a)$$

$$\langle\bar{\psi}(x)i\gamma_5\psi(x)\rangle \neq 0. \qquad (45b)$$

作 Fourier 变换,有

$$\frac{i}{2}\gamma_5\Gamma^{(2)}_{\psi,\bar{\psi}}(p+k) + \Gamma^{(2)}_{\psi,\bar{\psi}}\frac{i}{2}\gamma_5 = \frac{1}{2g}k_\mu\Gamma^{(3)}_{\psi,\bar{\psi};A_\mu}(p+k,-p;-k) + \Gamma^{(3)}_{\psi,\bar{\psi};\lambda}(p+k,-p;-k)\sigma_c, \qquad (46)$$

当 $k_\mu \to 0$,得费米子质量

$$m_f = z_\psi^{-1}\gamma_5\Gamma^{(3)}_{\psi,\bar{\psi};\lambda}(0,0;0)\sigma_c. \qquad (47)$$

z_ψ 为波函数的重正化常数. 同理,对(43)式中 A_c^μ 求导,类似可得

$$\sigma_c\Gamma^{(2)}_{A_\nu,\lambda}(p) - \frac{i}{2g}p_\mu\Gamma^{(2)}_{A_\nu,A_\mu}(p) = 0. \qquad (48)$$

由(48)式,可以看出,如果手征对称不是自发破缺,则规范场仅具有横分量. 由此可得规范场 A_μ 的质量

$$m_A^2 = \lim_{q\to 0} z_A^{-1} 2g\frac{q_\mu}{q^2}\Gamma^{(2)}_{A_\mu,\lambda}(q)\sigma_c. \qquad (49)$$

z_A 是重正化常数. 上面得到的结果和文献[4]的结果一致.

4 结论和讨论

本文从约束 Hamilton 系统理论出发,利用 F-S 量子化方法,给出了 Cornwall-Norton 和 Jackiw-Johnson 模型的路径积分量子化,基于约束 Hamilton 系统的正则 Ward 恒等式,分别导出了这两个系统的 Ward 恒等式,得到包括费米子和束缚态的质量谱,这样从另一个角度来研究,也给出文献[4]中相同的结果. 文献[4]是从位形空间中的生成泛函和 Ward 恒等式出发来讨论的,由于所研究的系统在相空间存在约束,本文用约束 Hamilton 系统理论,给出系统的路径积分量子化,得到相空间中 Green 函数的生成泛函,具有更一般的意义,上面所发展的应用正则 Ward 恒等式方法,可进一步推广到非 Abel 规范理论的动力学对称破缺研究中去. 特别是对于相空间路径积分中对动量不可积(或积分困难)的系统有重要意义.

参考文献(References)

1 Cornwall J M, Norton R E. Phys. Rev., 1973, **D8**:3338—3346
2 Jackiw R, Johnson K. Phys. Rev., 1973, **D8**:2386—2398
3 Nambu Y, Jona-Lasino G. Phys. Rev., 1961, **122**:345—358; 1961, **124**:246—254
4 SHEN Kun, KUANG Yu-Ping. Phys. Rev., 1998, **D57**:6386—6397
5 SHEN Kun, QIU Zhong-Ping. Phys. Rev., 1992, **D45**:3877—3879
6 SHEN Kun, QIU Zhong-Ping. Phys. Rev., 1993, **D48**:1801—1807
7 Baruch Rosoenstein, Brain J. Warr. Phys. Lett., 1989, **B219**:469—473
8 Eichten Estia J, Feinberg Frank L. Phys. Rev., 1974, **D10**:3254—3279
9 Heinz Pagels. Phys. Rev., 1980, **D21**:2336—2347
10 Cornwall John M, Jackiw R, Tomboulis E. Phys. Rev., 1974, **D10**:2428—2445
11 Lee T D, YANG C N. Phys. Rev., 1962, **128**:885—898
12 Schwinger J. Phys. Rev., 1962, **125**:397—398; 1962, **128**:2425—2429
13 LI Z P. High Energy Phys. and Nucl. Phys., 1994, **18**:265—273

14 LI Z P. High Energy Phys. and Nucl. Phys., 1995, **19**:405—412(in Chinese)
(李子平. 高能物理与核物理, 1995, **19**:405—412)
15 LI Z P. High Energy Phys. and Nucl. Phys., 2002, **26**:122—128(in Chinese)
(李子平. 高能物理与核物理, 2002, **26**:122—128)
16 LI Z P. High Energy Phys. and Nucl. Phys., 2002, **26**:230—238(in Chinese)
(李子平. 高能物理与核物理, 2002, **26**:230—238)
17 LI Z P. Int. J. Theor. Phys., 1995, **34**:523—543
18 LI Z P. Z. Phys., 1997, **C76**:181—189
19 Faddeev L D. Theor. Math. Phys., 1970, **1**:1—6
20 Senjanovic P. Ann. Phys., 1976, **100**:227—233

Canonical Ward Identity and Dynamical Mass Generation in Abelian Gauge Theory*

LONG Zheng-Wen[1;1)] LI Zi-Ping[2;2)]
1(Department of Physics, Guizhou University, Guiyang 550025, China)
2(Department of Applied Physics, Beijing Polytechnic University, Beijing 100022, China)

Abstract Based on theory of constrained Hamiltonian system, the Faddeev-Senjanovic Path-integral quantization for studying the quantization of Conwall-Norton and Jackiw-Johnson models were reexamined, the canonical Ward identities were derived. The mass spectra of both fermion and bound states were also obtained, which is in agreement with that results obtained by using another method.

Key words Canonical Ward identity, Constraints, Dynamical symmetry breaking, Abelian gauge theory

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1) E-mail: longshc@hotmail.com
2) E-mail: zpli@solaris.bjpu.edu.cn