# Study on High Order Modes of a $\beta$=0.45, $f$=350MHz Spoke Cavity*

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**Abstract** Intensive study has been done on low and medium energy section of proton and heavy ion accelerators. Many experiments show that spoke cavity is a very good candidate to connect low beta RFQ (radio frequency quadrapole) structure and high beta elliptical accelerating structure. A $\beta$=0.45, $f$=350MHz spoke cavity has been designed and a copper model spoke cavity has been fabricated at Peking University. In this paper, we put the emphasis on the analysis of HOMs (high order modes) of the spoke cavity.

**Key words** spoke cavity, high order mode, MAFIA program

## 1 Introduction

Compared to elliptical cavity, spoke cavity has lots of advantages. When beta value is lower than 0.4, elliptical cavity can hardly be used because of its poor mechanical properties, but spoke cavity works very well, e.g. the gradient of LANL (Los Alamos National Lab) $\beta$=0.175 spoke cavity reaches 11.6MV/m at 4K[1]. At medium $\beta$(about 0.5), though both of them can be chosen, spoke cavity can provide excellent mechanical stability, work at 4.2K in stead of 2K, and greatly decrease the heat load. So many big projects choose spoke cavity in stead of elliptical cavity in the medium energy section.

However, spoke cavity is a new developing accelerating structure. There are still lots of problems (e. g. multipacting, high order modes) to be studied. Since 2004, Peking University (PKU) began to study spoke cavity.

## 2 Design of a spoke cavity

According to some basic requirements, we have designed a $\beta$=0.45, $f$=350MHz spoke cavity. The length of a 2-gap spoke resonator is determined by the structure's $\beta$ and the desired mode of operation[2]: the length over both gaps is $2/3\,\beta\,\lambda$, the distance from gap-center to gap-center is $\beta\,\lambda/2$. The diameter of the cavity is of an order of $\lambda/2$. We take the cavity aperture radius as 3.0cm.

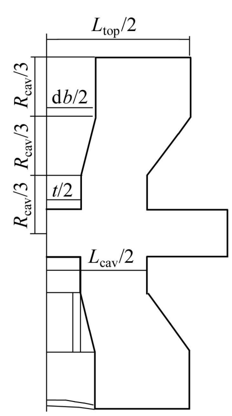


Fig. 1. Cross-section of the half spoke cavity.

The RF design of the spoke cavity is made by MICROWAVE STUDIO and MAFIA program[3]. In order to obtain low $E_p/E_{acc}$ and $B_p/E_{acc}$, the base of the spoke is taken as cylindrical and the cross section

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of the spoke at the center is race-rack. Fig. 1 is the cross-section of the half spoke cavity. Table 1 gives the frequency and geometry parameters of the spoke cavity, and Table 2 gives the RF parameters of the cavity[4].

Table 1. Dimensions of spoke cavity.

| $L_{cav}$/mm | $L_{top}$/mm | $L_{fl}$/mm | $G$/mm | $rbm$/mm | $t$/mm | $d$/mm | d$b$/mm | $R_{cav}$/mm | Frequency/MHz |
|---|---|---|---|---|---|---|---|---|---|
| 257 | 358 | 450 | 193 | 30 | 85.7 | 70 | 120 | 218 | 352 |

$L_{cav}$: cavity length, $L_{top}$: top cavity length, $L_{fl}$: flange to flange length, $G$: gap center to gep center, $rbm$: radius of the beam tube, $t$: spoke thickness at center, d$b$: spoke diameter at base, $R_{cav}$: cavity radius.

