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Electron Trapping in Multipole Magnet

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Abstract The electron cloud effect limits the performance of several accelerators with high beam current, such as SLAC and KEK B factories, the CERN SPS and the CERN PS. In this paper, the electron trapping in general $2n$ multipole magnet (n is integer) is studied, and we find that there exists electron trapping in the adiabatic region of the multiple magnet ($n>1$).

Key words electron cloud, guiding center motion, trapping

1 Introduction

An electron cloud is generated in the vacuum chamber by photoemission or beam induced mutipacting and subsequent electron accumulation during a bunch or bunch train passage. The electron cloud effect limits the performance of several accelerators with high beam current[1], such as SLAC and KEK B factories, the CERN SPS, the CERN PS. In the early days of the CERN ISR, the coupled oscillations of the proton and the trapped electron hindered the high current coasting beam operation. In 1997, an anomalous multibunch instability at CESR could be explained by the photoelectrons trapped in the pump leakage fields. In this paper, using Hamiltonian perturbation theory in noncanonical coordinates, we study the electron trapping in general $2n$ multipole magnet, and find that there exists electron trapping in the adiabatic region of magnet.

2 Hamiltonian perturbation theory in noncanonical coordinates[2]

An Hamiltonian phase space forms a natural symplectic manifold. In $2N+1$ dimensional extended phase, let us choose coordinates (q_i, p_i, t), $i=1,\cdots,N$. The Poincare-Cartan 1-form of particles is

$$\gamma = p_i \mathrm{d}q_i - H_c \mathrm{d}t, \tag{1}$$

where H_c is the Hamiltonian function, and the exterior derivative of γ is a 2-form

$$\omega = \mathrm{d}\gamma = \mathrm{d}p_i \wedge \mathrm{d}q_i - \mathrm{d}H_c \wedge \mathrm{d}t. \tag{2}$$

Without lossing generality, we choose the noncanonical variables $\{Z_\mu\}$, $\mu=1,\cdots,2N+1$, Usually $z_{2N+1}=t$, so the 1-form for particle is

$$\gamma = \gamma_i \mathrm{d}z_i - H\mathrm{d}t, \tag{3}$$

$$\gamma_i = \boldsymbol{p}\cdot\frac{\partial \boldsymbol{q}}{\partial z_i},\ H = H_c - \boldsymbol{p}\cdot\frac{\partial \boldsymbol{q}}{\partial t}.$$

Under the coordinate transformation $z=\{z_\mu\}\rightarrow \boldsymbol{Z}=\{Z_\mu\}$, γ becomes

$$\gamma = \Gamma_\mu \mathrm{d}Z_\mu, \tag{4}$$

$$\Gamma_\mu = \gamma_\sigma \frac{\partial z_\sigma}{\partial Z_\mu}.$$

For 1-form Lie perturbation, suppose γ can be expanded into series of a certain small dimensionless parameter ε

$$\gamma = \sum_{n=0}^{+\infty}\varepsilon^n \gamma^{(n)}. \tag{5}$$

Under the coordinate Lie transformation

$$\boldsymbol{Z} = Tz,$$
$$T = \cdots T_3 T_2 T_1,$$
$$T_n = \exp(\varepsilon^n L_n),$$

the 1-form $\gamma=\gamma_\mu \mathrm{d}z_\mu$ transforms according to the following law

$$\Gamma = T^{-1}\gamma + \mathrm{d}s, \tag{6}$$

$\Gamma=\Gamma_\mu \mathrm{d}Z_\mu$, and s represents the gauge transformation function in the phase space. Γ can be expanded into the series of ε

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$$\gamma = \sum_{n=0}^{+\infty} \varepsilon^n \Gamma^{(n)}, \tag{7}$$

$$\Gamma^{(0)} = \gamma^{(0)} + \mathrm{d}s_0, \tag{8}$$

$$\Gamma^{(1)} = \gamma^{(1)} - L_1 \gamma^{(0)} + \mathrm{d}s_1, \tag{9}$$

$$\Gamma^{(2)} = \gamma^{(2)} - L_1 \gamma^{(1)} + \left(\frac{1}{2}L_1^2 - L_2\right)\gamma^{(0)} + \mathrm{d}s_2. \tag{10}$$

Besides, the Euler-Largrangian equations for z_i can be derived from the 1-form $\gamma = \gamma_i \mathrm{d}z_i - H\mathrm{d}t$,

$$\omega_{ij}\frac{\mathrm{d}z_j}{\mathrm{d}t} = \frac{\partial H}{\partial z_i} + \frac{\partial \gamma_i}{\partial t}, \tag{11}$$

where ω_{ij} is the largrangian bracket, and ω_{ij} is invariant under the transformation $\gamma \to \gamma + \mathrm{d}s$.

