Multiple-Scale Perturbation Theory of Generalized Anharmonic Oscillator

CHENG Yan-Fu[1)] DAI Tong-Qing

(College of Electronic and Information Engineering, South-Central University for Nationalities, Wuhan 430074, China)

Abstract Classical and quantum oscillator of generalized anharmonicity is solved analytically up to the linear power of ε by using the multiple-scale perturbation method. The commutation relation of position and momentum operator can be simplified easily and the quantum solutions transformed into the classical form conveniently under the extreme conditions, which are different from the earlier multiple-scale perturbation theory. Moreover compared with the Taylor series solution, the frequency shifts in our solutions appear in the expression of oscillations of all orders in both classical and quantum cases, so multiple-scale perturbation method is more suitable for solving the weak-coupling anharmonic oscillation than the Taylor series approach.

Key words generalized anharmonic oscillator, multiple-scale perturbation theory, classical and quantum solution

1 Introduction

There are lots of problems in mathematical physics which are solved by using more than one methods. The anharmonic oscillator is perhaps the most common example among them. The equation of motion of a classical quartic anharmonic oscillator of unit mass and unit frequency is given by

$$\ddot{x}+x+\varepsilon x^3=0, \tag{1}$$

where ε is the anharmonic constant. Eq. (1) is termed as Duffing equation with out forcing and the exact solution in phase plane is available in terms of the elliptic functions. For positive anharmonic constant ($\varepsilon>0$), the solution is periodic with a fixed center whose period can be obtained in terms of the elliptic functions. However, this approach is not useful in several occasions. For example, the trajectory of a classical particle executing quartic anharmonic motion at a later time t is not determined from the initial conditions. To have more insight about the physical behavior, various approximate methods are proposed to obtain the solution of a classical quartic anharmonic oscillator[1, 2]. Straightforward application of perturbation theory to Eq. (1) gives rise to secular terms that increase unboundedly with time[1, 2] even for periodic motion. There are several methods that enable one to correct such unphysical behavior of the approximate solutions; among them[1, 2] the Lindstedt-Poincare technique, the method of renormalization, the multiple-scale perturbation theory and the Taylor series method. The quantum anharmonic oscillator problem is even more difficult due to the noncommuting nature of position and momentum operators. The first solution of a quantum quartic oscillator was given by Bender and Bettencourt using the multiple-scale perturbation theory[3, 4]. They obtained the frequency shift of the oscillator proportional to ε . This solution is equivalent to that of the first-order calculation under usual perturbation theory. Later on Auberson[5] obtained the second-order (ε^2) solution applying the multiple-scale perturbation theory

Received 27 February 2006, Revised 18 March 2006

1) E-mail: chengyf@scuec.edu.cn

to the same problem. Pathak and Mandal proposed a Taylor series method[6] and generalized the first-order results from quartic to higher anharmonicity[7]. The Taylor series method could give the solution of higher anharmonic oscillator, but the solution is so complex that it is not the best in this problem. We have already given the solution of classical and quantum oscillators of sextic anharmonicity by using the multiple-scale perturbation theory[8]. In this paper, we find the solution of generalized anharmonic oscillator by using the same method.

2 Classical case

The equation of motion of the classical generalized anharmonic oscillator of unit mass and unit frequency is given by

$$\ddot{x}+x+\varepsilon x^{2m-1}=0, \tag{2}$$

where $\varepsilon>0$ is the anharmonic constant. Multiple-scale analysis assumes the existence of many time scales $T_0=t$, $T_1=\varepsilon t$, $T_2=\varepsilon^2 t$, $\cdots$, which can be temporarily treated as independent variables. Here, we use only the two variables T_0 and T_1 and seek a perturbative solution to Eq. (2) of the form

$$x(t)=x_0(T_0,T_1)+\varepsilon x_1(T_0,T_1). \tag{3}$$

In addition, we replace the derivative with respect to time by:

$$\frac{\mathrm{d}}{\mathrm{d}t} = D_0+\varepsilon D_1,$$
$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} = D_0^2+2\varepsilon D_0 D_1,$$

where $D_n=\partial/\partial T_n$ is a partial derivatives with respect to the independent time scale T_n. On substituting Eq. (3) into Eq. (2) and collecting terms with equal powers of ε, we obtain

$$D_0^2 x_0+x_0 = 0, \tag{4}$$
$$D_0^2 x_1+x_1 = -2D_0D_1x_0-x_0^{2m-1}. \tag{5}$$

