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A Possible Relation between the Neutrino Mass Matrix and the Neutrino Mapping Matrix

R. Friedberg[1] T. D. Lee[1,2]

1 (Physics Department, Columbia University, New York, NY 10027, U.S.A.)

2 (China Center of Advanced Science and Technology(CCAST/World Lab.), Beijing 100080, China)

Abstract We explore the consequences of assuming a simple 3-parameter form, first without T-violation, for the neutrino mass matrix M in the basis ν_e, ν_μ, ν_τ with a new symmetry. This matrix determines the three neutrino masses m_1, m_2, m_3, as well as the mapping matrix U that diagonalizes M. Since U, without T-violation, yields three measurable parameters s_{12}, s_{23}, s_{13}, our form expresses six measurable quantities in terms of three parameters, with results in agreement with the experimental data. More precise measurements can give stringent tests of the model as well as determining the values of its three parameters. An extension incorporating T-violation is also discussed.

Key words neutrino mass operator, neutrino mapping matrix, T-violation

1 Neutrino mapping matrix without *T*-violation

In this paper we wish to explore further the connection between the neutrino mass operator $\mathscr{M}$ which contains three neutrino masses m_1, m_2, m_3 and the neutrino mapping matrix U, characterized by the standard four parameters θ_{12}, θ_{23}, θ_{13} and $e^{i\delta}$. For clarity, we first examine the special case that the T-violating phase parameter $\delta=0$. In terms of the mass eigenstates ν_1, ν_2 and ν_3 the neutrino mass operator is

$$\mathscr{M}=m_1\overline{\nu}_1\nu_1+m_2\overline{\nu}_2\nu_2+m_3\overline{\nu}_3\nu_3. \qquad (1.1)$$

Our assumption is that the same $\mathscr{M}$, when expressed in terms of ν_e, ν_μ and ν_τ, has a simple form with a new symmetry property:

$$\alpha(\overline{\nu}_\tau-\overline{\nu}_\mu)(\nu_\tau-\nu_\mu)+\beta(\overline{\nu}_\mu-\overline{\nu}_e)(\nu_\mu-\nu_e)+ m_0(\overline{\nu}_e\nu_e+\overline{\nu}_\mu\nu_\mu+\overline{\nu}_\tau\nu_\tau) \qquad (1.2)$$

also with three real parameters α, β and m_0. These three new parameters are to be determined by the mass eigenvalues m_1, m_2 and m_3. The transformation matrix U that brings $\mathscr{M}$ from (1.2) to (1.1) is the neutrino mapping matrix for $\delta=0$. (The general case when $\delta\neq0$ will be discussed in the next section.) Throughout the paper, we denote

$$\nu_i=\psi(\nu_i) \quad \text{and} \quad \overline{\nu}_i=\psi^\dagger(\nu_i)\gamma_4 \qquad (1.3)$$

with $\psi(\nu_i)$ a 4-component Dirac field operator, † denoting the hermitian conjugation and the index $i=1,\ 2,\ 3$ or e, μ, τ.

Since the neutrino mapping matrix U is independent of the overall mass-shift term m_0, in order for our hypothesis to be successful, there must be some special features about the first two terms in (1.2):

$$\alpha(\overline{\nu}_\tau-\overline{\nu}_\mu)(\nu_\tau-\nu_\mu)+\beta(\overline{\nu}_\mu-\overline{\nu}_e)(\nu_\mu-\nu_e). \qquad (1.4)$$

We note that (1.4) is invariant under the transformation

$$\nu_e\rightarrow\nu_e+z,\quad \nu_\mu\rightarrow\nu_\mu+z \quad \text{and} \quad \nu_\tau\rightarrow\nu_\tau+z \qquad (1.5)$$

with z a space-time independent constant element of the Grassmann algebra, anticommuting with the

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neutrino field operators ν_i. Thus, the usual equal-time anticommutation relations between the neutrino fields ν_i and their zero-mass free particle action-integral are invariant under (1.5). This symmetry is violated by the last m_0-dependent term in (1.2), as well as by T-violation, as we shall discuss later. The interesting case that z might be space-time dependent will not be discussed in this paper.

