Branching Ratio of Rare Decay $B^0(B_s) \to \gamma\nu\bar{\nu}$ *

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Abstract The three-body decay $B^0(B_s) \to \gamma\nu\bar{\nu}$ can occur via penguin and box diagrams in the Standard Model (SM). These channels are useful to determine the decay constant f_B (f_{B_s}) and B (B_s) meson wave function. Using the B meson wave function determined in hadronic B (B_s) decays, we calculate and get the branching ratio of order 10^{-9} and 10^{-8} for B^0 and B_s decay, respectively. They agree with previous calculations.

Key words wave function, rare B decay, branching ratio, decay constant

1 Introduction

The flavor changing neutral current process is one of the most important fields for testing the Standard Model (SM) at loop level and for establishing new physics beyond that. The rare B decays provide a direct and reliable tool for extracting information about the fundamental parameters of the Standard Model (SM), such as, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{td} and V_{ts}, if we know the value of the decay constant f_B from other methods. Conversely, we can determine the decay constant f_B if the CKM matrix elements are known.

Pure leptonic decays $B_s \to \mu^+\mu^-$ and $B_s \to e^+e^-$ are difficult to measure in experiments, since helicity suppression gives a very small branching ratio at the order $\mathscr{O}(10^{-9})$ and $\mathscr{O}(10^{-14})$, respectively[1]. For B^0 meson case the situation gets even worse due to the smaller CKM matrix element V_{td}. For decay $B_s \to \tau^+\tau^-$, although its branching ratio is about 10^{-7}[2], it is still hard for experiments due to the low efficiency of τ lepton measurements.

The $B^0(B_s) \to \nu\bar{\nu}$ decay is forbidden due to massless neutrino. Fortunately, having an extra real photon emitted, the radiative leptonic decays can escape from the helicity suppression, so that larger branching ratio of $B^0(B_s) \to \gamma\nu\bar{\nu}$ is expected. A preliminary work of this type decay was carried out with many different approaches both in SM[3—5] and beyond SM[6]. In the above work, it was shown that the diagrams with photon radiation from light quarks give the dominant contribution to the decay amplitude, that is inversely proportional to the constituent light quark mass. However the “constituent quark mass” is poorly understood. In this work, we calculate the branching ratio using B meson wave function which describes the valence quark momentum distribution. The wave function has been studied for many years[7] and used in calculating non-leptonic B decay[8]. Recently, this approach is also used to calculate radiative leptonic decay of charged B meson[9].

In the next section we analyze the relevant effective Hamiltonian for the $B^0(B_s) \to \gamma\nu\bar{\nu}$ decay. In Section 2, we give our analytical and numerical results, and then compare with other results. At last, we summarize this work in Section 3.

Received 11 September 2005, Revised 25 November 2005
* Supported by National Natural Science Foundation of China (10475085, 10135060)
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2 Effective Hamiltonian

Let us first look at the quark level process $b \to q\nu\bar{\nu}$, with $q = s$ or d, which is shown in Fig. 1. This is a flavor changing neutral current process, and both box and Z penguin diagrams contribute to this process. The effective Hamiltonian in SM is given[10]:

$$H = C(\bar{q}\gamma_\mu P_L b)(\bar{\nu}\gamma^\mu P_L \nu), \tag{1}$$

with $P_L = (1-\gamma_5)/2$. The coefficient C is

$$C = \frac{\sqrt{2}G_F\alpha}{\pi\sin^2\theta_w} V_{tb}V^*_{tq}\frac{x}{8}\left[\frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2}\ln x\right], \tag{2}$$

and $x = m_t^2/m_W^2$. From this expression, we can see that the coefficient C is sensitive to the mass of the particle in loop. If new particles exist, they should affect the Wilson Coefficient and change the branching ratio. That is why this kind of flavor changing neutral current processes are sensitive to new physics[6].

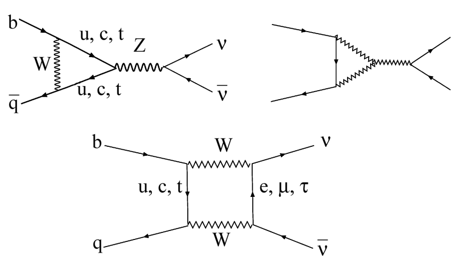

Fig. 1. Leading order Feynman diagrams in SM for $b \to q\nu\bar{\nu}$, with $q = s$ or d.

