Massive and massless neutrinos on unbalanced seesaws*

XING Zhi-Zhong(邢志忠)[1,2;1)]

1 (Institute of High Energy Physics, CAS, Beijing 100049, China)
2 (Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei 230026, China)

Abstract The observation of neutrino oscillations requires new physics beyond the standard model (SM). A SM-like gauge theory with p lepton families can be extended by introducing q heavy right-handed Majorana neutrinos but preserving its $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge symmetry. The overall neutrino mass matrix M turns out to be a symmetric $(p+q)\times(p+q)$ matrix. Given $p>q$, the rank of M is in general equal to $2q$, corresponding to $2q$ non-zero mass eigenvalues. The existence of $(p-q)$ massless left-handed Majorana neutrinos is an exact consequence of the model, independent of the usual approximation made in deriving the Type-I seesaw relation between the effective $p\times p$ light Majorana neutrino mass matrix M_ν and the $q\times q$ heavy Majorana neutrino mass matrix $M_{\rm R}$. In other words, the numbers of massive left- and right-handed neutrinos are fairly matched. A good example to illustrate this "seesaw fair play rule" is the minimal seesaw model with $p=3$ and $q=2$, in which one massless neutrino sits on the unbalanced seesaw.

Key words neutrino mass, type-I seesaw, majorana neutrinos

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1 Introduction

Very robust evidence for non-zero neutrino masses and large lepton flavor mixing has recently been achieved from solar[1], atmospheric[2], reactor[3] and accelerator[4] neutrino oscillation experiments. This great breakthrough opens a new window to physics beyond the standard model (SM). So far a number of theoretical scenarios have been proposed, either at low energy scales or at high energy scales, to understand why the masses of neutrinos are considerably smaller than those of charged leptons and quarks[5]. Among them, the seesaw mechanism[6] seems to be most elegant and natural. In particular, an appropriate combination of the seesaw mechanism and the leptogenesis mechanism[7] allows one to simultaneously account for the observed neutrino oscillations and the observed matter-antimatter asymmetry of the Universe.

The canonical (Type-I) seesaw idea is rather simple, indeed. By introducing three right-handed Majorana neutrinos to the SM and keeping its Lagrangian invariant under the $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge transformation, one may write out a normal Dirac neutrino mass term relevant to the electroweak symmetry breaking ($M_{\rm D}$) and an extra Majorana neutrino mass term irrelevant to the electroweak symmetry breaking ($M_{\rm R}$). Given $M_{\rm D}$ as the seesaw fulcrum at or close to the electroweak symmetry breaking scale ($\sim 10^2$ GeV), the smallness of three left-handed neutrino masses (<1 eV) is then attributed to the largeness of three right-handed neutrino masses ($>10^{13}$ GeV)[6]: $M_\nu \approx -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^{\rm T}$. Since both $M_{\rm D}$ and $M_{\rm R}$ are in general the rank-3 matrices, M_ν is also of rank 3 and thus has three non-vanishing mass eigenvalues.

Can massive and massless neutrinos coexist in a general seesaw scenario? Such a question makes sense for two simple reasons. On the one hand, current neutrino oscillation data do allow one of the light neutrinos to be massless or almost massless (e.g., either $m_1\to 0$ or $m_3 \to 0$[8]). On the other hand, it is conceptually interesting to distinguish between the neutrino with an exact zero mass and the neutrino with a vanishingly small mass. An affirmative answer to the above question has been observed in Refs. [9, 10]. The purpose of this short note is to have a new look at the properties of massive and massless neutrinos in the generalized Type-I seesaw mechanism. We

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1) E-mail: xingzz@ihep.ac.cn

shall consider a straightforward extension of the SM with p lepton families, q heavy right-handed Majorana neutrinos and the $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge symmetry. The overall neutrino mass matrix M in this model turns out to be a symmetric $(p+q)\times(p+q)$ matrix. Given $p>q$, the rank of M is in general equal to $2q$, corresponding to $2q$ non-zero mass eigenvalues. We demonstrate that the existence of $(p-q)$ massless left-handed Majorana neutrinos is an exact consequence of the model, independent of the usual approximation made in deriving the Type-I seesaw relation between the effective $p\times p$ light Majorana neutrino mass matrix M_ν and the $q\times q$ heavy Majorana neutrino mass matrix $M_{\rm R}$. We refer to this kind of seesaw, in which the number of left-handed neutrinos is larger than the number of right-handed neutrinos, as the unbalanced seesaw. The fact that the numbers of massive left- and right-handed Majorana neutrinos are fairly matched on unbalanced seesaws can be referred to as the "seesaw fair play rule". A well-known example is the minimal seesaw model with $p=3$ and $q=2$[8], in which one massless neutrino sits on the unbalanced seesaw. The stability of $m_i=0$ against radiative corrections from the seesaw scale down to the electroweak scale will also be stressed.

