Precise correction to parameter ρ in the littlest Higgs model

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Abstract In this paper tree-level violation of weak isospin parameter, ρ in the frame of the littlest Higgs model is studied. The potentially large deviation from the standard model prediction for the ρ in terms of the littlest Higgs model parameters is calculated. The maximum value for ρ for $f = 1$ TeV, $c = 0.05$, $c' = 0.05$ and $v' = 1.5$ GeV is $\rho = 1.2973$ which means a large enhancement than the SM.

Key words the littlest Higgs model, custodial symmetry violation

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1 Introduction

Despite the success of the Standard Model (SM)[1, 2], there still exist several problems such as the hierarchy problem[3, 4] which motivate much of the current research work about new physics beyond the SM.

Among the extended models beyond the SM, the little Higgs model offers a new solution to the "hierarchy problem" in which the Higgs boson is naturally light as a result of non-linearly realized symmetry[5—12]. The first successful little Higgs model was constructed by Arkani-Hamed, Cohen and Georgi, which can cancel the relevant quadratic divergences based on the pseudo-Goldstone idea[5]. After that more models were constructed such as the minimal moose $SU(3)^2/SU(3)$[6], $SU(6)/SP(6)$[7], $SU(5)/SO(5)$[8] and the general moose $SU(3)^n/SU(3)^k$[9].

The most economical model of them is the littlest Higgs (LH) model[10], which is based on an $SU(5)/SO(5)$ nonlinear sigma model[8]. It consists of a $SU(5)$ global symmetry, which is spontaneously broken down to $SO(5)$ by a vacuum condensate f. In the LH model, a set of new heavy gauge bosons (A_H, Z_H, W_H) and a new heavy vector-like quark (T) are introduced which just cancel the quadratic divergence induced by the SM gauge boson loops and the top quark loop, respectively[5—12]. Physicists expect that the LH model also can give reasonable explanations to the problem as well as the MSSM[13]. It should be mentioned that precise measurement of electroweak observables in the scale of the LH model is effected by bounds which refer to the model typicality. One of these bounds on electroweak correction comes from the custodial $SU(2)$ symmetry violation and ρ parameter in the LH model which bounds the scale of Λ[14—21] and also gives rise to the top and bottom quark masses[22]. The possibility of using ρ as a handle on limiting the Higgs self-coupling is suggested in Ref. [23] and the importance of correction to the weak isospin parameter is emphasized in Refs. [22, 24—27]. In the little Higgs model both of the $SU(2)$'s are gauged so the custodial symmetry explicitly has been broken and ρ shifts to a large deviation from the SM prediction and ρ=1 is no longer acceptable[28—30]. The aim of this paper is to derive an expression for ρ in terms of the LH model parameters based on the corrections in LH model presented in Refs. [14, 17, 28—31]. We will find that the deviation of ρ parameter is more than what is illustrated in Ref. [20].

In Section 2 we briefly introduce the littlest Higgs model, in Section 3 we derive the ρ based on the model parameters and in Section 4 the presented plots show the numerical results.

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2 The related theory of the LH model

In the littlest Higgs model the vacuum expectation value (VEV) associated with the spontaneous symmetry breaking at the scale $\Lambda_s \sim 4\pi f$[17, 28]. The vacuum expectation value (VEV) breaks the $SU(5)$ global symmetry into its subgroup $SO(5)$ and breaks the local gauge symmetry $[SU(2)\otimes U(1)]^2$ into its diagonal subgroup $SU(2)_L\otimes U(1)_Y$ at the same time, which is identified as the SM electroweak gauge group. As we expect, the breaking of the gauge symmetry $[SU(2)\otimes U(1)]^2$ into its diagonal subgroup $SU(2)_L\otimes U(1)_Y$ gives rise to heavy gauge bosons W′ and B′, and the remaining unbroken subgroup $SU(2)_L\otimes U(1)_Y$ introduces the massless gauge bosons W and B.

The spontaneous gauge symmetry breaking gives rise to mass terms of order f for the gauge boson with the field rotation to the mass eigenstates given by:

$$\begin{aligned} &W = sW_1 + sW_2, \quad W' = cW_1 - sW_2, \\ &B = s'B_1 + c'B_2, \quad B' = -c'B_1 + s'B_2, \end{aligned} \tag{1}$$

where c and c' are the mixing angles between two $SU(2)$'s and $U(1)$'s in the LH model and are given by:

$$c = \frac{g_1}{\sqrt{g_1^2+g_2^2}}, \qquad c' = \frac{g_1'}{\sqrt{g_1'^2+g_2'^2}}. \tag{2}$$

The SM gauge coupling g and g' can be expressed as $g = g_1 s = g_2 c$ and $g' = g_1' s' = g_2' c'$ respectively. At scale f, the SM gauge fields remain massless, and the heavy gauge bosons are massive:

$$\begin{aligned} m_{W'} &= \frac{f}{2}\sqrt{g_1^2+g_2^2} = \frac{g}{2sc} f, \\ m_{B'} &= \frac{f}{2\sqrt{5}}\sqrt{g_1'^2+g_2'^2} = \frac{g'}{2\sqrt{5}s'c'} f. \end{aligned} \tag{3}$$

For Higgs doublet and triplet fields, VEV's are parameterized as υ and υ' respectively that $\upsilon = 2\frac{m_W}{g}$[17].