We have already mentioned that the pure leptonic ($\nu\bar{\nu}$) decay is forbidden due to helicity conservation. However, when a photon is emitted from any charged line of b or q quark, this pure leptonic processes turn into radiative ones and helicity suppression does not exist anymore. At quark level the process $B_{s(d)} \to \gamma\nu\bar{\nu}$ is described by the same diagrams as $b \to q\gamma\nu\bar{\nu}$ shown in Fig. 2. Incidentally, we should note the following peculiarities of this process:

(1) when a photon emitted from internal charged particles (W or top quark), the above mentioned process will be suppressed by a factor m_b^2/m_W^2 (see Ref. [3]), in comparison to the process $b \to q\nu\bar{\nu}$, one can neglect the contribution of such diagrams.

(2) The Wilson coefficient C is the same for the processes $b \to q\gamma\nu\bar{\nu}$ and $b \to q\nu\bar{\nu}$ as a consequence of the extension of the Low's low energy theorem (for more detail see Ref. [11]).

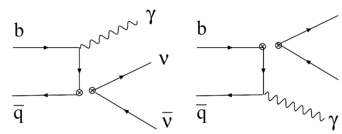

Fig. 2. Feynman diagrams for $b \to q\gamma\nu\bar{\nu}$ using effective four fermi operators.

So, when the photon emitted from initial b or light quark line, there are only two diagrams contributing to the process $b \to q\gamma\nu\bar{\nu}$. From Fig. 2, the corresponding decay amplitude turns out to be

$$\begin{aligned}\mathscr{A} \;=\; & -\frac{e}{6}C\;\bar{q}[\not{\epsilon}^*_\gamma \frac{\not{p}_\gamma - \not{p}_q + m_q}{(p_q\cdot p_\gamma)}\gamma_\mu P_L + \\ & P_R\gamma_\mu\frac{\not{p}_b - \not{p}_\gamma + m_b}{(p_b\cdot p_\gamma)}\not{\epsilon}^*_\gamma]b(\bar{\nu}\gamma^\mu P_L\nu).\end{aligned} \tag{3}$$

3 Analytical and numerical results

In order to calculate analytic formulas of the decay amplitude, we use the wave functions $\Phi_{M,\alpha\beta}$ decomposed in terms of spin structure. In the summation procedures, the B meson is treated as a heavy-light system. Thus, the B meson light-cone matrix element can be decomposed as[12]:

$$\Phi_{B,\alpha\beta} = \frac{i}{\sqrt{2N_c}}\left\{(\not{P}_B\gamma_5)_{\alpha\beta}\phi_B^A + \gamma_{5\alpha\beta}\phi_B^P\right\}, \tag{4}$$

where $N_c = 3$ is color degree of freedom, P_B is the corresponding momentum of the B meson, ϕ_B^A and ϕ_B^P are Lorentz scalar distribution amplitudes. As heavy quark effective theory leads to $\phi_B^P \simeq M_B\phi_B^A$, then B meson's wave function can be expressed by

$$\Phi_{B,\alpha\beta}(x) = \frac{i}{\sqrt{2N_c}}[\not{P}_B + M_B]\gamma_{5\alpha\beta}\phi_B(x). \tag{5}$$

In the above formula, the function ϕ_B describes the momentum distribution amplitude. Since b quark is much heavier than the light quark in B meson, there is a sharp peak in the small x region for the light quark momentum fraction,

$$\phi_B(x) = N_B x^2(1-x)^2\exp\left[-\frac{M_B^2\,x^2}{2\omega_b^2}\right]. \tag{6}$$

Table 1. Comparison of results from different approaches.

mode	our results	quark model	pole model	sum rule	light front
$Br(B^0\to\gamma\nu\bar{\nu})$	0.74×10^{-9}	1.7×10^{-9}	2.1×10^{-9}	4.2×10^{-9}	1.4×10^{-9}
$Br(B_s\to\gamma\nu\bar{\nu})$	2.4×10^{-8}	3.5×10^{-8}	1.8×10^{-8}	7.5×10^{-8}	2.0×10^{-8}

It satisfies the normalization relation:

$$\int_0^1\phi_B(x)\mathrm{d}x=\frac{f_B}{2\sqrt{2N_c}}, \tag{7}$$

with f_B being the B meson decay constant. This choice of B meson's wave function is almost a best fit from the B meson non-leptonic two body decays[8].

