Paris $N\bar{N}$ Potential and the Possible Resonance States of $p\bar{p}$ *

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Abstract By solving the complex Schrödinger equation with the phenomenological Paris optical potential between nucleon(N) and antinucleon($\bar{N}$), the near-threshold binding and resonant behaviors of the p-$\bar{p}$ system are studied. As the result, a $^{13}P_0$ resonance state of $p\bar{p}$ is obtained. It is found that the energy and width of the state are compatible with the BES data in which the observed enhancement in the $J/\psi \to \gamma p\bar{p}$ decay was fitted by a Breit-Wigner (B-W) formula either in the S-wave or in the P-wave. The indication of the results and the suggestion for the further theoretical investigation are discussed.

Key words $N\bar{N}$ interaction, resonance state, optical potential

It is well known that the deuteron is an only loosely bound NN state. The properties of the state can accurately be measured in experiment. Whether a bound state or a resonant state of $N\bar{N}$ can be formed through the strong interaction is still an open problem. This is a long standing problem and there are many controversial experimental findings and theoretical claims. In fact, the theoretical investigation on the possible $N\bar{N}$ bound state started before the discovery of antiproton in 1955[1]. In 1949, Fermi and Yang[2] proposed that pion may be a tightly bound state of $N\bar{N}$. In 1961, Nanbu and Jona-Lasinio[3] found that based on their chiral symmetry model, there might exist not only a near zero-mass pion as the $N\bar{N}$ bound state but also a scalar resonance of twice proton mass. Soon after, it was noted[4] that the $N\bar{N}$ picture is difficult to explain the observed pattern of the meson spectrum. As stated by Sapiro et al. in 1970's[5], the $N\bar{N}$ states were no more associated with "ordinary" light mesons, but with a specific type of mesons. These mesons have masses near the value of the $N\bar{N}$ threshold and specific decay properties. They are called Baryonium. In 1980's, the operation of the Low Energy Antiproton Ring (LEAR) provided a good chance to study the $N\bar{N}$ interaction at the low energy. In particular, in terms of the high precision $N\bar{N}$ data provided by LEAR, one has been trying to understand NN and $N\bar{N}$ interactions. As a result, some key issues on underlying annihilation dynamics have been yielded and many phenomenological potential models for the $N\bar{N}$ system have been developed. Unfortunately, the early evidence for narrow states associated with monochromatic γ emission could not be confirmed. The narrow $N\bar{N}$ bound states in the mass range of 1.1—1.8GeV seem to be ruled out by the experiments[6].

Recently, a near-threshold enhancement in the proton-antiproton($p\bar{p}$) invariant mass spectrum of the radiative decay $J/\psi \to \gamma p\bar{p}$ observed by the BES collaboration[7] sheds new light on the investigation of the $N\bar{N}$ interaction at the low energy and re-attracts one's eyes on the binding behavior

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of $N\bar{N}$.

As suggested by the BES collaboration, this enhancement can be fitted by a weighted S-wave B-W function with a peak mass $M = 1859^{+3}_{-10}(\text{stat})^{+5}_{-25}(\text{sys})$ MeV and a total width $\Gamma < 30$MeV. Its spin and parity quantum numbers are $J^{PC} = 0^{-+}$. This enhancement may also be fitted by a P-wave B-W function with a mass $M = 1876.4 \pm 0.9$MeV and a width $\Gamma = 4.6 \pm 1.8$MeV. The corresponding quantum numbers are $J^{PC} = 0^{++}$. Up to now, no well-established mesons can be associated with such states. In fact, the Belle collaboration has also reported similar phenomena, namely the enhancement in the $p\bar{p}$ invariant mass distribution near $2m_p$, in the $B^+ \to K^+ p\bar{p}$[8] and $\bar{B}^0 \to D^0 p\bar{p}$[9] decays.