2 Unbalanced seesaws

Let us consider a simple extension of the SM with p lepton families and q heavy right-handed Majorana neutrinos. The Lagrangian of this electroweak model is required to be invariant under the $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge transformation. To be explicit, the lepton mass terms can be written as

$$-\mathscr{L}=\overline{l_{\rm L}}Y_{\rm l}e_{\rm R}H+\overline{l_{\rm L}}Y_\nu N_{\rm R}H^{\rm c}+\frac{1}{2}\overline{N^{\rm c}_{\rm R}}M_{\rm R}N_{\rm R}+{\rm h.c.}\,, \quad (1)$$

where $l_{\rm L}$ denotes the left-handed lepton doublets; $e_{\rm R}$ and $N_{\rm R}$ stand respectively for the right-handed charged-lepton and Majorana neutrino singlets; H is the Higgs-boson weak isodoublet (with $H^{\rm c}\equiv {\rm i}\sigma_2 H^*$); $M_{\rm R}$ is the $q\times q$ heavy Majorana neutrino mass matrix; $Y_{\rm l}$ and Y_ν are the coupling matrices of charged-lepton and neutrino Yukawa interactions. After spontaneous gauge symmetry breaking, the neutral component of H acquires the vacuum expectation value $v\approx 174$ GeV. Then we arrive at the $p\times p$ charged-lepton mass matrix $M_{\rm l}=vY_{\rm l}$ and the $p\times q$ Dirac neutrino mass matrix $M_{\rm D}=vY_\nu$. Eq. (1) turns out to be

$$-\mathscr{L}'=\overline{e_{\rm L}}M_{\rm l}e_{\rm R}+\frac{1}{2}\overline{(\nu_{\rm L}\,N^{\rm c}_{\rm R})}\begin{pmatrix}0 & M_{\rm D}\\ M^{\rm T}_{\rm D} & M_{\rm R}\end{pmatrix}\begin{pmatrix}\nu^{\rm c}_{\rm L}\\ N_{\rm R}\end{pmatrix}+{\rm h.c.}\,, \quad (2)$$

where e, $\nu_{\rm L}$ and $N_{\rm R}$ represent the column vectors of p charged-lepton fields, p left-handed neutrino fields and q right-handed neutrino fields, respectively. In obtaining Eq. (2), we have made use of the relation $\overline{\nu_{\rm L}}M_{\rm D}N_{\rm R}=\overline{N^{\rm c}_{\rm R}}M^{\rm T}_{\rm D}\nu^{\rm c}_{\rm L}$ as well as the properties of $\nu_{\rm L}$ (or $N_{\rm R}$) and $\nu^{\rm c}_{\rm L}$ (or $N^{\rm c}_{\rm R}$)[11]. Note that the mass scale of $M_{\rm R}$ can naturally be much higher than the electroweak scale v, because those right-handed Majorana neutrinos are $SU(2)_{\rm L}$ singlets and their corresponding mass term is not subject to the magnitude of v. The overall neutrino mass matrix

$$M=\begin{pmatrix}0 & M_{\rm D}\\ M^{\rm T}_{\rm D} & M_{\rm R}\end{pmatrix} \quad (3)$$

is a symmetric $(p+q)\times(p+q)$ matrix and can be diagonalized by the transformation

$$U^\dagger M U^*=\begin{pmatrix}m_1 & & & & & \\ & \ddots & & & & \\ & & m_{\rm p} & & & \\ & & & M_1 & & \\ & & & & \ddots & \\ & & & & & M_{\rm q}\end{pmatrix}\,, \quad (4)$$

where U is a unitary matrix, m_i (for $i=1,\cdots,p$) denote the masses of p left-handed Majorana neutrinos, and M_j (for $j=1,\cdots,q$) denote the masses of q right-handed Majorana neutrinos. If the mass scale of $M_{\rm R}$ is considerably higher than that of $M_{\rm D}$, one may obtain the effective light neutrino mass matrix

$$M_\nu\approx -M_{\rm D}M^{-1}_{\rm R}M^{\rm T}_{\rm D} \quad (5)$$

as an extremely good approximation[12]. In this Type-I seesaw scenario, the mass eigenvalues of M_ν and $M_{\rm R}$ are m_i (for $i=1,\cdots,p$) and M_j (for $j=1,\cdots,q$), respectively, to a high degree of accuracy. Of course, $m_i\ll v$ and $M_j\gg v$ naturally hold. Our concern is whether some of m_i can in general be vanishing.