For simplicity, we consider the B meson at rest and use the light-cone coordinate $(p^+,p^-,\boldsymbol{p}_\perp)$ to describe the momenta of the meson and quark, where $p^\pm=\frac{1}{\sqrt{2}}(p^0\pm p^3)$ and $p_\perp=(p^1,p^2)$. Using this coordinate we can take the B_q, $\nu\bar{\nu}$ (momentum sum) and photon's momenta as

$$\begin{aligned}&P_B=\frac{M_B}{\sqrt{2}}(1,1,\boldsymbol{0}_\perp);\quad P_{\nu\bar{\nu}}=\frac{M_B}{\sqrt{2}}(1,r^2,\boldsymbol{0}_\perp);\\&P_\gamma=\frac{M_B}{\sqrt{2}}(0,1-r^2,\boldsymbol{0}_\perp),\end{aligned} \tag{8}$$

with $r^2=P^2_{\nu\bar{\nu}}/M^2_B$. The momenta of b and q quark in B meson are $p_b=(1-x)P_B$, $p_q=xP_B$. Using the above convention, the amplitude for $B_q\to\gamma\nu\bar{\nu}$ decay is written by:

$$\begin{aligned}A=&\frac{\sqrt{6}eC}{6}\Big[\mathrm{i}C_1\epsilon_{\alpha\beta\mu\nu}\varepsilon^{*\alpha}P_\gamma^\beta P_B^\nu+\\&C_2(P_{\gamma\mu}\varepsilon^*_\nu-P_{\gamma\nu}\varepsilon^*_\mu)\ P_B^\nu\Big](\bar{\nu_1}\gamma^\mu p_L\nu_2),\end{aligned} \tag{9}$$

with

$$C_1=\int_0^1\frac{\phi_B(x)}{p_b\cdot P_\gamma}+\int_0^1\frac{\phi_B(x)}{p_q\cdot P_\gamma}, \tag{10}$$

$$C_2=\int_0^1\frac{\phi_B(x)}{p_q\cdot P_\gamma}-\int_0^1\frac{\phi_B(x)}{p_b\cdot P_\gamma}. \tag{11}$$

After squaring the amplitude and performing the phase space integration over one of the two Dalitz variables, and summing over three generation of neutrinos, we get the differential decay width versus the photon energy E_γ:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma}=\frac{6C^2\alpha}{(12\pi)^2}(C_1'^2+C_2'^2)(M_B-2E_\gamma)E_\gamma, \tag{12}$$

with

$$C_1'^2=\left(\int_0^1\frac{\phi_B(x)}{1-x}\mathrm{d}x+\int_0^1\frac{\phi_B(x)}{x}\mathrm{d}x\right)^2, \tag{13}$$

$$C_2'^2=\left(\int_0^1\frac{\phi_B(x)}{1-x}\mathrm{d}x-\int_0^1\frac{\phi_B(x)}{x}\mathrm{d}x\right)^2. \tag{14}$$

By integrating over the variable E_γ, we get the decay width:

$$\Gamma=\frac{M_B^3C^2\alpha}{(24\pi)^2}(C_1'^2+C_2'^2). \tag{15}$$

In this work, we use the following parameters[7, 13]:

$$\begin{aligned}&\omega_b=0.4,\ f_B=0.19\ \text{GeV},\ \tau_{B^0}=1.54\times10^{-12}\text{s};\\&\omega_{b_s}=0.5,\ f_{B_s}=0.24\ \text{GeV},\ \tau_{B_s}=1.46\times10^{-12}\text{s};\\&G_F=1.66\times10^{-5}\text{GeV}^{-2};\ \sin^2\theta_\omega=0.23;\ \alpha=\frac{1}{132};\\&V_{tb}=0.999,\quad V_{td}=0.0074,\quad V_{ts}=0.041.\end{aligned} \tag{16}$$

Using these parameters, we get the branching ratios:

$$\begin{aligned}&Br(B^0\to\gamma\nu\bar{\nu})=0.7\times10^{-9},\\&Br(B_s\to\gamma\nu\bar{\nu})=2.4\times10^{-8}.\end{aligned} \tag{17}$$

Just as we mentioned above, many approaches have been used to analyze these processes such as constituent quark model[3], pole model[3, 14], QCD sum rule[4], and light front approach[5]. Here we compare our results with them in Table 1.

