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S Wave Kπ Scattering and Dynamically Generated Scalar Resonances*

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Abstract Based on a chiral unitary approach (ChUA), *S* wave Kπ scattering is studied. We consider Kπ and Kη coupled channels on isospin bases. With only one parameter, the *S* wave Kπ scattering phase shift data below 1.2GeV can be described, and the resultant *S* wave scattering lengths also coincide with the experimental values well. In this approach, a scalar resonance corresponding to the κ can be generated dynamically. Its mass and width are about 725MeV and 594MeV, respectively. A resonance corresponding to the scalar $K_0^*(1430)$ can also be generated qualitatively.

Key words *S* wave Kπ scattering, chiral unitary approach, κ

1 Introduction

The standard method describing low energy Kπ scattering is the chiral perturbation theory (ChPT)[1—3]. The most general Lagrangian is an expansion in powers of the momenta of the Goldstone bosons and the light quark masses. Although the ChPT is very successful in describing low energy meson-meson interactions, the perturbative character determines that it can only be used with a momentum below several hundred MeV. On the other hand, the unitary relation is only respected in a perturbative manner. Furthermore, a perturbative expansion cannot generate a resonance to any finite order. This also sets a applicable limit in the ChPT. In recent years, a chiral unitary approach (ChUA) was proposed to describe the meson-meson interactions[4]. In this approach, coupled-channel factorized Bethe-Salpeter equations (BSE), whose kernel was the lowest order chiral amplitudes, were used to unitarize the scattering amplitudes. The phase shift data below 1.2GeV for the I=0, 1 channels could be well-described, and the scalar mesons σ, $f_0(980)$ and $a_0(980)$ could be generated dynamically as poles in the second Riemann sheet of the full scattering amplitudes. The Kπ scattering were studied using similar methods in Refs. [5—7]. However, the number of parameters are large due to involving the nest-to-leading order chiral Lagrangian. There is also a singularity problem in the conventional cut-off method calculating the loop integral used in Ref. [4] due to the three-momentum cut-off[8].

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In this paper, we will use the ChUA to study the S wave Kπ scattering. The S wave Kπ phase shifts and scattering lengths will be calculated. We will find poles in the second Riemann sheet of the full amplitudes for $I=\frac{1}{2}$ also.

2 Formalism

The starting point is the lowest order chiral Lagrangian

$$\mathscr{L}^{(2)}=\frac{1}{12f^2}\mathrm{Tr}[(\partial_\mu\Phi\Phi-\Phi\partial_\mu\Phi)^2+M\Phi^4], \quad (1)$$

where f=92.4MeV is the pion decay constant, Φ represents the pseudoscalar meson octet

$$\Phi=\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}, \quad (2)$$

and $M=\mathrm{diag}\{m_\pi^2,m_\pi^2,2m_{\mathrm{K}}^2-m_\pi^2\}$.

Two channels will be considered for S wave $I=\frac{1}{2}$ Kπ scattering, i.e., Kπ and Kη, and they will be labelled as 1 and 2, respectively. Only Kπ is in the $I=\frac{3}{2}$ channel. If we approximate η by η_8, the scattering amplitudes can be obtained from Eq. (2), and after the S wave projection, one obtain for the $I=\frac{1}{2}$ channel,

$$\begin{aligned} V_{11}^{\frac{1}{2}}(s) &= -\frac{1}{8f^2}\left(5s-2(m_\pi^2+m_{\mathrm{K}}^2)-\frac{3(m_\pi^2-m_{\mathrm{K}}^2)^2}{s}\right), \\ V_{12}^{\frac{1}{2}}(s) &= \frac{1}{12f^2}\left(m_\pi^2-10m_{\mathrm{K}}^2+3m_\eta^2+\right. \\ &\quad 18\sqrt{\left(m_{\mathrm{K}}^2+\frac{\lambda(s,m_{\mathrm{K}}^2,m_\eta^2)}{4s}\right)}\times \\ &\quad \left.\sqrt{\left(m_{\mathrm{K}}^2+\frac{\lambda(s,m_{\mathrm{K}}^2,m_\pi^2)}{4s}\right)}\right), \\ V_{22}^{\frac{1}{2}}(s) &= \frac{1}{24f^2}\left(9s+4m_\pi^2-18m_{\mathrm{K}}^2-6m_\eta^2+\right. \\ &\quad \left.\frac{9(m_{\mathrm{K}}^2-m_\eta^2)^2}{s}\right), \end{aligned} \quad (3)$$

where $\lambda(s,m_1^2,m_2^2)=(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)$ is the Kaellen function. And for the $I=\frac{3}{2}$ channel,

$$V^{\frac{3}{2}}(s)=\frac{1}{2f^2}(s-m_\pi^2-m_{\mathrm{K}}^2). \quad (4)$$

The full scattering amplitudes can be written as an algebraic Bethe-Salpeter equation (BSE) in ChUA[4]