Expression (1.4) can be generalized to an equivalent form with three real parameters a, b and c:

$$a(\overline{\nu}_\tau-\overline{\nu}_\mu)(\nu_\tau-\nu_\mu)+b(\overline{\nu}_\mu-\overline{\nu}_e)(\nu_\mu-\nu_e)+c(\overline{\nu}_e-\overline{\nu}_\tau)(\nu_e-\nu_\tau). \tag{1.6}$$

The corresponding neutrino mass operator is

$$a(\overline{\nu}_\tau-\overline{\nu}_\mu)(\nu_\tau-\nu_\mu)+b(\overline{\nu}_\mu-\overline{\nu}_e)(\nu_\mu-\nu_e)+ c(\overline{\nu}_e-\overline{\nu}_\tau)(\nu_e-\nu_\tau)+m_0\sum_i\overline{\nu}_i\nu_i. \tag{1.7}$$

It is clear that (1.6) is also invariant under the transformation (1.5). The same invariance can also be expressed in terms of the transformation between the constants a, b and c, with

$$a\rightarrow a+\lambda,\quad b\rightarrow b+\lambda,\quad \text{and}\quad c\rightarrow c+\lambda. \tag{1.8}$$

As we shall prove, the form of the neutrino mapping matrix U remains unchanged under the transformation (1.8).

Since the relative phases between ν_e, ν_μ and ν_τ are unphysical, we may transform

$$\nu_e\rightarrow-\nu_e,\quad \nu_\mu\rightarrow-\nu_\mu\quad \text{and}\quad \nu_\tau\rightarrow\nu_\tau, \tag{1.9}$$

so that (1.7) is written in a less symmetric form, with

$$\mathscr{M} = a(\overline{\nu}_\tau+\overline{\nu}_\mu)(\nu_\tau+\nu_\mu)+b(\overline{\nu}_\mu-\overline{\nu}_e)(\nu_\mu-\nu_e)+ c(\overline{\nu}_e+\overline{\nu}_\tau)(\nu_e+\nu_\tau)+m_0\sum_i\overline{\nu}_i\nu_i. \tag{1.10}$$

The sole purpose of using this less symmetric expression of $\mathscr{M}$ is to have the resulting neutrino mapping matrix U in the standard form given by the particle data group[1]. We write (1.10) as

$$\mathscr{M}=(\overline{\nu}_e\ \overline{\nu}_\mu\ \overline{\nu}_\tau)(m_0+\overline{M})\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \tag{1.11}$$

where

$$\overline{M}=\begin{pmatrix} b+c & -b & c\\ -b & a+b & a\\ c & a & c+a\end{pmatrix}. \tag{1.12}$$

The neutrino mapping matrix U is defined by

$$U^\dagger(m_0+\overline{M})U=\begin{pmatrix} m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}. \tag{1.13}$$

Introduce a 3×1 column matrix

$$\phi_2\equiv\sqrt{\frac{1}{3}}\begin{pmatrix}1\\ 1\\ -1\end{pmatrix}. \tag{1.14}$$

One can readily verify that

$$\overline{M}\phi_2=0; \tag{1.15}$$

i.e., ϕ_2 is an eigenvector of $\overline{M}$ with eigenvalue 0. Let ϕ_1 and ϕ_3 be the other two real normalized eigenvectors of $\overline{M}$. Since

$$\tilde{\phi}_i\phi_j=\delta_{ij}, \tag{1.16}$$

with $\sim$ denoting the transpose, the neutrino mapping matrix U is

$$U=(\phi_1\ \phi_2\ \phi_3), \tag{1.17}$$

which, on account of (1.14) and (1.16), is given by

$$U=\begin{pmatrix} \sqrt{\frac{2}{3}}\cos\frac{\theta}{2} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}}\sin\frac{\theta}{2}\\ -\sqrt{\frac{1}{6}}\cos\frac{\theta}{2}+\sqrt{\frac{1}{2}}\sin\frac{\theta}{2} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{6}}\sin\frac{\theta}{2}+\sqrt{\frac{1}{2}}\cos\frac{\theta}{2}\\ \sqrt{\frac{1}{6}}\cos\frac{\theta}{2}+\sqrt{\frac{1}{2}}\sin\frac{\theta}{2} & -\sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{6}}\sin\frac{\theta}{2}+\sqrt{\frac{1}{2}}\cos\frac{\theta}{2}\end{pmatrix}, \tag{1.18}$$

in the approximation that the T-violating parameter $\delta=0$, with the angle $\theta/2$ denoting the azimuthal orientation of ϕ_1, ϕ_3 around the fixed eigenvector ϕ_2. Except for minor notational differences, the above U is the same expression first obtained by Harrison and Scott[2].