Since the triplet Higgs boson mass $M_\Phi^2 = \frac{2m_H^2 f^2}{\upsilon^2}\frac{1}{\left[1-(4\upsilon' f/\upsilon^2)^2\right]}$ must be positive, we can find a relation between υ and υ' as $\frac{\upsilon'}{\upsilon} < \frac{\upsilon}{4f}$[28].

The relevant expressions of the masses of new gauge bosons and the couplings for our calculation can be found in Ref. [28]. The masses of charged gauge bosons W_L and W_Z are expressed in terms of the LH model parameters as:

$$\begin{aligned} M^2_{W_L^\pm} &= m_W^2\left[1-\frac{\upsilon^2}{f^2}\left(\frac{1}{6}+\frac{1}{4}(c^2-s^2)^2\right)+4\frac{\upsilon'^2}{\upsilon^2}\right], \\ M^2_{W_H^\pm} &= m_W^2\left(\frac{f^2}{s^2c^2\upsilon^2}-1\right), \end{aligned} \tag{4}$$

and the neutral gauge bosons masses are expressed as:

$$\begin{aligned} M^2_{A_L} &= 0, \\ M^2_{A_H} &= m_Z^2 s_W^2\left(\frac{f^2}{5s'^2c'^2\upsilon^2}-1+\frac{x_H c_W^2}{4s^2c^2s_W^2}\right), \\ M^2_{Z_L} &= m_Z^2\Bigg[1-\frac{\upsilon^2}{f^2}\Bigg(\frac{1}{6}+\frac{1}{4}(c^2-s^2)^2+ \\ &\qquad \frac{5}{4}(c'^2-s'^2)^2\Bigg)+8\frac{\upsilon'^2}{\upsilon^2}\Bigg], \\ M^2_{Z_H} &= m_W^2\left(\frac{f^2}{s^2c^2\upsilon^2}-1-\frac{x_H s_W^2}{s'^2c'^2c_W^2}\right), \end{aligned} \tag{5}$$

in which;

$$x_H = \frac{5}{2}gg'\frac{scs'c'\left(c^2s'^2+s^2c'^2\right)}{5g^2s'^2c'^2-g'^2s^2c^2},$$

that m_Z and m_W are the masses of neutral and charged gauge bosons predicted by the SM respectively, and at the tree-level it keeps the relation $c_W = \cos\theta_W = m_W/m_Z$, where θ_W is the weak mixing angle in the SM and $s_W^2 = 1-c_W^2$.

3 ρ in terms of the LH model parameters

Precise measurement of electroweak observables in the scope of the LH model requires modification on ρ parameter as one of the bounds which comes from the LH model typicality. In the SM custodial symmetry is conserved and $\rho = \frac{m_W^2}{m_Z^2 c_W^2}=1$. In the LH model both of the $SU(2)$'s are gauged therefore the relation of $\rho=1$ is modified at the tree level under shadow of extra input parameter, vacuum expectation value (VEV) of the Higgs triplet (υ')[32, 33]. Now we derive the ρ in the LH model which is defined by $\rho = \frac{M^2_{W_L}}{M^2_{Z_L}c_\theta^2}$[17, 29, 34] which $M^2_{W_L}$ and $M^2_{Z_L}$ are given by Eq. (4) and Eq. (5) respectively and c_θ is the effective leptonic mixing angle. The correction to the weak isospin parameter is independent of the choice of the gauge coupling[14, 28]. For gauge bosons the coupling to fermions is in the form of $i\gamma^\mu(g_V+g_A\gamma^5)$ in which g_V and g_A are the vector and axial vector couplings respectively. Now we can find s_θ from $\frac{g_V}{g_A} = 4s_\theta - 1$[14, 15]. For the $Z_L e\bar{e}$ coupling, g_V and g_A are given as below[28];

$$\begin{aligned} g_V^e = \frac{g}{2c_W}\Bigg\{&(-1/2+2s_W^2)+ \\ &\frac{\upsilon^2}{f^2}\left[-c_W x_Z^{W'}\frac{c}{2s}+\frac{s_W x_Z^{B'}}{s'c'}\left(2y_e-\frac{9}{5}+\frac{3}{2}c'^2\right)\right]\Bigg\}, \end{aligned} \tag{6}$$