These phenomena have provoked various theoretical speculations. In Ref. [10], possible quantum numbers and decay modes of the state was discussed. Based on the general symmetry consideration and available experimental information, the authors concluded that the quantum number of the state is very likely to be $J^{PC} = 0^{-+}$, $I^G = 0^+$. By use of the toy model[11], the enhancement was explained by either an S-wave bound state or a P-wave resonance of $p\bar{p}$. In the linear σ model study[12], the resultant binding energy and the lifetime of such a state were consistent with the BES result. The effect of the final state interaction through one pion exchange was studied in Ref. [13]. As a result, the enhancement can be partly reproduced. In terms of the scattering lengthes obtained by fitting the LEAR $p\bar{p}$ scattering data, both the enhancement in the $J/\psi \to \gamma p\bar{p}$ decay and the flat behavior in the $J/\psi \to \pi^0 p\bar{p}$ decay can be explained[14]. By employing a Skyrmion-type potential, the resultant $p\bar{p}$ bound state with 0^{-+} was consistent with the BES finding[15]. In the constituent quark model study, an S-wave bound state with $I = 1$ and $J^{PC} = 0^{-+}$ was obtained by using the resonating group approach[16]. Moreover, in Ref. [17], it is shown that the enhancement can be fitted as a cusp. Although the enhancement in the $J/\psi \to \gamma p\bar{p}$ can more or less be explained, a critical explanation is still needed.

In this paper, we study the possible resonant behavior of $p\bar{p}$ from another angle. Among some well-known phenomenological $N\bar{N}$ potentials[18] which well-describe a huge amount of highly accurate scattering data of $N\bar{N}$, we select the Paris optical potential[19] to serve this aim.

In 1982, based on the Paris NN optical potential, Cote et al.[19] constructed an optical potential, called Paris $N\bar{N}$ potential, to describe the low energy $N\bar{N}$ interaction. The real part of the potential is obtained by the G-parity transformation of the Paris NN potential in the long- and medium-ranges supplemented with a phenomenological short-range part. The multi-channel effect of the $N\bar{N}$ interaction is effectively described by the imaginary part of the optical potential, namely the absorption potential, which is energy and state dependent.

In terms of the Paris $N\bar{N}$ potential[20—22], the poles of the S-matrix was studied, the scattering phase shifts were calculated and the possible $p\bar{p}$ resonances and bound states were obtained via Argand diagrams. Recently, Loiseau et al.[23] claimed that either the near threshold quasi-bound state $^{11}S_0$ or the well-established $^{13}P_0$ resonance can describe the $J/\psi \to \gamma p\bar{p}$ data obtained by the BES collaboration.

Now, by using the Paris $N\bar{N}$ potential in various versions, we calculate the masses and the widthes of the possible quasi-bound states and resonances of the $p\bar{p}$ system.

The general form of the Paris $N\bar{N}$ potential is written as

$$V_{N\bar{N}} = U_{N\bar{N}} - \mathrm{i}\,W_{N\bar{N}}. \tag{1}$$

The details of the real part potential $U_{N\bar{N}}$ and the imaginary part potential $W_{N\bar{N}}$ can be found in the appendix of the Ref. [21]. The major improvement in newer versions is the parameter values fixed by the newly obtained data especially for the short range interaction. Considering the BES results mentioned above, we only study the binding behavior of the $^{11}S_0$ and $^{13}P_0$ states. The corresponding in the $^{11}S_0$ state potential curve and the potential curve with the centrifugal barrier in the $^{13}P_0$ state, are plotted in Figs. 1 and 2, respectively. In these figures, the solid, dashed and dotted curves represent the Paris94, Paris99 and Paris03 potentials, respectively. The general feature of these phenomenological $N\bar{N}$ potentials is that they have very strong strengths and relative narrow widths, especially for the imaginary parts.

With these complex potentials, we solve the Schrödinger equation for the $^{11}S_0$ and $^{13}P_0$ states of the $p\bar{p}$ system, numerically. The reduced radial equation for such a problem reads

$$u''(r) + \left[k^2 - \frac{l(l+1)}{r^2} - v(r)\right]u(r) = 0, \tag{2}$$

with the boundary condition

$$\begin{cases} u(0) = 0, \\ u(R)\mathscr{O}'_l(kR) - u'(R)\mathscr{O}_l(kR) = 0. \end{cases} \tag{3}$$