Table 2. RF parameters of $\beta$=0.45, $f$=350MHz spoke cavity.

| $F$/MHz | $E_p/E_{acc}$ | $B_p/E_{acc}/(\mathrm{G\cdot MV^{-1}\cdot m})$ | $r/Q/\Omega$ | $T$@$\beta$=0.45 | $Q(R_s$=60n$\Omega)$ | $P_{cw}$(4K)@7.5MV/m |
|---|---|---|---|---|---|---|
| 350 | 2.90 | 87 | 251 | 0.852 | $1.65\times10^9$ | 8.99W |

The electromagnetic parameters show that the spoke cavity has good accelerating performance. Its high shunt impedance means the spoke cavity needs much lower heat load than elliptical cavity.

## 3 Simulation of high order modes

Spoke cavity is coaxial like resonator, and its accelerating mode is TEM (transverse electromagnetic) mode, but it also has many high order modes (TE (transverse electric) modes or TM (transverse magnetic) modes). For high current beams, the study of HOMs is important. First we carried out some calculations of the high order modes with MAFIA program.

From the consideration of the danger to the beam and the dissipated power of the high order modes, we study the modes with frequency lower than $3f_0$ ($f_0$ is the frequency of the accelerating mode). Table 3 lists the results of 13 modes from MAFIA calculation.

M1 is the accelerating mode and the other 12 high order modes are divided into three types. 1) There is an electric field along the center axis of the beam accelerating gaps; 2) The field along the center axis of the accelerating gaps is very weak, but there is a longitudinal electric field away from the center axis; 3) The electric field among the whole accelerating gaps is very weak.

The first type has 4 modes: M2, M4, M11 and M13. M2 has a relative high $r/Q$ (9.2Ω). As the frequency increases, the $r/Q$ of the modes M4, M11, M13 decreases. M2 and M4 have high electric field along the axis, and should be excited easily; M11 and M13 has weak field along the axis and should not be excited easily. Most modes belong to the second type, e.g. M3, M6, M8, M9, M10 and M12. $r/Q$ of these modes are very small ($10^{-3}$—$10^{-2}\Omega$). Here we will use $V_c^*$ or $(r/Q)^*$ (explanation seen in Table 3), the $(r/Q)^*$ of M6, M8 and M3 are 20.5Ω, 16.7Ω, and 6.4Ω, respectively. $(r/Q)^*$ for the other modes are smaller than 2Ω. Modes with high $(r/Q)^*$ value can be excited easily. The third type has M5 and M7. Among the whole accelerating gaps, the electric field is very weak, so both their $r/Q$ and $(r/Q)^*$ are very small and these modes can not be easily excited.

Table 3. RF parameters of 13 modes calculated from MAFIA.

| mode | $F$ /MHz | $V_c$ /V | $V_c^*$ /V | $Q_0$ $(\times10^4)$ | $r/Q$ /Ω | $(r/Q)^*$ /Ω |
|---|---|---|---|---|---|---|
| M1 | 352.2 | 1.565 | | 2.04 | 251 | |
| M2 | 442.0 | 0.344 | | 3.169 | 9.62 | |
| M3 | 562.7 | $8.42\times10^{-3}$ | 0.317 | 2.91 | $8.7\times10^{-4}$ | 6.4 |
| M4 | 651 | 0.226 | | 3.27 | 2.8 | |
| M5 | 723.9 | 0.136 | | 2.88 | 0.9 | |
| M6 | 779.3 | 0.025 | 0.667 | 3.72 | $\sim0$ | 20.5 |
| M7 | 821 | $2.82\times10^{-3}$ | | 3.07 | $\sim0$ | |
| M8 | 837.5 | 0.021 | 0.623 | 4.13 | $\sim0$ | 16.7 |
| M9 | 865.5 | $7.55\times10^{-3}$ | 0.220 | 4.23 | $\sim0$ | 1.8 |
| M10 | 869.9 | $6.40\times10^{-3}$ | 0.170 | 3.34 | $\sim0$ | 1.2 |
| M11 | 996.7 | 0.146 | 0.208 | 3.91 | 0.77 | |
| M12 | 1006.4 | $8.07\times10^{-4}$ | 0.178 | 4.37 | $\sim0$ | 1.13 |
| M13 | 1067.3 | 0.147 | 0.256 | 2.88 | 0.73 | |

For some modes, the electric field along the beam center axis is almost zero, so the voltage is 0. Here we integrate the electric field along the axis 30mm away from the beam center and get $V_c^*$, and the effective shunt impedance is $\left(\dfrac{r}{Q}\right)^*=\dfrac{V_c^{*2}}{\omega U}$.