We assume that the solution of Eq. (4) is

$$x_0=\frac{1}{\sqrt{2}}(A\exp(-\mathrm{i}T_0)+\bar{A}\exp(\mathrm{i}T_0)), \tag{6}$$

where $A=A(T_1)$ is an integral constant and $\bar{A}$ is conjugate of A. On substituting Eq. (6) into Eq. (5), we obtain

$$D_0^2x_1+x_1=\mathrm{i}\sqrt{2}[(D_1A)\mathrm{e}^{-\mathrm{i}T_0}-(D_1\bar{A})\mathrm{e}^{\mathrm{i}T_0}]-\sum_{r=0}^{2m-1}\frac{C_{2m-1}^r}{2^{m-1/2}}A^r\bar{A}^{2m-r-1}\mathrm{e}^{\mathrm{i}(2m-2r-1)T_0}. \tag{7}$$

To avoid a secular term in T_0, we eliminates all contributions on Eq. (7) that are proportional to $\exp(\pm\mathrm{i}T_0)$. This leads to the solvability conditions:

$$\mathrm{i}\sqrt{2}D_1A = \frac{C_{2m-1}^m}{2^{m-1/2}}A^m\bar{A}^{m-1}, \tag{8}$$
$$\mathrm{i}\sqrt{2}D_1\bar{A} = -\frac{C_{2m-1}^m}{2^{m-1/2}}A^{m-1}\bar{A}^m. \tag{9}$$

Using Eq. (8) and Eq. (9) yields $D_1(A\bar{A})=0$. If we write $A=B\mathrm{e}^{-\mathrm{i}\beta}$ and substitute this form into Eq. (9), we obtain

$$\beta=\frac{C_{2m-1}^m}{2^m}(B\bar{B})^{m-1}T_1. \tag{10}$$

Hence the zeroth order solution can be given by

$$x_0=\frac{1}{\sqrt{2}}(B\mathrm{e}^{-\mathrm{i}\alpha}+\bar{B}\mathrm{e}^{\mathrm{i}\alpha}), \tag{11}$$

where $\alpha=T_0+\beta$. The second order difference Eq. (7) can be written in the following form

$$D_0^2x_1+x_1 = -\sum_{r=0}^{m-2}\frac{C_{2m-1}^r}{2^{m-1/2}}[B^r\bar{B}^{2m-r-1}\mathrm{e}^{\mathrm{i}(2m-2r-1)\alpha}+B^{2m-r-1}\bar{B}^r\mathrm{e}^{-\mathrm{i}(2m-2r-1)\alpha}]. \tag{12}$$

The solution of Eq. (12) is given by

$$x_1 = \sum_{r=0}^{m-2}\frac{C_{2m-1}^r}{2^{m-1/2}(2m-2r)(2m-2r-2)}\times[B^r\bar{B}^{2m-r-1}\mathrm{e}^{\mathrm{i}(2m-2r-1)\alpha}+\mathrm{c.c}]. \tag{13}$$

On substituting Eq. (11) and Eq. (13) into Eq. (3), we have the solution of multiple-scales of perturbation for generalized classical anharmonic oscillator.

If we introduce the initial conditions $x_0(0,0)=F$, $\dot{x}_0(0,0)=0$ then $B=F/\sqrt{2}$. Eq. (11) can be written as

$$x_0=F\cos(\Omega t), \tag{14}$$

where the shifted frequency is given by

$$\Omega=1+\varepsilon\frac{C_{2m-1}^m}{2^{2m-1}}F^{2m-2}. \tag{15}$$

Finally Eq. (3) can be reduced to

$$x = F\cos\Omega t+\varepsilon\sum_{r=0}^{m-2}\frac{C_{2m-1}^rF^{2m-1}}{2^{2m}(m-r)(m-r-1)}\times\cos[(2m-2r-1)\Omega t]. \tag{16}$$

We calculate several specific results from general expressions. For $m=2$, which the classical quartic anharmonic oscillator, we have

$$\begin{aligned}\Omega &= 1+\varepsilon\frac{3}{8}F^2,\\ x &= F\cos\Omega t+\varepsilon\frac{1}{32}F^3\cos 3\Omega t\ .\end{aligned}\qquad(17)$$

For $m=3$ we have

$$\begin{aligned}\Omega &= 1+\varepsilon\frac{5}{16}F^4,\\ x &= F\cos\Omega t+\varepsilon\frac{1}{384}F^5(\cos 5\Omega t+15\cos 3\Omega t).\end{aligned}\qquad(18)$$