Next we return to the transformation (1.8), under which $\overline{M}$ of (1.12) transforms as

$$\overline{M} \to \overline{M} + \lambda \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

Since

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \phi_2 = 0,$$

the neutrino mapping matrix U remains given by (1.18). Setting

$$\lambda = -c, \tag{1.19}$$

we have

$$\begin{aligned} a &\to \alpha = a - c, \\ b &\to \beta = b - c, \\ c &\to 0. \end{aligned} \tag{1.20}$$

The corresponding neutrino mass operator $\mathscr{M}$ of (1.7) becomes (1.2). With the additional phase convention (1.9), $\mathscr{M}$ of (1.10) reduces to

$$\begin{aligned} \mathscr{M} \ =\ & \alpha(\overline{\nu}_\tau + \overline{\nu}_\mu)(\nu_\tau + \nu_\mu) + \\ & \beta(\overline{\nu}_\mu - \overline{\nu}_e)(\nu_\mu - \nu_e) + m_0 \sum_i \overline{\nu}_i \nu_i, \end{aligned} \tag{1.21}$$

which has only three parameters α, β and m_0. Of course, the mass operator (1.21) is a special case of the mass operator (1.10), which has 4 parameters a, b, c and m_0. It is of interest that they shares the same neutrino mapping matrix U given by (1.18), provided that $a-c=\alpha$ and $b-c=\beta$. Yet, the neutrino masses m_1, m_2 and m_3 in the two cases can be different, as can be readily seen by examining the trace of $\overline{M}$ given by (1.12). Therefore, the full physical contents of (1.21) and (1.10) are not the same. This is especially important when we generalize the model to include T-violation in the next section.

For the remaining part of this section, we shall explore further the physical consequences of our model, using only the more restrictive form (1.21) with three real parameters α, β and m_0.

It is instructive to re-derive (1.18) in a more elementary way. Write (1.21) as

$$\mathscr{M} = (\overline{\nu}_e\ \overline{\nu}_\mu\ \overline{\nu}_\tau)\ (\alpha M_\alpha + \beta M_\beta + m_0) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \tag{1.22}$$

with

$$M_\alpha = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{1.23}$$

and

$$M_\beta = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{1.24}$$

The matrix $\alpha M_\alpha + \beta M_\beta$ in (1.22) will be diagonalized in two steps. Introduce first a real orthogonal matrix[3, 4] U_0 by setting $\theta = 0$ in (1.18); i.e.,

$$U_0 = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \tag{1.25}$$

The matrix U_0 diagonalizes M_α, with

$$M'_\alpha = U_0^\dagger M_\alpha U_0 = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.26}$$

and transforms M_β to

$$M'_\beta = U_0^\dagger M_\beta U_0 = \frac{1}{2} \begin{pmatrix} 3 & 0 & -\sqrt{3} \\ 0 & 0 & 0 \\ -\sqrt{3} & 0 & 1 \end{pmatrix}. \tag{1.27}$$

Their sum $\alpha M'_\alpha + \beta M'_\beta$ can then be readily diagonalized with another real orthogonal transformation matrix

$$U_1 = \begin{pmatrix} \cos\frac{\theta}{2} & 0 & -\sin\frac{\theta}{2} \\ 0 & 1 & 0 \\ \sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{pmatrix} \tag{1.28}$$

with

$$\sin\theta = \left[(2\alpha-\beta)^2 + 3\beta^2\right]^{-\frac{1}{2}} \sqrt{3}\beta, \tag{1.29}$$

$$\cos\theta = \left[(2\alpha-\beta)^2 + 3\beta^2\right]^{-\frac{1}{2}} (2\alpha-\beta), \tag{1.30}$$

and therefore

$$\tan\theta = \frac{\sqrt{3}\beta}{2\alpha-\beta}. \tag{1.31}$$

The resulting transformation matrix $U = U_0 U_1$ satisfies

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{1.32}$$

and is given by (1.18). The corresponding masses m_1, m_2 and m_3 are related to α, β and m_0 by

$$m_1 = \alpha+\beta-\left(\alpha-\frac{\beta}{2}\right)\left[1+\frac{3\beta^2}{(2\alpha-\beta)^2}\right]^{\frac{1}{2}}+m_0, \tag{1.33}$$

$$m_2 = m_0 \tag{1.34}$$

and

$$m_3=\alpha+\beta+\left(\alpha-\frac{\beta}{2}\right)\left[1+\frac{3\beta^2}{(2\alpha-\beta)^2}\right]^{\frac{1}{2}}+m_0. \tag{1.35}$$

The matrix U depends only on one parameter θ, which in turn is determined by the ratio β/α.