According to $r/Q$ or $(r/Q)^*$, we put our emphasis on 4 dangerous modes: M2, M8, M6 and M4. In next section we will testify this through experiments. The good news is that there are no TM modes (electric field perpendicular to beam tube axis) according to the calculation. So the beam loss induced by transverse force of TM modes is avoided.

## 4 RF measurements of the resonant modes

RF parameters (frequency $f$, quality factor $Q_0$, field distribution) of all the modes are measured by computer-controlled vector network analyzer (see in Fig. 2). Quality factor is derived from loaded quality factor and coupling factor. Field distribution along the axis is measured through bead perturbation.

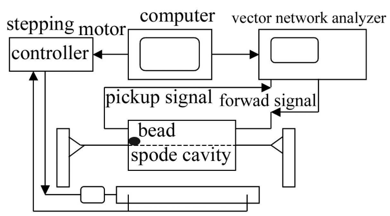


Fig. 2. Block diagram of test bench for RF measurements of the resonant modes.

In order to excite the second type modes, we did not put the antennas in the center of the beam tube, but 5mm away from the axis (parallel to the axis). The position of pickup antenna to the center axis has the same direction as that of input antenna to the center axis. Because different modes have different field distribution, some modes have strong longitudinal field at the excursion in $y$ axis, but some others at the excursion in $z$ axis ($x$ axis is the beam direction). Therefore, we choose excursion of the antennas in two different directions to excite the high order modes. Fig. 3 shows the frequency sweeping results when the antennas are at the excursion in $y$ axis and $z$ axis. Table 4 and Table 5 give the RF parameters.

Table 4. Parameters of the excited modes when the antennas are at the excursion in $y$ axis.

| mode | measured $f$/MHz | calculated $f$/MHz | $Q_0$ | $Q_0$ from MAFIA |
|---|---|---|---|---|
| M1 | 351.8 | 352.2 | 4363 | 20400 |
| M2 | 440.1 | 442.0 | 7728 | 31700 |
| M4 | 651.3 | 651.0 | 4163 | 4163 |
| M8 | 838.4 | 837.5 | 11426 | 41300 |
| M10 | 870.8 | 869.9 | 6024 | 33400 |

Table 5. Parameters of the excited modes when the antennas are at the excursion in $z$ axis.

| mode | measured $f$/MHz | calculated $f$/MHz | $Q_0$ | $Q_0$ from MAFIA |
|---|---|---|---|---|
| M1 | 351.8 | 352.2 | 4933 | 20400 |
| M2 | 440.1 | 442.0 | 6636 | 31700 |
| M4 | 651.3 | 651.0 | 4676 | 32700 |
| M5 | 722.1 | 723.9 | 5780 | 28800 |
| M6 | 776.7 | 779.3 | 10894 | 37200 |
| M8 | 838.4 | 837.5 | 11802 | 32700 |
| M9 | 863.2 | 865.5 | 4966 | 42300 |
| M10 | 870.8 | 869.9 | 7610 | 33400 |