3 Guiding center motion of electron in equilibrium magnetic field[3]

The motion of the charged particles in electromagnetic fields is one of the oldest topics. The trapping of particle in magnetic configuration and the particle diffusion depend on the long time behavior of particles. In order to integrate the long time particle motion, we need to make an expansion of the motion equations in gyroradius, and average out the rapid particles gyro phase motion to obtain the equations for the guiding center motion. The problem of the guiding center is essentially the perturbative solution of the motion of the charged particle in given electromagnetic field. In the following, we study the guiding center motion of the charged particles in equilibrium magnetic field using the 1-form perturbation method in the natural unit system.

The 1-form of charged particle in equilibrium magnetic field is

$$\gamma = (\boldsymbol{A}(\boldsymbol{x}') + \boldsymbol{v}) \cdot \mathrm{d}\boldsymbol{x}' - \frac{1}{2}v^2\mathrm{d}t, \tag{12}$$

$\boldsymbol{A}(\boldsymbol{x}')$ is the equilibrium magnetic vector potential, and $\boldsymbol{v}$ the velocity vector of particle. Expand γ into the series of the small parameter $\varepsilon_B \sim \frac{\rho}{L_B}$($\rho$ is the Larmor radius of particle, L_B is the inhomogeneity scale of magnetic field)

$$\gamma = \gamma_0 + \varepsilon_B\gamma_1 + \varepsilon_B^2\gamma_2 + \cdots, \tag{13}$$

$$\gamma_0 = \boldsymbol{A}\cdot\mathrm{d}\boldsymbol{x}', \tag{14}$$

$$\gamma_1 = \boldsymbol{v}\cdot\mathrm{d}\boldsymbol{x}' - \frac{1}{2}v^2\mathrm{d}t, \tag{15}$$

$$\boldsymbol{v} = u\hat{b} + w\hat{c}, \tag{16}$$

$$\hat{a} = \cos\theta\hat{e}_1 - \sin\theta\hat{e}_2, \tag{17}$$

$$\hat{b} = \frac{\boldsymbol{B}}{B},\ \boldsymbol{B} = \nabla\times\boldsymbol{A}, \tag{18}$$

$$\hat{c} = -\sin\theta\hat{e}_1 - \cos\theta\hat{e}_2, \tag{19}$$

where $\hat{e}_1, \hat{e}_2, \hat{b}$ form a local right hand orthogonal system, and $\theta = \mathrm{arctg}\frac{\boldsymbol{v}\cdot\hat{e}_1}{\boldsymbol{v}\cdot\hat{e}_2}$ is the gyro angle of particle motion. In order to remove the γ dependence on θ, we make Lie transformation $\gamma \to \Gamma$, Γ is

$$\Gamma = \Gamma_0 + \varepsilon_B\Gamma_1 + \varepsilon_B^2\Gamma_2 + \cdots, \tag{20}$$

where $\Gamma_0, \Gamma_1, \Gamma_2, \Gamma$ are

$$\Gamma_0 = \boldsymbol{A}\cdot\mathrm{d}\boldsymbol{x}, \tag{21}$$

$$\Gamma_1 = u\hat{b}\cdot\mathrm{d}\boldsymbol{x} - \left(\frac{1}{2}u^2 + \mu B\right)\mathrm{d}t, \tag{22}$$

$$\Gamma_2 = -\left\{\mu\left(\boldsymbol{R} + \frac{1}{2}\hat{b}(\hat{b}\cdot\nabla\times\hat{b})\right)\right\}\cdot\mathrm{d}\boldsymbol{x} + \mu\mathrm{d}\theta, \tag{23}$$

$$\Gamma = \left\{\boldsymbol{A} + u\hat{b} - \mu\left[\boldsymbol{R} + \frac{1}{2}(\hat{b}\cdot\nabla\times\hat{b})\hat{b}\right]\right\}\cdot \mathrm{d}\boldsymbol{x} + \mu\mathrm{d}\theta - \left(\frac{1}{2}u^2 + \mu B\right)\mathrm{d}t, \tag{24}$$

respectively, and $\mu = \frac{w^2}{2B}$, $\boldsymbol{R} = (\nabla\hat{e}_2)\cdot\hat{e}_1$.