In the standard parametric representation, the matrix element U_{13} is $s_{13}=\sin\theta_{13}$ when $e^{i\delta}=1$, with the experimental bound[1]

$$s_{13}^2=0.9\begin{array}{l}+2.3\\-0.9\end{array}\times 10^{-2}. \tag{1.36}$$

From (1.18), U_{13} is $-\sqrt{\frac{2}{3}}\sin\frac{\theta}{2}$. It follows then

$$\sin^2\frac{\theta}{2}=\frac{3}{2}\ s_{13}^2\ll 1. \tag{1.37}$$

Thus, by using (1.29)—(1.31) we see that

$$\left(\frac{\beta}{\alpha}\right)^2\ll 1, \tag{1.38}$$

which together with (1.33)—(1.35) yield the conclusion that m_1 and m_2 are very close, forming a doublet, and m_3 is the singlet. Their mass differences are given by approximate expressions:

$$m_2-m_1 = -\frac{3}{2}\beta+O\left(\frac{\beta^2}{\alpha}\right), \tag{1.39}$$

$$m_3-m_2 = 2\alpha+\frac{1}{2}\beta+O\left(\frac{\beta^2}{\alpha}\right) \tag{1.40}$$

and

$$m_3-\frac{1}{2}(m_1+m_2)=2\alpha-\frac{1}{4}\beta+O\left(\frac{\beta^2}{\alpha}\right). \tag{1.41}$$

From $m_1<m_2$, we conclude

$$\beta<0. \tag{1.42}$$

Furthermore, ν_3 is heavier or lighter than the doublet ν_1 and ν_2 depending on the sign of α, with

$$\begin{array}{lll} \alpha>0 & \text{for} & m_3>m_1 \text{ or } m_2, \\ \alpha<0 & \text{for} & m_3<m_1 \text{ or } m_2. \end{array} \tag{1.43}$$

Neglecting $O(\beta/\alpha)$ corrections, we have from (1.34), (1.39) and m_1 positive,

$$m_0>\frac{3}{2}|\beta| \tag{1.44}$$

and

$$\delta m^2\equiv m_2^2-m_1^2=\left(m_0-\frac{3}{4}|\beta|\right)3\ |\beta|. \tag{1.45}$$

Thus

$$\delta m^2>\frac{9}{4}\ \beta^2. \tag{1.46}$$

For

$$\Delta m^2\equiv m_3^2-\frac{1}{2}\ (m_2^2+m_1^2) \tag{1.47}$$

we find, neglecting $O(\beta^2)$,

$$\Delta m^2=4\alpha\ (\alpha+m_0)+\left(\frac{1}{2}m_0-2\alpha\right)|\beta|. \tag{1.48}$$

The experimental values for δm^2 and Δm^2 are given by[1]

$$\delta m^2=7.92\,(1\pm 0.09)\times 10^{-5}\text{eV}^2 \tag{1.49}$$

and

$$|\Delta m^2|=2.4\left(1\begin{array}{l}+0.21\\-0.26\end{array}\right)\times 10^{-3}\ \text{eV}^2. \tag{1.50}$$

Their ratio is

$$\frac{\delta m^2}{|\Delta m^2|}=3.3\left(1\begin{array}{l}+0.23\\-0.28\end{array}\right)\times 10^{-2}. \tag{1.51}$$

Next, we analyze first the case that the singlet ν_3 is of a lower mass than the doublet masses; i.e., $\alpha<0$. In that case, since $m_3>0$, (1.26) yields

$$m_3=m_0-2|\alpha|-\frac{1}{2}|\beta|+O\left(\frac{\beta^2}{\alpha}\right)>0;$$

therefore

$$m_0>2|\alpha|. \tag{1.52}$$

Neglecting $O(\beta/\alpha)$ corrections in (1.45) and (1.48), we have

$$\left|\frac{\delta m^2}{\Delta m^2}\right|=\frac{3}{4}\left|\frac{\beta}{\alpha}\right|\frac{m_0}{m_0-|\alpha|}, \tag{1.53}$$

which gives

$$\frac{3}{2}\left|\frac{\beta}{\alpha}\right|>\left|\frac{\delta m^2}{\Delta m^2}\right|>\frac{3}{4}\left|\frac{\beta}{\alpha}\right|. \tag{1.54}$$