The solution of sextic anharmonic oscillator exactly coincides with the previous results[8]. For $m=4$ we have

$$\begin{aligned}\Omega &= 1+\varepsilon\frac{35}{128}F^6,\\ x &= F\cos\Omega t+\frac{\varepsilon F^7}{3072}(\cos 7\Omega t+14\cos 5\Omega t+\\ &\quad 126\cos 3\Omega t).\end{aligned}\qquad(19)$$

The difference between our results and the Taylor series method[7] is that all the variables in trigonometric function are Ωt in the former while only zeroth-order variable is Ωt in the latter. In conclusion, the method of multiple-scale could give correct solutions of classical generalized anharmonic oscillator.

3 Quantum case

The equation of motion of a quantum generalized anharmonic oscillator is given by

$$\ddot{X}+X+\varepsilon X^{2m-1}=0.\qquad(20)$$

We now use the method of multiple-scale to Eq. (20) and only consider the first-order solution. We write

$$X(t)=X_0(T_0,T_1)+\varepsilon X_1(T_0,T_1).\qquad(21)$$

This equation is analogous to Eq. (3) but here X_0 and X_1 are operator functions. On subsisting Eq. (21) into Eq. (20), collect the coefficients of ε^0 and ε^1, we obtain

$$D_0^2X_0+X_0 = 0,\qquad(22)$$

$$D_0^2X_1+X_1 = -2D_0D_1X_0-X_0^{2m-1}.\qquad(23)$$

In quantum case, the position operator $X(t)$ and the momentum operator $\dot{X}(t)$ satisfy the commutation relation

$$[X(t),\dot{X}(t)]=\mathrm{i},\qquad(24)$$

where

$$\dot{X}(t)=D_0X_0+\varepsilon(D_1X_0+D_0X_1).\qquad(25)$$

On subsisting Eq. (21) and Eq. (25) into Eq. (24) and collecting terms with equal powers of ε, we obtain the following relation

$$[X_0,D_0X_0]=\mathrm{i},\qquad(26)$$

$$[X_0,D_1X_0+D_0X_1]+[X_1,D_0X_0]=0,\qquad(27)$$

$$[X_1,D_1X_0+D_0X_1]=0.\qquad(28)$$

We also assume that the solution of Eq. (22) is the following complex form

$$X_0=\frac{1}{\sqrt{2}}[A^\dagger(T_1)\mathrm{e}^{\mathrm{i}T_0}+\mathrm{e}^{-\mathrm{i}T_0}A(T_1)],\qquad(29)$$

where $A^\dagger$ is the conjugate operator of A. In the condition of $T_0=0$, on substituting Eq. (29) into Eq. (26) the following relation is given by

$$[A(T_1),A^\dagger(T_1)]=1,\qquad(30)$$

where A and $A^\dagger$ are similar to the usual bosonic annihilation and creation operators. We substitute Eq. (29) into Eq. (23) and obtain

$$\begin{aligned}D_0^2X_1+X_1 &= -\mathrm{i}\sqrt{2}[D_1A^\dagger\mathrm{e}^{\mathrm{i}T_0}-\mathrm{e}^{-\mathrm{i}T_0}D_1A]-\\ &\quad\left[\frac{1}{\sqrt{2}}(A^\dagger\mathrm{e}^{\mathrm{i}T_0}+\mathrm{e}^{-\mathrm{i}T_0}A)\right]^{2m-1}.\end{aligned}\qquad(31)$$

We first construct a normal ordered expansion of $(A^\dagger\mathrm{e}^{\mathrm{i}T_0}+\mathrm{e}^{-\mathrm{i}T_0}A)^{2m-1}$, hence

$$\begin{aligned}&(A^\dagger\mathrm{e}^{\mathrm{i}T_0}+\mathrm{e}^{-\mathrm{i}T_0}A)^{2m-1}=\\ &\sum_{r=0}^{m-1}t_{2r}C_{2m-1}^{2r}:(A^\dagger\mathrm{e}^{\mathrm{i}T_0}+\mathrm{e}^{-\mathrm{i}T_0}A)^{2m-2r-1}:\end{aligned}\qquad(32)$$

with

$$t_{2r}=\begin{cases}\dfrac{(2r-1)!}{2^{r-1}(r-1)!} & \text{for}\quad r>1\\ 1 & \text{for}\quad r=0,1\end{cases},\qquad(33)$$