$$g_A^{\rm e}=\frac{g}{2c_{\rm W}}\left\{1/2+\frac{v^2}{f^2}\left[c_{\rm W}x_{\rm Z}^{\rm W'}\frac{c}{2s}+\frac{s_{\rm W}x_{\rm Z}^{\rm B'}}{s'c'}\left(-\frac{1}{5}+\frac{1}{2}c'^2\right)\right]\right\}, \tag{7}$$

in which $x_{\rm Z}^{\rm B'}=-\frac{5}{2s_{\rm W}}s'c'(c'^2-s'^2)$ and $x_{\rm Z}^{\rm W'}=-\frac{1}{2c_{\rm W}}sc(c^2-s^2)$. Then we will arrive at:

$$4s_\theta^2-1=(4s_{\rm W}^2-1)+\frac{2v^2}{f^2}\left[s_{\rm W}^2c^2(c^2-s^2)-c_{\rm W}^2(c'^2-s'^2)(-2+5c'^2)\right]. \tag{8}$$

Now we can extract c_θ^2 as fallows;

$$c_\theta^2=1-s_{\rm W}^2-\frac{v^2}{2f^2}\left[s_{\rm W}^2c^2(c^2-s^2)-c_{\rm W}^2(c'^2-s'^2)(-2+5c'^2)\right]. \tag{9}$$

On the other hand we need to give some correction to the effective Fermi coupling $G_{\rm f}$ in the LH model. So we use the effective Lagrangian of the charged current interaction as below[14, 28];

$$\begin{aligned}L_{\rm c}=&gW_{{\rm L}\mu}^aJ^{a\mu}\left(1+\frac{c^2(s^2-c^2)h^2}{f^2}\right)+\\&g'B_{{\rm L}\mu}J_{\rm Y}^\mu\left(1-\frac{5c'^2(s'^2-c'^2)h^2}{f^2}\right)+\\&gW_{{\rm L}\mu}^3J_{\rm Y}^\mu\frac{5(s'^4-c'^4)h^2}{f^2}-\\&g'B_{{\rm L}\mu}J^{3\mu}\frac{c^2(s^2-c^2)h^2}{f^2}-\\&J_\mu^aJ^{a\mu}\frac{2c^4}{f^2}-J_\mu^{\rm Y}J^{{\rm Y}\mu}\frac{10c'^4}{f^2}.\end{aligned} \tag{10}$$

Then by integrating out the $W_{\rm L}$ bosons, we will get the expression below for the effective four-fermion operator;

$$-\frac{g^2}{2M_{\rm W}^2}J^{+\mu}J_\mu^-\left[1+\frac{c^2(s^2-c^2)v^2}{f^2}\right]-J^{+\mu}J_\mu^-\frac{2c^4}{f^2}=-2\sqrt{2}G_{\rm f}J^{+\mu}J_\mu^-, \tag{11}$$

where $J^\pm=\frac{1}{2}(J^1\pm{\rm i}J^2)$. After considering the W mass correction, we finally obtain the corrected expression of $G_{\rm f}$ in the LH model as[14, 34]:

$$\frac{1}{\sqrt{2}G_{\rm f}}=v^2\left(1+\frac{v^2}{4f^2}+4\frac{v'^2}{f^2}\right). \tag{12}$$

By inverting Eq. (12) to obtain v in terms of $G_{\rm f}$, f and v' we will get;

$$v^2=\frac{1}{\sqrt{2}G_{\rm f}}\left(1-\frac{1}{4\sqrt{2}G_{\rm f}f^2}-4\frac{v'^2}{f^2}\right). \tag{13}$$

For more simplifying we assign A, B, C and D as below:

$$A=\frac{1}{6}+\frac{1}{4}(c^2-s^2)^2, \tag{14}$$

$$B=s_{\rm W}^2c^2(c^2-s^2)-c_{\rm W}^2(c'^2-s'^2)(-2+5c'^2), \tag{15}$$

$$C=\frac{\sqrt{2}G_{\rm f}v'^2}{\left(1-\frac{1}{4\sqrt{2}G_{\rm f}f^2-4\frac{v'^2}{f^2}}\right)}, \tag{16}$$

$$D=\frac{1}{\sqrt{2}G_{\rm f}f^2}\left(1-\frac{1}{4\sqrt{2}G_{\rm f}f^2}-4\frac{v'^2}{f^2}\right), \tag{17}$$

So finally the expression for ρ in terms of model parameters, mixing angel $c_{\rm W}$ and $G_{\rm f}$ using Eqs. (6), (11) and (13) is:

$$\rho=c_{\rm W}^2\times\frac{1-CA+4D}{\left(1-DA-\frac{5}{4}D(c'^2-s'^2)^2+8C\right)\left(1-s_{\rm W}^2-\frac{1}{2}DB\right)}. \tag{18}$$

4 Numerical results

The fallowing input parameters has been used: $m_{\rm Z}$ = 91.1876 GeV and $m_{\rm W}$=80.425 GeV to determine the mixing angle $c_{\rm W}$, and $G_{\rm f}=1.16637\times10^{-5}$ GeV^{-2}. For evaluation of ρ in the framework of LH model the following model parameters have been used: the global symmetry breaking scale f in range of 1 TeV< f <5 TeV[30], the Higgs triplet vacuum expectation value v' (GeV), and the cosine mixing angles for charged and neutral gauge boson c and c'.