Combining this expression with (1.51), we find

$$4.4\times10^{-2} > \left|\frac{\beta}{\alpha}\right| > 2.2\times10^{-2}. \tag{1.55}$$

On the other hand, from (1.29) and to the same accuracy, we have

$$\sin^2\theta = \frac{3\beta^2}{4\alpha^2}, \tag{1.56}$$

which on account of (1.36) gives

$$\frac{\beta^2}{\alpha^2} = \left(0.72 \begin{array}{c} +1.84 \\ -0.72 \end{array}\right)\times10^{-1}. \tag{1.57}$$

While (1.55) is barely consistent with (1.57), the compatibility depends on that, within one standard of deviation, (1.57) is also consistent with $\beta^2/\alpha^2=0$ (i.e., $s_{13}^2=0$). Thus, this "compatibility" between (1.51) and (1.57) is definitely not a comfortable one. A more accurate determination of U_{13} may well rule out the case that ν_3 can be lighter than the doublet ν_1, ν_2. Within our model, we also made a similar analysis for the case that the singlet ν_3 is heavier than the doublet ν_1, ν_2. In that case, $\alpha>0$ and the situation is quite different; there is no incompatibility between (1.51) and (1.57).

Remark. We note that if $\beta=0$ in (1.21) then there is only one term

$$\alpha(\overline{\nu}_\tau+\overline{\nu}_\mu)(\nu_\tau+\nu_\mu) \tag{1.58}$$

that is relevant for the determination of the mapping matrix; correspondingly, in the mass operator (1.22) we need only to consider αM_α, with M_α given by (1.23). Introducing a 45^0 rotation matrix

$$R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \tag{1.59}$$

we have

$$\tilde{R}_1 M_\alpha R_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \tag{1.60}$$

Because of the degeneracy in its first two eigenvalues, $\tilde{R}_1 M_\alpha R_1$ commutes with any unitary matrix of the form

$$\begin{pmatrix} \begin{matrix} & u & \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{pmatrix}, \tag{1.61}$$

where u is a 2×2 unitary matrix. Thus there is a one-parameter family of solutions for the neutrino mass eigenstates.

The situation is quite different when

$$\left|\frac{\beta}{\alpha}\right| = 0+. \tag{1.62}$$

As mentioned before, because of the invariance (1.5) and the phase convention (1.9),

$$\nu_2 = \sqrt{\frac{1}{3}}(\nu_e+\nu_\mu-\nu_\tau) \tag{1.63}$$

is a mass eigenstate. Furthermore, the transformation matrix

$$U_0 = R_1 R_2 \tag{1.64}$$

is completely determined, with

$$R_2 = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.65}$$

which is a rotation of angle$=\sin^{-1}\sqrt{\frac{1}{3}}$. For β/α small but nonzero, the mapping matrix U deviates from U_0 through the small parameter θ, as given by (1.18).

2 Neutrino mapping matrix with *T*-violation

We generalize the neutrino mass operator $\mathscr{M}$ by inserting phase factors $\mathrm{e}^{\pm i\eta}$ into (1.6), replacing it by

$$a(\overline{\nu}_\tau-\overline{\nu}_\mu)(\nu_\tau-\nu_\mu)+b(\overline{\nu}_\mu-\overline{\nu}_e)(\nu_\mu-\nu_e)+ \\ c(\mathrm{e}^{-i\eta}\overline{\nu}_e-\overline{\nu}_\tau)(\mathrm{e}^{i\eta}\nu_e-\nu_\tau) \tag{2.1}$$

where a, b, c and η are all real. When $\eta=0$, (2.1) becomes (1.6), and is invariant under the symmetry (1.5). Furthermore, if $\mathrm{e}^{i\eta}\neq\pm1$, T-invariance is also violated. As in (1.6), in order to conform to the standard form of the neutrino mapping matrix U given by the particle data group[1], we make the phase transformation $\nu_e\rightarrow-\nu_e$, $\nu_\mu\rightarrow-\nu_\mu$ and $\nu_\tau\rightarrow\nu_\tau$, the mass operator (1.10) is then replaced by