where the notation $:(A^\dagger\mathrm{e}^{\mathrm{i}T_0}+\mathrm{e}^{-\mathrm{i}T_0}A)^{2m-2r-1}:$ is simply a binomial expansion in which powers of the $A^\dagger$ are always kept to the left of the powers of the A. Hence we have

$$\begin{aligned}:(A^\dagger\mathrm{e}^{\mathrm{i}T_0}+\mathrm{e}^{-\mathrm{i}T_0}A)^{2m-2r-1}: &= \sum_{k=0}^{2m-2r-1}C_{2m-2r-1}^{k}\times\\ &\quad A^{\dagger k}A^{2m-2r-1-k}\mathrm{e}^{-\mathrm{i}(2m-2r-2k-1)T_0}.\end{aligned}\qquad(34)$$

According to Eq. (32) and Eq. (34), Eq. (31) becomes

$$D_0^2X_1+X_1=-\mathrm{i}\sqrt{2}[D_1A^\dagger \mathrm{e}^{\mathrm{i}T_0}-\mathrm{e}^{-\mathrm{i}T_0}D_1A]-\frac{1}{2^{m-1/2}}\sum_{r=0}^{m-1}\sum_{k=0}^{2m-2r-2}t_{2r}C_{2m-1}^{2r}C_{2m-2r-1}^{k}\times A^{\dagger k}A^{2m-2r-k-1}\mathrm{e}^{-\mathrm{i}(2m-2r-2k-1)T_0}. \quad (35)$$

To eliminate a secular term, we must set the coefficients of $\exp(\pm \mathrm{i}T_0)$ to zero and obtain the solvability donditions

$$\mathrm{i}\sqrt{2}D_1A-\sum_{r=0}^{m-1}\frac{t_{2r}C_{2m-1}^{2r}C_{2m-2r-1}^{m-r-1}}{2^{m-1/2}}A^{\dagger(m-r-1)}A^{m-r}=0,$$
$$\mathrm{i}\sqrt{2}D_1A^\dagger+\sum_{r=0}^{m-1}\frac{t_{2r}C_{2m-1}^{2r}C_{2m-2r-1}^{m-r-1}}{2^{m-1/2}}A^{\dagger(m-r)}A^{m-r-1}=0. \quad (36)$$

To solve Eq. (36), we begin by multiplying the first equation on the left by $A^\dagger$ and the second equation on the right by A. Adding the resulting two equations and simplifying, we get $D_1(A^\dagger A)=0$. Hence the operator $N=A^\dagger A$ is independent of the variable T_1. We again assume that $A=\mathrm{e}^{-\mathrm{i}\beta}B$ and substitute this expression and its conjugate into the first and second equation the system Eq. (36), separetely. It is easy to find the following relation

$$\frac{\mathrm{d}\beta}{\mathrm{d}T_1}=\frac{\mathrm{d}\beta^\dagger}{\mathrm{d}T_1}=\sum_{r=0}^{m-1}\frac{(2m-1)!}{2^{m+r}r!(m-r)!(m-r-1)!}\times (B^\dagger \mathrm{e}^{\mathrm{i}\beta^\dagger})^{m-r-1}(\mathrm{e}^{-\mathrm{i}\beta}B)^{m-r-1}\ . \quad (37)$$

Hence $\beta^\dagger=\beta$ is the Hermite operator and B is independent of the variable T_1. When we introduce $N=A^\dagger A=B^\dagger B$ which is the number operator, we obtain the following relation

$$N=B^\dagger B=AA^\dagger-1=\mathrm{e}^{-\mathrm{i}\beta}N\mathrm{e}^{\mathrm{i}\beta}. \quad (38)$$

We thus know that $\mathrm{e}^{\pm\mathrm{i}\beta}$ is commutate with N and B satisfies the commutation relations $[B,B^\dagger]=1$. By using the following relation

$$f(N)B=Bf(N-1),\quad f(N)B^\dagger=B^\dagger f(N+1), \quad (39)$$

we have

$$(B^\dagger \mathrm{e}^{\mathrm{i}\beta})^k(\mathrm{e}^{-\mathrm{i}\beta}B)^k=\prod_{l=0}^{k-1}(N-l). \quad (40)$$

On substituting Eq. (40) into Eq. (37), we obtain

$$\beta=\gamma_{2m-1}(N)T_1, \quad (41)$$

where

$$\gamma_{2m-1}(N)=\sum_{r=0}^{m-1}\frac{(2m-1)!}{2^{m+r}r!(m-r)!(m-r-1)!}\times\prod_{l=0}^{m-r-2}(N-l) \quad (42)$$

and $\prod_{l=0}^{s}(N-l)=1$, (if $s<0$).