$$T(s)=(1-V(s)G(s))^{-1}V(s), \quad (5)$$

where $G(s)$ is a diagonal matrix with the i-th diagonal element being the two-meson loop integral

$$G(s)=\mathrm{i}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{1}{q^2-m_1^2+\mathrm{i}\varepsilon}\frac{1}{(p_1+p_2-q)^2-m_2^2+\mathrm{i}\varepsilon}, \quad (6)$$

where p_1 and p_2 are the four-momenta of the two initial particles respectively, and m_1 and m_2 are the masses of the two particles appearing in the loop. It has been shown that by this method, the scattering matrix satisfies the unitary relation[5, 6, 8]. The conventional method calculating the loop integral in ChUA is the three-momentum cut-off method. In this method, a three-momentum cut-off is used as a parameter to fit the data. However, we found that there would be an artificial singularity corresponding to the cut-off in the loop function, and this singularity would have impacts even at several hundred MeV below its location[8]. To avoid this problem, the dimensional regularization with a subtraction constant will be used to deal with the loop integral[6]

$$\begin{aligned} G(s) &= \frac{1}{16\pi^2}\left\{a(\mu)+\log\frac{m_1^2}{\mu^2}+\frac{\Delta-s}{2s}\log\frac{m_1^2}{m_2^2}+\right. \\ &\quad \frac{\sigma}{2s}[\log(s-\Delta+\sigma)+\log(s+\Delta+\sigma)- \\ &\quad \left.\log(-s+\Delta+\sigma)-\log(-s-\Delta+\sigma)]\right\}, \end{aligned} \quad (7)$$

where $a(\mu)$ is a subtraction constant, μ is the regularization scale, $\sigma=[-(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)]^{\frac{1}{2}}$, and $\Delta=m_1^2-m_2^2$. The result is independent of μ because the change caused by a change of μ can be absorbed in a corresponding change of the subtraction constant $a(\mu)$.

3 S wave phase shifts

In the following, we choose to calculate at $\mu=m_{\mathrm{K}}$. The single parameter is the subtraction constant $a(m_{\mathrm{K}})$. It will be determined from fitting to the S wave phase shift data for $I=\frac{1}{2}$ and $I=\frac{3}{2}$, respectively. The experimental data are taken from

Refs. [9—11]. The fitted results are plotted in Fig. 1. One can see the fit is good below 1.2GeV.

The parameters from fitting to the S wave phase shift data are determined to be

$$a(m_{\rm K})^{\frac{1}{2}} = -1.278 \pm 0.014, \qquad a(m_{\rm K})^{\frac{3}{2}} = -4.646 \pm 0.083. \tag{8}$$

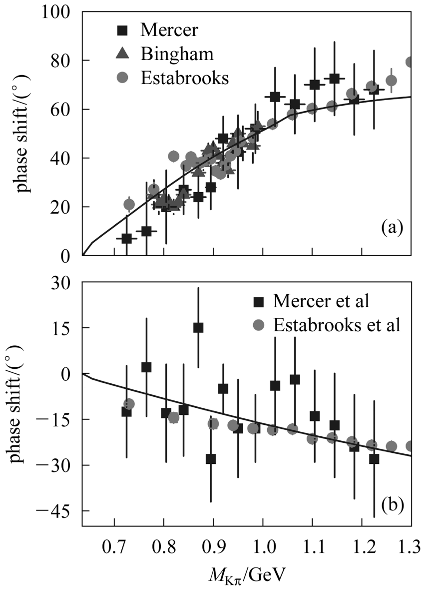

Fig. 1. S wave Kπ phase shifts with the two coupled channels. The upper figure is for the $I=\frac{1}{2}$ channel, and the lower one is for the $I=\frac{3}{2}$ channel.

4 Scattering lengths

Using our normalization, the real part of the total partial wave amplitudes near threshold can be expanded in the form

$$\mathrm{Re}T_l^I(s) = -8\pi\sqrt{s}q^{2l}(a_l^I + q^2 b_l^I + O(q^4)). \tag{9}$$

The quantities a_l^I are referred to scattering lengths, and b_l^I effective ranges. For S wave Kπ scattering, the scattering lengths should be

$$a_0^I = -\frac{1}{8\pi(m_\pi + m_{\rm K})}\mathrm{Re}\,t_0^I((m_\pi + m_{\rm K})^2). \tag{10}$$

Using the central values of the parameters given in Eq. (8), the results can be worked out as listed in Table 1. For comparison, we show the results of the lowest order ChPT in the third column, the next-to-leading order (NLO) ChPT in the fourth column as well as the mean experimental values from Refs. [12—14] in the fifth column, and the newest analysis of the data using Roy-Steiner equations in the last column[15]. Our results are very similar to the NLO ChPT results and compatible with the experimental ones.