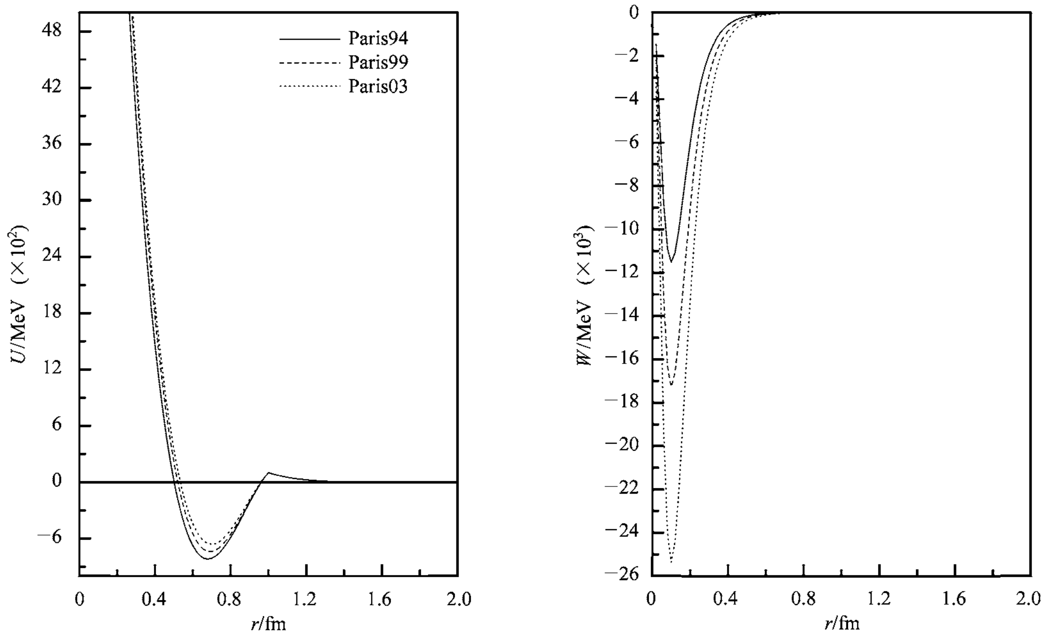

Fig.1.　Paris potential in $^{11}S_0$ state. U and W denote the real and imaginary parts of the potential.

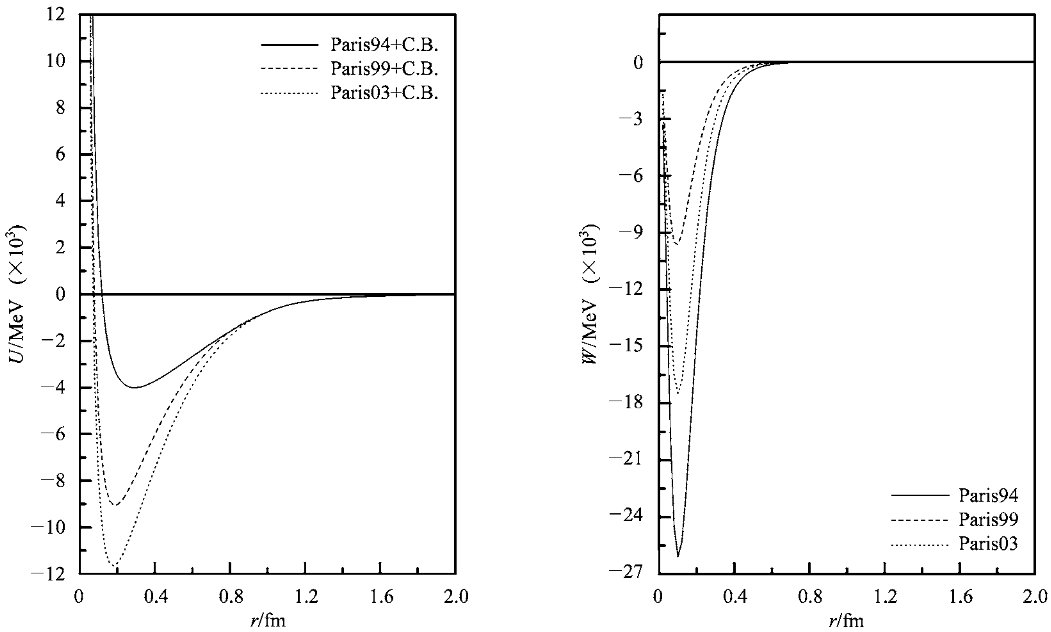

Fig.2.　Paris potential in $^{13}P_0$ state. U and W denote the real and imaginary parts of the potential.