We focus on the $p>q$ case, since the $p<q$ case is less motivated from the viewpoint of maximum simplicity and predictability in building a seesaw model and interpreting the experimental data. Given $p>q$, the rank of M_ν is determined by that of $M_{\rm R}$ through the seesaw relation $M_\nu\approx -M_{\rm D}M^{-1}_{\rm R}M^{\rm T}_{\rm D}$. Namely, M_ν must be of rank q in general1). Because the number

1) Here "in general" means that any contrived textures of $M_{\rm D}$, which might reduce the rank of M_ν from q to a smaller integer, are not taken into account. Without loss of generality, $M_{\rm R}$ can always be taken to be diagonal and positive. In this basis, a too special texture of $M_{\rm D}$ is usually disfavored in order to simultaneously account for current neutrino oscillation data and the cosmological baryon number asymmetry[5, 8].

of non-zero eigenvalues of a symmetric matrix is equal to the rank of this matrix[13], we can conclude that M_ν has $(p-q)$ vanishing mass eigenvalues. Note that this statement relies on the Type-I seesaw relation which directly links $M_{\rm R}$ to M_ν. Taking account of the approximation made in deriving this seesaw formula (no matter how good it is), we have to clarify whether the $(p-q)$ mass eigenvalues of M_ν are exactly vanishing or only vanishingly small. A reliable proof or disproof of the above statement should be independent of the approximate seesaw relation.

So what we need to do is to calculate the rank of M in Eq. (3). Taking

$$M_{\rm D} = \begin{pmatrix} D_{11} & \cdots & D_{1q} \\ \vdots & \ddots & \vdots \\ D_{p1} & \cdots & D_{pq} \end{pmatrix} ,$$
$$M_{\rm R} = \begin{pmatrix} R_{11} & \cdots & R_{1q} \\ \vdots & \ddots & \vdots \\ R_{q1} & \cdots & R_{qq} \end{pmatrix} , \tag{6}$$

where $R_{ij}=R_{ji}$ (for $i,j=1,\cdots,q$), we write out the explicit expression of M:

$$M = \begin{pmatrix} 0 & \cdots & 0 & D_{11} & \cdots & D_{1q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & D_{p1} & \cdots & D_{pq} \\ & & & & & \\ D_{11} & \cdots & D_{p1} & R_{11} & \cdots & R_{1q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ D_{1q} & \cdots & D_{pq} & R_{q1} & \cdots & R_{qq} \end{pmatrix} . \tag{7}$$

By definition, the rank of M is the number of non-zero rows in the reduced row echelon form of M. The latter can be calculated by using the method of Gauss elimination. Because the upper-left $p\times p$ sub-matrix is a zero matrix, it is easy to convert the upper-right $p\times q$ sub-matrix (i.e., $M_{\rm D}$) into a reduced row echelon form in which the first $(p-q)$ rows are full of zero elements. In contrast, the lower-right $q\times q$ sub-matrix (i.e., $M_{\rm R}$) is of rank q. The rank of M turns out to be $p-(p-q)+q=2q$, corresponding to $2q$ non-zero mass eigenvalues. In other words, q of the p light Majorana neutrinos must be massive, and the remaining $(p-q)$ light Majorana neutrinos must be exactly massless.

If a seesaw scenario includes unequal numbers of light (left-handed) and heavy (right-handed) Majorana neutrinos, it can be referred to as an unbalanced seesaw scenario. When the number of light neutrinos is larger than that of heavy neutrinos, such an unbalanced seesaw is actually balanced because all the redundant light neutrinos are massless. That is, the number of massive left-handed Majorana neutrinos is fairly equal to the number of heavy right-handed Majorana neutrinos. We refer to this interesting observation, which is independent of the approximation made in deriving the Type-I seesaw formula, as the "seesaw fair play rule". One can see later on that such a rule is not only conceptually appealing but also applicable to an instructive and phenomenologically-favored model, the minimal seesaw model[8].

3 Further discussions

To be realistic, one has to fix $p=3$ for the number of left-handed neutrinos. Then only $q=1$ and $q=2$ are of interest for the discussion of unbalanced seesaw scenarios. The $q=1$ case is not favored in the Type-I seesaw framework, because it requires two left-handed Majorana neutrinos to be massless and thus cannot accommodate two independent neutrino mass-squared differences observed in solar and atmospheric neutrino oscillations (i.e., $\Delta m^2_{21}=m^2_2-m^2_1\approx 8\times10^{-5}$ eV^2 and $\Delta m^2_{32}=m^2_3-m^2_2\approx \pm2.5\times10^{-3}$ eV^2[14]). On the other hand, the $q=2$ case is compatible with current experimental data and has been referred to as the minimal seesaw model[8] for the study of both neutrino mixing and baryogenesis via leptogenesis.