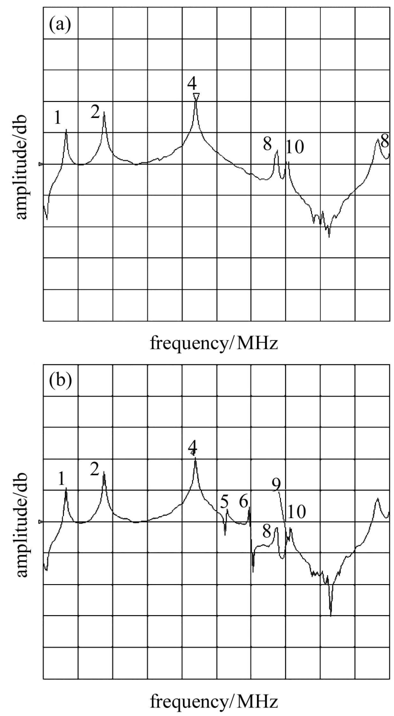


Fig. 3. (a) Sweeping result when the antennas are at the 5mm excursion in $y$ axis; (b) Sweeping result when the antennas are at the 5mm excursion in $z$ axis.

We can see that the measured frequencies coincide with the calculated frequencies. Because the model cavity is not clean and there are also many errors of the instruments during measurements, the measured quality factor is much smaller than the calculated one.

The field distributions of the excited modes are measured with bead perturbation method. According to the perturbation equation of metallic bead[5]

$$\frac{\omega_0'-\omega_0}{\omega_0}=-\frac{3}{4}\frac{\Delta V}{W_0}\left(\varepsilon E^2-\frac{1}{2}\mu H^2\right), \qquad (1)$$

Where, $\varepsilon$ and $\mu$ are the permittivity and permeability inside the cavity. $\omega_0$ and $\omega_0'$ are the frequencies before and after the perturbation. $W_0$ is the average stored energy. From Eq. (1), we know that frequency shifts when a perturbing bead is inside the cavity. When the bead moves along the axis, we can get the frequency shift in this direction. From Eq. (2), we can get the field distribution along the axis. The field distribution in our experiment is achieved by measuring the phase shift, because the frequency shift is proportional to the phase shift around the resonant frequency[6].

$$E(z)=\left[\frac{\Delta f(z)}{\Delta f_{\mathrm{p}}}\right]^{1/2}E_{\mathrm{p}}. \tag{2}$$

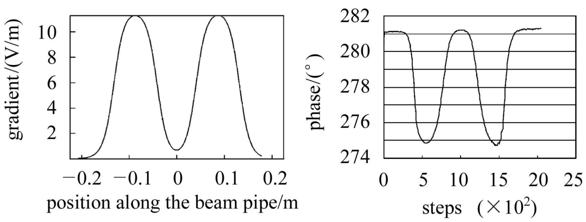


Fig. 4. (Left)calculated field distribution along the axis for M2; (Right) Measured field distribution along the axis for M2.

Through the field distribution along the beam axis, we confirm the previously mentioned modes (M2, M4, M6 and M8) are easily excited. The measured field distribution is coincident with the calculated one from MAFIA program. Here we just give one example. Fig. 4 clearly show the coincidence.

## 5 Conclusion

A $\beta$=0.35, $f$=350MHz spoke cavity has been designed at PKU. The RF parameters of the TEM mode show that the cavity has a good performance. To study the high order modes of spoke cavity, we fabricated a copper model cavity and measured the parameters of all the modes and the field profiles of the excited modes. The measurements have the coincident results to the simulation, confirming the dangerous modes derived from the calculation. This work can be a theoretical guide to the next HOM damping work.

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# $\beta$=0.45, $f$=350MHz spoke腔高阶模的研究*

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**摘要**　质子或者离子在中低能段的加速结构是目前研究的热点，其中许多实验结果表明spoke腔是连接RFQ加速结构和椭球腔加速结构的最具潜力的桥梁. 但是这种新型结构仍然存在许多问题有待解决，为此，北京大学开展了spoke型质子加速腔的研究，完成了$\beta$=0.45，频率为350MHz spoke腔的设计和铜模型腔的加工. 重点对spoke腔高阶模进行了分析，模拟计算找出了几种危害模式，并完成了实验测量，取得了实验测量与模拟计算相一致的结果.

**关键词**　spoke腔　高阶模　MAFIA程序

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