In the equation, $k^2 = 2\mu E/\hbar^2$, $v(r) = 2\mu V_{N\bar{N}}(r)/\hbar^2$, μ stands for the reduced mass of the N-$\bar{\text{N}}$ system and R is a distance beyond the range of the $N\bar{N}$ potential. The outgoing Coulomb wave can be written as

$$\mathscr{O}_l = \mathscr{G}_l + \mathrm{i}\mathscr{F}_l, \qquad (4)$$

with $\mathscr{G}_l$ and $\mathscr{F}_l$ being the irregular and regular Coulomb functions, and the asymptotic behavior of $\mathscr{O}_l$ can be expressed as

$$\mathscr{O}_l(kR) \sim \exp[\mathrm{i}(kR - \eta \ln 2kR - l\frac{\pi}{2} + \sigma_l)], \qquad (5)$$

with η being the Sommerfeld parameter and σ_l being the Coulomb phase shift.

The results for the $^{11}S_0$ state and $^{13}P_0$ state are listed in Tables 1 and 2, respectively. For comparison, the results given by the Paris group are also listed in the tables.

Table 1. The results for the quasi-bound state $^{11}S_0$.

	Paris03		Paris99		Paris94	
	E_{Re} /MeV	width /MeV	E_{Re} /MeV	width /MeV	E_{Re} /MeV	width /MeV
Paris group	-4.8	52.5	-69	46	none	none
our results	none	none	none	none	none	none

Table 2. The results for the resonant state $^{13}P_0$.

	Paris03		Paris99		Paris94	
	E_{Re} /MeV	width /MeV	E_{Re} /MeV	width /MeV	E_{Re} /MeV	width /MeV
Paris group	1873	10.7	1876	4.8	1876	10.4
our results	1877	4.3	1880	30.8	1881	31

From above tables, we see that although there are some differences between ours and the Paris group's, both of them are compatible with the BES data. However, in our calculation where the Schrödinger equation is solved, no $^{11}S_0$ quasi-bound state solution can be obtained, but in the Paris group result obtained via the analysis of the pole property of the S-matrix, a quasi-bound state $^{11}S_0$ for either Paris99 or Paris03 potential is obtained. The resultant mass and width of the $^{11}S_0$ state is close to the values given by BES. Moreover, in both calculations conducted in different ways, a resonant state $^{13}P_0$ in the case of either Paris94 or Paris98 or Paris03 version of potential can be obtained. Although the mass and width of the state in our case is slightly larger than those obtained by the Paris group, these results are still close to those given by BES. We provide following arguments to explain the difference.

Firstly, the method used for investigating the binding behavior of the $p\bar{p}$ system is different. In the Paris group case, the properties of the possible state are obtained by analyzing the property of the $p\bar{p}$ scattering with various versions of the Paris $N\bar{N}$ potential. In our case, a stationary bound state Schrödinger equation with the complex Paris potential is solved for the binding behaviors of the $p\bar{p}$ system.

However, due to the existence of the complex potential, the hermitian condition of the Hamiltonian, the normalization condition of the wave function and the conservation of the probability are all broken. Thus, the states studied are no longer stable and generally cannot be described by the stationary equation. These states are sometimes called as the quasi-stationary states. The pioneer work about the quasi-bound state was the theory of the α-particle decay of heavy nuclei by Gamow[24] and by Gorney and Condon[25]. In that theory, the asymptotic solution of the Schrödinger equation of the quasi-bound state given by Gamow is $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$. When energy has a complex value, namely $E = E_{Re} - i\frac{\Gamma}{2}$, the wave function increases exponentially with increasing r. Hence the validity of Gamow's procedure was criticized. Zeldovich[26] developed a regularization method, generalized the normalization condition in the perturbation sense and proved that the Gamow's procedure was valid only when $\Gamma \ll E_{Re}$ (the detailed discussion can be found in Ref.[27]). He also pointed that the theory of quasi-stationary state is far to be complete.