Figure 1 shows the variation of ρ as the functions of the global symmetry breaking scale f when the other model parameters are fixed as c=0.5, c'=0.7 and v'=15 GeV. As we see the ρ value decreases very sharply by increasing f from 1 to 2.5 TeV. For f >2.5 TeV the curve decreases slightly to reach the minimum value ρ=1.01. So ρ is very sensitive to f.

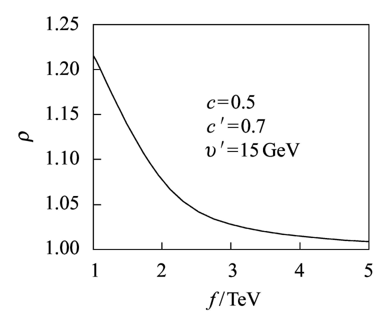

Fig. 1. ρ as the functions of global symmetry breaking scale f in case of c=0.5, c'=0.7 and v'=15 GeV.

Figure 2 shows the ρ under variation of c while f=1 TeV, c'=0.7 and v'=15 GeV, ρ has a minimum value 1.214 when c=0.654 and for c more and less than this value ρ has a very sharp increase to the maximum value 1.23. So ρ is also very sensitive to c parameter.

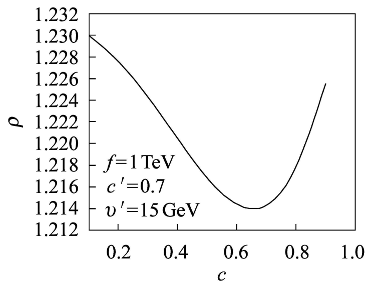

Fig. 2. ρ as the functions of c while f=1 TeV, c'=0.7 and v'=15 GeV.

Figure 3 shows the variation of ρ as a function of c' when the other model parameters are fixed as f=1 TeV, c=0.1 and v'=15 GeV. As we can see ρ is not too sensitive to c' because the figure shows that the difference between the maximum and minimum value of ρ is just 0.03 and the curve is very smooth.

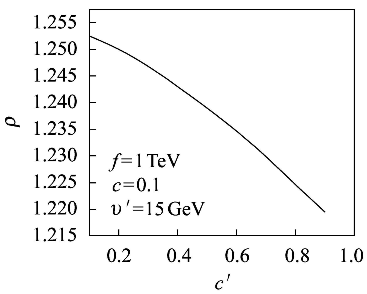

Fig. 3. ρ as a function of c' when the other model parameters are fixed as f=1 TeV, c=0.1 and v'=15 GeV.

Figure 4 shows the variation of ρ as a function of v' when the other model parameters are fixed as f=1 TeV, c=0.1 and c'=0.1 and by increasing v' the value of ρ will decrease. In this figure we realize that ρ is sensitive to v' parameter but not as much as f and c parameters as we noticed in Figs. 1 and 2.

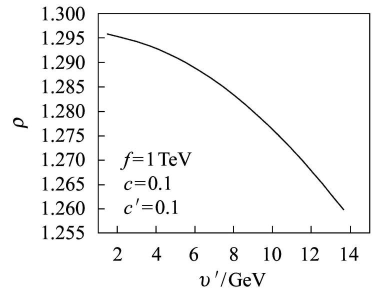

Fig. 4. ρ as a function of v' when the other model parameters are fixed as f=1 TeV, c=0.1 and c'=0.1.

5 Summary

The littlest Higgs model is a very interesting extension of the SM. It can be an alternative candidate of new physics beyond the SM which solves the little hierarchy problem. The LH model predicts a set of new particles and modifies the SM-like gauge boson couplings to other SM-like particles. The deviation of ρ value from unit as a conclusion of symmetry breaking in the LH in this paper has been calculated and it is realized that it is very sensitive to the global symmetry breaking scale f, c and v' and not very sensitive to c'. The maximum value of $\Delta\rho(=\rho{-}1)$ for f=1 TeV, c=0.05, c'=0.05 and v'=1.5 GeV is 0.2973 which is a large deviation from the SM prediction (ρ=1).

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