$$\mathscr{M} = a(\overline{\nu}_\tau+\overline{\nu}_\mu)(\nu_\tau+\nu_\mu)+b(\overline{\nu}_\mu-\overline{\nu}_e)(\nu_\mu-\nu_e)+ c(e^{-i\eta}\overline{\nu}_e+\overline{\nu}_\tau)(e^{i\eta}\nu_e+\nu_\tau)+m_0\sum_i\overline{\nu}_i\nu_i, \quad (2.2)$$

which can be written as

$$\mathscr{M}=(\overline{\nu}_e\ \overline{\nu}_\mu\ \overline{\nu}_\tau)\ M\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \quad (2.3)$$

where

$$M=aM_a+bM_b+cM_c+m_0 \quad (2.4)$$

with

$$M_a=\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} \quad (2.5)$$

$$M_b=\begin{pmatrix}1&-1&0\\-1&1&0\\0&0&0\end{pmatrix}, \quad (2.6)$$

identical to M_α and M_β given by (1.23) and (1.24), and

$$M_c=\begin{pmatrix}1&0&e^{-i\eta}\\0&0&0\\e^{i\eta}&0&1\end{pmatrix}. \quad (2.7)$$

As in (1.25)—(1.27), we first perform the U_0 transformation. Let

$$M_c'\equiv\tilde{U}_0M_cU_0=\begin{pmatrix}\frac{1}{6}(5+4\cos\eta) & \frac{1}{3}\sqrt{\frac{1}{2}}(1+e^{i\eta}-2e^{-i\eta}) & \frac{1}{2}\sqrt{\frac{1}{3}}(1+2e^{-i\eta})\\ \frac{1}{3}\sqrt{\frac{1}{2}}(1+e^{-i\eta}-2e^{i\eta}) & \frac{2}{3}(1-\cos\eta) & \sqrt{\frac{1}{6}}(-1+e^{-i\eta})\\ \frac{1}{2}\sqrt{\frac{1}{3}}(1+2e^{i\eta}) & \sqrt{\frac{1}{6}}(-1+e^{i\eta}) & \frac{1}{2}\end{pmatrix}. \quad (2.8)$$

Next, we apply the U_1 transformation given by (1.28), and write

$$\tilde{U}_1\tilde{U}_0MU_0U_1=H_0+ch \quad (2.9)$$

where H_0 is diagonal, given by

$$H_0=\begin{pmatrix}\mu_1&0&0\\0&\mu_2&0\\0&0&\mu_3\end{pmatrix} \quad (2.10)$$

with μ_1, μ_2, μ_3 the same ones in (1.33)—(1.35), except for the replacement of α, β by a, b; i.e.,

$$\mu_1 = a+b-\left(a-\frac{b}{2}\right)\left[1+\frac{3b^2}{(2a-b)^2}\right]^{\frac{1}{2}}+m_0,$$
$$\mu_2 = m_0 \quad (2.11)$$

and

$$\mu_3=a+b+\left(a-\frac{b}{2}\right)\left[1+\frac{3b^2}{(2a-b)^2}\right]^{\frac{1}{2}}+m_0.$$

In (2.9)

$$h=\tilde{U}_1M_c'U_1. \quad (2.12)$$

Since U_0 and U_1 are real and symmetric, h is a hermitian.

It is useful to decompose h into real and imaginary parts:

$$h=h^R+ih^I \quad (2.13)$$

where

$$h^I=\sin\eta\begin{pmatrix}0 & \sqrt{\frac{1}{2}}\cos\frac{\theta}{2}+\sqrt{\frac{1}{6}}\sin\frac{\theta}{2} & -\sqrt{\frac{1}{3}}\\ -\sqrt{\frac{1}{2}}\cos\frac{\theta}{2}-\sqrt{\frac{1}{6}}\sin\frac{\theta}{2} & 0 & -\sqrt{\frac{1}{6}}\cos\frac{\theta}{2}+\sqrt{\frac{1}{2}}\sin\frac{\theta}{2}\\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{6}}\cos\frac{\theta}{2}-\sqrt{\frac{1}{2}}\sin\frac{\theta}{2} & 0\end{pmatrix} \quad (2.14)$$