In constituent quark model, the non-relativity character is considered. If we replace our B meson distribution amplitude in Eq. (6) by a δ function $(f_B/2\sqrt{6})\ \delta(x-m_q/m_B)$, our formula will return to the constituent quark model in Ref. [3]. Since we have poor knowledge about quark mass up to now, our new calculation is surely an improvement. In Ref. [3], the authors also calculate these processes in the pole model, and the results are similar to the quark model case. From Table 1, one can also see that most of the methods get similar results except the QCD sum rule approach, whose result is larger than others. We hope the experiments in future can test these different methods.

In Fig. 3 and Fig. 4, we figure out the differential decay rate of $B^0(B_s)\to\gamma\nu\bar{\nu}$ versus photon energy E_γ.

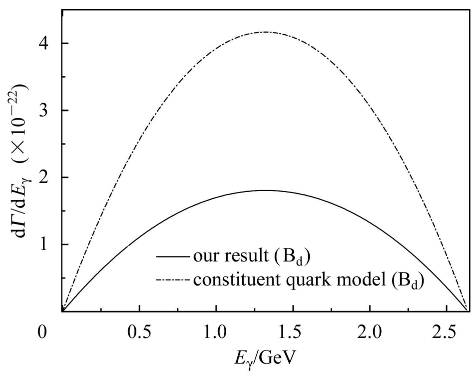

Fig. 3. Differential decay rate of $B^0 \to \gamma\nu\bar{\nu}$ versus the photon energy E_γ.

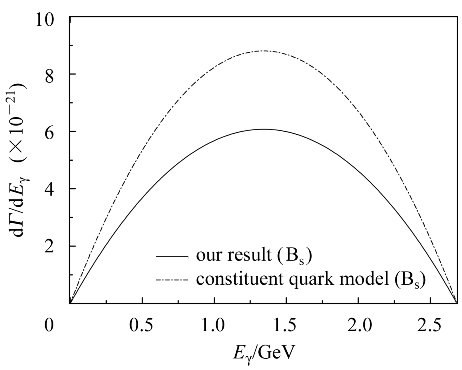

Fig. 4. Differential decay rate of $B_s \to \gamma\nu\bar{\nu}$ versus the photon energy E_γ.

We also display the photon energy spectrum from constituent quark model[1)]. From these figures, we find our results are smaller than the constituent quark one, but the shape of the spectrum is the same. If normalized decay rate is used, the two lines will become only one, since the function is very simple

$$f(x) = 24x(1-2x), \tag{18}$$

which can be extracted from Eq. (12).

Of course, there are also uncertainties in our calculation. The most largest uncertainty comes from the heavy meson wave function. The high order contribution, the high Fock states for wave function are also not included, because they are not quite clear now. We hope that the non-leptonic B meson decay can offer more information in the near future.

4 Summary

In this work, we calculate the branching ratios in SM for $B_s \to \gamma\nu\bar{\nu}$ to be 10^{-8} and for $B^0 \to \gamma\nu\bar{\nu}$ to be 10^{-9} using B meson wave function constrained by non-leptonic B decays. These decay channels are useful to determine the decay constants f_B and B meson wave function. After calculation, we find our leading order results are at the same order as other approaches but a little smaller. These rare decays are sensitive to any new physics contributions which can be measured by future experiment such as LHC-b.

We thank SHEN Y L and ZHU Jin for the help on the program, and also thank YU X Q and WANG Wei for helpful discussions.

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1)The results of constituent quark model are from updated parameters.

稀有衰变 $B^0(B_s)\to\gamma\nu\bar{\nu}$ 分支比的计算*

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摘要　在标准模型中, 三体稀有衰变 $B^0(B_s)\to\gamma\nu\bar{\nu}$ 只有通过箱图和企鹅图才可以发生. 这个过程对于确定B介子的衰变常数及其波函数有着较重要的物理意义, 由于这些衰变道的分支比较小, 因此也是探测新物理理论的比较好的场所. 利用B介子强衰变确定的波函数, 得到 $B^0(B_s)\to\gamma\nu\bar{\nu}$ 的分支比的数量级是 $10^{-9}(10^{-8})$, 这些结果可以在未来的实验上得到检验.

关键词　波函数　B介子稀有衰变　分支比　衰变常量

2005 – 09 – 11 收稿,　2005 – 11 – 25 收修改稿
*国家自然科学基金(10475085, 10135060)资助
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