In our case, the numerical method used is based on the "Gamow Function"[28]. Thus, the consistency of the solutions by different methods is conditional.

The stability of the numerical solution in solving the Schrödinger equation for quasi-bound state is another problem. It comes from two sources. The first one is the boundary condition at large value of r. As mentioned above, the Gamow function is an exponentially increased function. When Γ is not much smaller compared with E_{Re}, the real part of the wave function becomes extraordinarily large at a moderately large r, and consequently the numerical result becomes unstable due to loss of significant figures. Then the choice of R_M where the strong interaction is sufficiently small so that it can safely be neglected would evidently affect the resultant mass and width. The second one is the extraordinary behavior of the Paris $N\bar{N}$ potential. The strengths of the potentials, including both real and imaginary parts, are usually in the order of 10GeV. It also makes the calculation unstable.

In order to fit scattering data of $N\bar{N}$, the Paris potential is state and energy dependent. Because the mass of state studied is close to the $p\bar{p}$ threshold energy, the energy dependence of the potential is not so serious. Therefore, in studying quasi-bound state, we neglect the tiny energy dependence in the potential. It also causes the inconsistency in solving the Schrödinger equation.

It should be mentioned that in the strict sense of the term, the potential should be derived from a fundamental theory. However, due to the complexity of the annihilation processes, one cannot fully include these channels. As an approximation, an imaginary part of the potential and a phenomenological short-range interaction are employed to compensate such an effect. Therefore, the short-range part of the

Paris $N\bar{N}$ potentials in various versions are quit different. It should also cause the difference in result. Unfortunately, in the results, the effect of different short-range interactions is not clearly shown. This might be due to the uncertainty of the numerical calculation, or because in the quasi-binding case, the inter-baryon interaction in the distance around a baryon radius is more important. Obviously, to obtain a definite understanding of the $p\bar{p}$ interaction, a complete calculation based on the QCD theory is desired.

In summary, by using the Paris $N\bar{N}$ potential which well describes a huge amount of $p\bar{p}$ scattering data at low energies, a resonant state $^{13}P_0$ which is near the threshold of $p\bar{p}$ is obtained by solving the Schrödinger equation. The calculated mass and width of the state is compatible with the result given by the Paris group and the P-wave fitting result obtained by the BES collaboration. In our calculation, we do not find any quasi-bound state solution in the $^{11}S_0$ channel, which is also consistent with the result in the Paris94 case. Although the results in our calculation and in the Paris group's calculation are basically compatible with the BES data, whether the threshold enhancement can be explained as a quasi-bound state is still doubtable.

In our opinion, if we use a three-body-decay mechanism $J/\psi \to \gamma p\bar{p}$ plus the final state p-$\bar{p}$ interaction which can explain existent low energy p-$\bar{p}$ scattering data can explain the threshold enhancement in the $J/\psi \to \gamma p\bar{p}$ decay, such an enhancement should be due to the final state p-$\bar{p}$ interaction. Otherwise, a cascade decay process $J/\psi \to \gamma X$, then $X \to p\bar{p}$ combined with the final state p-$\bar{p}$ interaction should be further considered. If this mechanism can explain the enhancement, the X state should be a resonantly produced quasi-bound $p\bar{p}$ state or resonant $p\bar{p}$ state. This is our undergoing calculation.

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$N\bar{N}$ 巴黎势和可能的 $p\bar{p}$ 共振态*

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摘要 通过求解 $N\bar{N}$ 巴黎光学势的复薛定谔方程,研究了 $N\bar{N}$ 系统的近阈束缚和共振行为,得到了一个 $p\bar{p}$ 系统的 $^{13}P_0$ 共振态.发现其能量和宽度与最近 BES 用 Breit-Wigner(B-W)公式分析 $J/\psi\to\gamma p\bar{p}$ 衰变的实验数据给出的结果相容.讨论了这一结果的含义及进一步研究的建议.

关键词 $N\bar{N}$ 相互作用 共振态 光学势

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