Table 1. The results for the S wave Kπ scattering lengths.

	ChUA	$O(p^2)$	$O(p^4)$[3]	Exp.[12—14]	Rse.[15]
$m_\pi a_0^{\frac{1}{2}}$	0.171	0.142	0.17	0.13—0.24	0.224±0.022
$m_\pi a_0^{\frac{3}{2}}$	−0.056	−0.071	−0.05	−0.13—−0.05	−0.0448±0.0077

5 Poles in the second Riemann sheet

For one channel scattering, a pole in the second Riemann sheet of full amplitude corresponds to a resonance. For two channels, the situation is somewhat more complicated, and the pole corresponding to a resonance may be changed to the third Riemann sheet due to a new open channel below. For details, refer to Ref. [16]. In our case, the only concerned part in analytic continuation to the second Riemann sheet is the loop function $G(s)$. Using the Shwartz reflection principle, one can get[4]

$$G^{\rm II}(\sqrt{s}+{\rm i}\varepsilon) = G^{\rm I}(\sqrt{s}+{\rm i}\varepsilon) - 2{\rm i}\,{\rm Im}\,G^{\rm I}(\sqrt{s}+{\rm i}\varepsilon), \tag{11}$$

and the imaginary part of $G^{\rm I}(\sqrt{s}+{\rm i}\varepsilon)$ is given by $-q_{\rm cm}/(8\pi\sqrt{s})$ with $q_{\rm cm}$ being the three-momentum of a meson in the center-of-mass frame of the two mesons.

After analytic continuation, one can find a pole in the second Riemann sheet of the full amplitude for $I=\frac{1}{2}$ at $0.725-{\rm i}0.297$GeV using the central value of $a(K)$ in Eq. (8). This pole exists considering only one channel Kπ, and the Kη channel has effect only on the pole positions. Thus, one can conclude that this pole corresponds to a resonance. Due to its position, one can associate the pole to the long controversial scalar meson κ. In this way, the mass and width of the κ are predicted at about 0.725GeV and 0.594GeV, respectively.

Trying to search poles in higher energy region, another pole in the second Riemann sheet for $I=\frac{1}{2}$ is found at $1.253-\mathrm{i}0.465$GeV. If one uses the physical values of f_K and f_η, i.e., f_K=113.0MeV and f_η=110.9MeV[17], instead of using the same value f_π for all of the mesons, the pole position will be changed to 1.411−i0.702GeV. Certainly, the parameter $a(m_K)$ should be re-determined from fitting in this case. This pole can be associated to the $K_0^*(1430)$ in PDG. The mass and width of the $K_0^*(1430)$ are 1412±6MeV and 294±23MeV, respectively[17]. From comparison, deviations of the pole position from the experimental values of the mass and width of the $K_0^*(1430)$ exist, especially the width. This can be understood because the phase shift data can be described only below 1.2GeV in our model. Thus, we cannot say anything with quantitative precision above 1.2GeV. However, a qualitative pole is found as illustrated. This should be because that the coupling of the $K_0^*(1430)$ with Kπ is very large.

6 Summary

In this paper, we use the chiral unitary approach to study S wave Kπ scattering. The scattering amplitudes are unitarized using a set of coupled-channel algebraic BSEs. The lowest order chiral amplitudes are used as their kernels. For $I=1/2$, both of Kπ and Kη are considered, and for $I=3/2$, only Kπ is involved. With a single parameter for each isospin, the S wave Kπ scattering phase shifts below 1.2GeV are described well, and the resultant scattering lengths are consistent with experimental values and are very close to the ones calculated using the NLO ChPT. In the second Riemann sheet of the full scattering amplitude for $I=1/2$, two poles are found at about 0.725−i0.297GeV and 1.253−i0.465GeV, respectively. The first pole can be associated to the broad scalar meson κ. In this way, the mass and width of the κ are predicted at about 0.725GeV and 0.594GeV, respectively. The second pole can be associated to the $K_0^*(1430)$ state. However, because the data can be described below 1.2GeV, the second pole should only be a qualitative one rather than a quantitative description of the $K_0^*(1430)$ state.

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S波Kπ散射以及动力学产生标量介子*

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摘要 考虑Kπ和Kη两个耦合道, 用手征幺正方法研究了S波Kπ散射. 结果表明, 只用一个参数, 1.2GeV以下的散射相移数据就可以得到很好的描述, 并且散射长度也和实验值符合很好. 此方法可以动力学产生标量介子κ, 并预言其质量和宽度分别大约是752MeV和594MeV. 可以定性地产生对应于$K_0^*(1430)$的标量介子.

关键词 S波Kπ散射 手征幺正方法 κ

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