When we substitute the above results into Eq. (35), we obtain

$$D_0^2X_1+X_1=-\frac{1}{2^{m-1/2}}\sum_{k=0}^{m-2}\sum_{r=0}^{m-k-2}t_{2r}C_{2m-1}^{2r}\times C_{2m-2r-1}^{m-r-k-2}[A^{\dagger(m-r-k-2)}A^{m-r+k+1}\times \mathrm{e}^{-\mathrm{i}(2k+1)T_0}+\mathrm{c.c}]. \quad (43)$$

The special solution of Eq. (43) is

$$X_1=\sum_{r=0}^{m-2}[P_{mk}\mathrm{e}^{-\mathrm{i}(2k+3)T_0}+Q_{mk}\mathrm{e}^{\mathrm{i}(2k+3)T_0}], \quad (44)$$

where

$$Q_{mk}=\sum_{r=0}^{m-k-2}J_{mkr}A^{\dagger(m-r+1+k)}A^{m-r-2-k} \quad (45)$$

and

$$J_{mkr}=\frac{2^{-(m+r+3/2)}(2m-1)!}{r!(m-r-k+2)!(m-r-k+1)!(k+1)(k+2)}. \quad (46)$$

By using Eq. (40) and the following equation

$$A^{\dagger k}=[B^\dagger \mathrm{e}^{\mathrm{i}\gamma(N)T_1}]^k=B^{\dagger k}\exp\left(\mathrm{i}\sum_{l=0}^{k-1}\gamma(N+l-1)T_1\right),$$

Eq. (45) can be reduced as

$$Q_{mk}=O_{mk}\exp\left(\mathrm{i}\sum_{n=0}^{2k+2}\gamma(N+n)T_1\right). \quad (47)$$

where

$$O_{mk}=\sum_{r=0}^{m-k-2}J_{mkr}B^{\dagger 2k+3}\prod_{l=0}^{m-k-r-3}(N-l)$$

and $P_{mk}=Q_{mk}^\dagger$. Hence in the quantum case the solution of the first order multiple-scale perturbation of generalised anharmonic oscillator is

$$X=\frac{1}{\sqrt{2}}B^\dagger \mathrm{e}^{\mathrm{i}\Omega_m t}+\sum_{k=0}^{m-2}O_{mk}\mathrm{e}^{\mathrm{i}\Omega_{mk}t}+\mathrm{c.c}, \quad (48)$$

where

$$\Omega_m=1+\varepsilon\gamma_{2m-1}(N),$$
$$\Omega_{mk}=(2k+3)+\varepsilon\sum_{n=0}^{2k+2}\gamma_{2m-1}(N+n), \quad (49)$$

We calculate sevral specific results from our general expression. For $m=2$ we have

$$\Omega_2=1+\varepsilon\frac{3}{4}(N+1).$$

Therefore the solution of the quantum quartic anharmonic oscillator is

$$X=\frac{1}{\sqrt{2}}B^{\dagger}\mathrm{e}^{\mathrm{i}\Omega_2 t}+\frac{\varepsilon}{16\sqrt{2}}B^{\dagger 3}\mathrm{e}^{\mathrm{i}3[1+\varepsilon\frac{3}{4}(N+2)]t}+\text{c.c.} \quad (50)$$

Similarly for $m=3$ we have

$$\Omega_3=1+\varepsilon\frac{5}{8}(2N^2+4N+3).$$

The solution of quantum sextic oscillator is

$$X = \frac{1}{\sqrt{2}}B^{\dagger}\mathrm{e}^{\mathrm{i}\Omega_3 t}+\frac{\varepsilon}{96\sqrt{2}}[B^{\dagger 5}\mathrm{e}^{\mathrm{i}5(1+\varepsilon\upsilon)t}+ 15B^{\dagger 3}(N+2)\mathrm{e}^{\mathrm{i}3(1+\varepsilon\nu)t}]+\text{c.c.} \quad (51)$$