4 Electron trapping in 2n multipoles

The guiding center motion equation for $\boldsymbol{x}$ and u can be derived from 1-form Eg.(24):

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \frac{\boldsymbol{B}^*}{\hat{b}\cdot\boldsymbol{B}^*}\frac{\partial H}{\partial u} + \frac{1}{\hat{b}\cdot\boldsymbol{B}^*}\hat{b}\times\nabla H, \tag{25}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{1}{\hat{b}\cdot\boldsymbol{B}^*}\boldsymbol{B}^*\cdot\nabla H, \tag{26}$$

where $\boldsymbol{B}^* = \nabla\times\left(\boldsymbol{A} + u\hat{b} - \mu\left[\boldsymbol{R} + \frac{1}{2}(\hat{b}\cdot\nabla\times\hat{b})\hat{b}\right]\right)$, $H = \frac{1}{2}u^2 + \mu B$. In the general multipoles expansion, $\boldsymbol{A}$ is independent of z coordinate, and $\boldsymbol{A} = A_z\hat{z}$. We choose $\hat{e}_2 = \hat{z}$, then $\boldsymbol{R}$ and $\hat{b}\cdot\nabla\times\hat{b}$ vanish, so the guiding center motion equation can be simplified as

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \hat{b}u + u^2\frac{\nabla\times\hat{b}}{B} + \frac{\mu}{B}\hat{b}\times\nabla B, \tag{27}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\mu}{B}\boldsymbol{B}\cdot\nabla B. \tag{28}$$

In cylindrical coordinate (r, φ, z), $\boldsymbol{x} = r\hat{r} + z\hat{z}$. Further, the guiding center motion equations can be expressed as

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \hat{b}u , \quad (29)$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = u^2 \frac{(\nabla \times \hat{b}) \cdot \hat{z}}{B} + \frac{\mu}{B}(\hat{b} \times \nabla B) \cdot \hat{z} , \quad (30)$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\mu}{B}\boldsymbol{B} \cdot \nabla B . \quad (31)$$

The field of normal $2n$ poles magnet is

$$\boldsymbol{B} = k_n r^{n-1}(\sin n\varphi\hat{r} + \cos n\varphi\hat{\varphi}), \quad (32)$$

After taking Eq.(31) into the Eqs.(28),(29),(30), the guiding center motion equations in the adiabatic region of magnet now are

$$\frac{\mathrm{d}r}{\mathrm{d}t} = u\sin n\varphi , \quad (33)$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{u}{r}\cos n\varphi , \quad (34)$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = (n-1)\cos n\varphi\left(-\frac{\mu}{r} - u^2\frac{1}{k_n r^n}\right) , \quad (35)$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\mu k_n (n-1) r^{n-2}\sin n\varphi . \quad (36)$$

Finally, from Eqs.(32),(33),(34),(35), in the adiabatic region of multipole magnet ($r > 0$, $n > 1$), we can get

$$\frac{\mathrm{d}z}{\mathrm{d}t} = (n-1)\frac{r_0^n}{k_n r^{2n}}(\mu B - 2E_0) , \quad (37)$$

where $E_0 = \frac{1}{2}u^2 + \mu B$ is the total energy of electron (constant), r_0 = constant. As for $n = 1$, $\frac{\mathrm{d}z}{\mathrm{d}t} = 0$, this is the dipole case. In this case, the electron trapping don't occur, many electrons are confined to the vicinity of the pipe wall and are lost quickly, neither gain significant amounts of energy nor directly harm the beam. But for $n > 1$, $\frac{\mathrm{d}z}{\mathrm{d}t} < 0$, the guiding center of electron drifts in a certain longitudinal direction, so electrons are trapped in the magnetic configuration until they drift out of the magnet. Hence, they may take some active role in the collective instabilities.

5 Discussions

In this paper, using the Hamiltonian perturbation theory in the noncanonical coordinates, we have studied the electron trapping in general $2n$ multipole magnet. Our theory only applies for the adiabatic region of magnet, i.e. in the region with $\frac{\rho}{L_B} \ll 1$, where the trapping occurs. In the other region of magnet, the motion of electron is more complex.

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多极场的电子云俘获效应研究

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摘要 电子云效应限制了几台加速器的高束流密度运行,例如SLAC和KEK的B工厂,CERN的SPS与PS. 本文运用辛流形上的1-form李摄动法研究了$2n$多极场的电子云俘获效应,结果发现在多极磁铁($n > 1$)的绝热区存在电子俘获.

关键词 电子云 引导中心 俘获

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