and the matrix elements of h^R are given by

$$h^R_{11} = \frac{1}{3}\left[2+\frac{1}{2}\cos\theta+(1+\cos\theta)\cos\eta\right]+ \sqrt{\frac{1}{3}}\left(\frac{1}{2}+\cos\eta\right)\sin\theta,$$

$$h^R_{22} = \frac{2}{3}(1-\cos\eta),$$

$$h^R_{33} = \frac{1}{3}(2+\cos\eta)-\frac{1}{6}(1+2\cos\eta)\cos\theta- \frac{1}{2}\sqrt{\frac{1}{3}}(1+2\cos\eta)\sin\theta, \tag{2.15}$$

$$h^R_{12} = h^R_{21}=\frac{1}{3}\sqrt{\frac{1}{2}}\left(\cos\frac{\theta}{2}-\sqrt{3}\sin\frac{\theta}{2}\right)(1-\cos\eta),$$

$$h^R_{13} = h^R_{31}=\frac{1}{6}(\sqrt{3}\cos\theta-\sin\theta)(1+2\cos\eta)$$

and

$$h^R_{23}=h^R_{32}=-\sqrt{\frac{1}{6}}(\cos\frac{\theta}{2}+\frac{1}{\sqrt{3}}\sin\frac{\theta}{2})(1-\cos\eta).$$

The presence of h^I violates T-invariance.

We note from (2.14) that the element

$$\mathrm{i}\,h^I_{13}=-\mathrm{i}\sqrt{\frac{1}{3}}\sin\eta \tag{2.16}$$

is of particular importance for testing T-invariance. Furthermore, there are at least three cases to be considered:

i) $|c|\ll|b|$; then T-violation is much smaller than the present upper limit, regardless of η.

ii) $|c|\sim O[|b|]$ but $|\sin\eta|\ll 1$; then T-violation is again very small on account of the prefactor $\sin\eta$ in (2.14).

iii) $|c|\sim O[|b|]$ and $|\sin\eta|\sim O[1]$; then T-violation can be close to the present upper limit.

The diagonalization of the 3×3 matrix (2.9) is simplified in case i). In that case, $|c|$ is much less than $|b|$ and $|a|$. The mass eigenstates and the correction to the neutrino mapping matrix can be readily obtained by using the standard first order perturbation formula.

Another simple case is $|\eta|\ll 1$, which includes the above case ii). Decompose (2.7) into a sum

$$M_c=(M_c)_0+\Delta \tag{2.17}$$

with

$$(M_c)_0=\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix} \tag{2.18}$$

and

$$\Delta=\begin{pmatrix}0&0&\mathrm{e}^{-\mathrm{i}\eta}-1\\0&0&0\\\mathrm{e}^{\mathrm{i}\eta}-1&0&0\end{pmatrix}. \tag{2.19}$$

Correspondingly, (2.4) can be written as

$$M=M_0+c\Delta \tag{2.20}$$

with

$$M_0=aM_a+bM_b+c(M_c)_0+m_0. \tag{2.21}$$

M_0 can be diagonalized by the same unitary matrix (1.18), with the angle θ given by (1.29)—(1.31), in which α and β are given by (1.20). For $|\eta|\ll 1$, Δ is small; the neutrino mapping matrix U can then be derived by using (2.20) and treating $c\Delta$ as a small perturbation.

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中微子质量矩阵和中微子转换矩阵间的一种可能的关系

R. Friedberg[1] 李政道[1,2]

1 (Physics Department, Columbia University, New York, NY 10027, U.S.A.)

2 (中国高等科学技术中心 北京 100080)

摘要 我们探讨了以 ν_e, ν_μ, ν_τ 为基的, 具有一种新的对称性的中微子质量矩阵 M. 首先在没有 T (时间)破坏的前提下, 假定该质量矩阵具有一个简单的三参数形式. 这一矩阵确定了 3 种中微子的质量 m_1, m_2, m_3 以及使 M 对角化的转换矩阵 U. 因为无 T 破坏的 U 给出 3 个可测量参数 s_{12}, s_{23}, s_{13}, 我们的形式用 3 个参数表示 6 个可测量的物理量, 其结果与实验数据符合得很好. 更精确的测量将对模型给出严格的检验, 并确定这 3 个参数的值. 本文还推广讨论了包含 T 破坏的情况.

关键词 中微子质量算符 中微子转换矩阵 T(时间)破坏

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