where

$$\upsilon = \frac{5}{8}(2N^2+12N+3),$$
$$\nu = \frac{5}{24}(6N^2+24N+31).$$

For $m=4$, we have

$$\Omega_4=1+\varepsilon\frac{35}{16}(N^3+3N^2+5N+3).$$

Hence the solution of quantum octic oscillator is

$$X = \frac{1}{\sqrt{2}}B^{\dagger}\mathrm{e}^{\mathrm{i}\Omega_4 t}+\frac{\varepsilon}{384\sqrt{2}}[B^{\dagger 7}\mathrm{e}^{\mathrm{i}7(1+\varepsilon\chi)t}+ 2B^{\dagger 5}(N+3)\mathrm{e}^{\mathrm{i}5(1+\varepsilon\kappa)t}+ 126B^{\dagger 3}(N^2+4N+10)\mathrm{e}^{\mathrm{i}3(1+\varepsilon\lambda)t}]+\text{c.c.} \quad (52)$$

where

$$\chi=\frac{35}{16}(N^3+12N^2+62N+120),$$
$$\kappa=\frac{35}{16}(N^3+9N^2+35N+51),$$
$$\lambda=\frac{35}{16}(N^3+6N^2+16N+16).$$

The above results are more concise than the solution of the Taylor series approach and comparable with the classical results.

4 Concluding remarks

The first difference between classical solution and quantum is that in quantum we want to consider the commutation relations. We have used the commutation relation Eq. (26) which is the zeroth order form of the canonical equal time commutation relation Eq. (24). It is easy to verify that our solutions satisfy the first order form of the commutation relation Eq. (27) and the second order form Eq. (28). Therefore in the powers of ε the expansion of the equal time commutation relation could be used to study the problem of multiple-scales perturbation analyses of anharmonic oscillator.

The second difference we want to know is by comparing the quantum results with the classical results. On introducing the initial conditions that $X_0(0,0)=Q_0$, $D_0X_0(0,0)=P_0$, we have

$$[Q_0,P_0]=\mathrm{i} \quad (53)$$

and then from Eq. (29) we have

$$Q_0=\frac{1}{\sqrt{2}}(B^{\dagger}+B), \quad P_0=\frac{\mathrm{i}}{\sqrt{2}}(B^{\dagger}-B). \quad (54)$$

It is clear that $B^{\dagger}$ and B are creation and annihilation operators of unperturbed field, respectively. Because $N=A^{\dagger}A=B^{\dagger}B$ is the number operator, the quantum solution can be translated into classical solution in the condition of $N\gg 1$. For example, for $m=2$, when $N\gg 1$ we have $\Omega_2\approx\left(1+\varepsilon\frac{3}{4}(N+2)\right)$. If introducing $B=F/\sqrt{2}$ Eq. (50) can be translated into Eq. (17). Similarity for $m=3,4,\cdots$ the quantum solution can also be translated into classical solution.

In conclusion we have studied the time evolution of position for classical and quantum oscillator of generalized anharmonicity. In particular our quantum results can be transformed into the classical solution easily which is better than the other one.

References

1 Nayfeh A H. Introduction to Perturbation Techniques. New Yok: Wikey, 1981

2 Fernández F M. Introduction to Perturbation Theory in Quantum Mechanics. Noca Ratom: CRC Press, 2000

3 Bender C M, Bettencourt L M A. Phys. Rev., 1996, **D54**: 7710

4 Bender C M, Bettencourt L M A. Phys. Rev. Lett., 1996, **54**: 4114

5 Auberson G, Capdequi P M. Phys. Rev., 2002, **A65**: 1

6 Pathak A, Mandal S. Phys. Lett., 2001, **286**: 261

7 Pathak A, Mandal S. Phys. Lett., 2002, **298**: 259

8 CHENG Yan-Fu, DAI Tong-Qing. HEP & NP, 2006, **30**(6): 513 (in Chinese)
(程衍富, 戴同庆. 高能物理与核物理, 2006, **30**(6): 513)

广义非简谐振子的多尺度微扰理论

程衍富[1)]　戴同庆

(中南民族大学电子信息工程学院　武汉　430074)

摘要　应用多尺度微扰理论到广义非简谐振子, 得到了一阶经典和量子微扰解. 特别是我们的量子解在极限条件下能方便地转变为经典解, 并且坐标和动量算符的对易关系的简化十分自然. 与Taylor级数解相比较, 无论是在经典还是在量子解中频率移动都出现在各阶振动表达式中, 所以多尺度微扰解是弱耦合非简谐振动的较好解法.

关键词　广义非简谐振子　多尺度微扰理论　经典和量子解

2006 – 02 – 27 收稿,　2006 – 03 – 18 收修改稿

1) E-mail: chengyf@scuec.edu.cn