According to the "seesaw fair play rule", there must exist one massless neutrino in the minimal seesaw model. One may also get at this point by calculating the determinant of the 5×5 neutrino mass matrix M, in which the Dirac neutrino mass matrix $M_{\rm D}$ is 3×2 and the right-handed Majorana neutrino mass matrix $M_{\rm R}$ is 2×2. It is very straightforward to prove ${\rm Det}M=0$. Since $|{\rm Det}M|=m_1m_2m_3M_1M_2$ holds, one of m_i (for $i=1,2,3$) must be vanishing. The solar neutrino oscillation experiment has fixed $m_2>m_1$[14], and thus we are left with two distinct possibilities:

1) $m_1=0$, corresponding to a normal neutrino mass hierarchy. Taking account of current experimental data, we can easily obtain $m_2=\sqrt{\Delta m^2_{21}}\approx 8.9\times10^{-3}$ eV and $m_3=\sqrt{\Delta m^2_{21}+|\Delta m^2_{32}|}\approx 5.1\times10^{-2}$ eV.

2) $m_3=0$, corresponding to an inverted neutrino mass hierarchy. Taking account of current experimental data, we arrive at $m_1=\sqrt{|\Delta m^2_{32}|-\Delta m^2_{21}}\approx 4.9\times10^{-2}$ eV and $m_2=\sqrt{|\Delta m^2_{32}|}\approx 5.0\times10^{-2}$ eV.

Note that it is possible to build viable neutrino models[15] to accommodate both a special neutrino mass spectrum with $m_1=0$ or $m_3=0$ and the (nearly) tri-bimaximal neutrino mixing pattern [16]. Some of such models can even provide a natural interpretation of the cosmological baryon number asymmetry via (resonant) leptogenesis.

It is worth mentioning that $m_1=0$ (or $m_3=0$) is stable against radiative corrections from the seesaw scale (usually measured by the lightest right-handed Majorana neutrino mass M_1) down to the electroweak scale (usually characterized by the Z^0 mass M_{Z} or simply the vacuum expectation value of the neutral Higgs field v) in the minimal seesaw model, at least at the one-loop level[17]. This observation is also expected to be true for a general unbalanced seesaw scenario with $p>q$; namely, the zero masses of left-handed Majorana neutrinos in such a scenario are insensitive to radiative corrections between the scales M_{Z} and M_1. Therefore, it makes sense to study the phenomenology of unbalanced seesaw models in which massive and massless neutrinos coexist.

4 Summary

To summarize, we have considered a SM-like $SU(2)_{\mathrm{L}}\times U(1)_{\mathrm{Y}}$ gauge theory with p lepton families and q heavy right-handed Majorana neutrinos. Given $p>q$, we have shown that the overall $(p+q)\times(p+q)$ neutrino mass matrix M is in general of rank $2q$, corresponding to $2q$ non-zero mass eigenvalues. An important emphasis is that the existence of $(p-q)$ massless left-handed Majorana neutrinos is an exact consequence of the model, independent of the usual approximation made in deriving the Type-I seesaw relation between the effective $p\times p$ light Majorana neutrino mass matrix M_ν and the $q\times q$ heavy Majorana neutrino mass matrix M_{R}. In other words, the numbers of massive left- and right-handed neutrinos are fairly matched in such an unbalanced seesaw scenario. We have taken the minimal seesaw model (with $p=3$ and $q=2$) as a simple but realistic example, in which one massless left-handed neutrino coexists with two massive left-handed neutrinos, to illustrate this "seesaw fair play rule".

Since the seesaw mechanism is a particularly natural, concise and appealing mechanism to understand the smallness of left-handed Majorana neutrino masses, its potential properties deserve further investigation. The main point of this note is that massless and massive neutrinos can coexist in an unbalanced seesaw scenario, if the number of heavy right-handed Majorana neutrinos is smaller than that of light left-handed Majorana neutrinos. Whether one of the light neutrinos is really massless or not remains an open question, but it is certainly a meaningful question and should be answered experimentally in the future. On the theoretical side, it is also of interest to explore a complete seesaw picture for neutrino mass generation, lepton flavor mixing, CP violation and leptogenesis with mismatched numbers of light and heavy Majorana neutrinos.

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