Isospin and mixed symmetry structure in ^{24}Mg*

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Abstract The interacting boson model with isospin (IBM-3) has been used to study the isospin excitation states and electromagnetic transitions for ^{24}Mg nucleus. The mixed symmetry states at low spin are also analyzed. The theoretical calculations are in agreement with experimental data. The present calculations indicate that the 3_1^+ state is the lowest mixed symmetry state.

Key words IBM-3, energy level, isospin, mixed symmetry states, electromagnetic transitions

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1 Introduction

The study of the lighter nucleus is very important. During recent years, Long G. L., Sun Y.[1] studied the superdeformed band in ^{36}Ar by the projected shell model and predicted the electromagnetic transitions. Later this prediction has been verified. Falih H.Al-Khudair,Y. S. Li and G. L. Long[2] studied the properties of the 0_2^+ state and isospin excitation in the $N=Z$ nucleus ^{68}Se by the interacting boson model (IBM-3). It is very important for understanding the nuclei nature in the lighter nucleus. ^{24}Mg lies in the lighter nuclei region and is one even-even $N=Z$ nucleus. Many researchers have studied the ^{24}Mg nucleus theoretically with different models such as the self-consistent 3D-cranking model, and the microscopic three-cluster model[3, 4]. By making use of the interacting boson model (IBM-3), we study the isospin excitation states, the electromagnetic transitions and the mixed symmetry states at low spin for ^{24}Mg nucleus. The interacting boson model (IBM) is one algebraic model describing the nucleus collective motion, which was introduced by Arima and Iachello. In the Bohr and Mottelson theory (BBM), the nucleus is anentity with geometry shape and the nucleus collective motion is studied by using dynamic variables of the transformation parameters. The IBM reveals the new dynamic symmetric characters and establishes the wave functions based on the degrees of freedom of boson and group method. IBM model is a very effective phenomenological model for describing low-lying collective motions of nuclei. In the original version (IBM-1), only one kind of boson is considered, and it has been successful in describing various properties of medium and heavy even-even nuclei[5—10]. The proton and neutron in the heavy nuclei lie in different major shells and the bosons are further classified into proton-boson and neutron-boson in IBM-2. In IBM-2, mixed symmetry in the proton and neutron degrees of freedom has been predicted[11] and discovered in electron-scattering experiment[12]. Many theoretical investigations of mixed symmetry state have been carried out (see, for example Refs. [13,14]). For lighter nuclei, the valence protons and valence neutrons are filling the same major shell and the isospin should be taken into account, so the IBM has been extended to the interacting boson model with isospin (IBM-3)[15]. In the IBM-3, three types of bosons including proton-proton boson $(\mathrm{s}_\pi, \mathrm{d}_\pi)$, neutron-neutron boson $(\mathrm{s}_\nu, \mathrm{d}_\nu)$ and proton-neutron boson $(\mathrm{s}_\delta, \mathrm{d}_\delta)$. The three s-boson and three d-boson form the isospin $T=1$ triplet. The microscopic fundament of IBM-3 is based on the shell model[16—18]. The dynamical symmetry group for IBM-3 is $U(18)$, which starts with $U_{sd}(6)\times U_c(3)$ and must contain $SU_T(2)$ and $O(3)$ as subgroups because the isospin and the angular momentum are good quantum numbers. The natural chains of IBM-3 group $U(18)$ are the following[19]

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$$U(18)\supset(U_c(3)\supset SU_T(2))\times (U_{sd}(6)\supset U_d(5)\supset O_d(5)\supset O_d(3)),$$
$$U(18)\supset(U_c(3)\supset SU_T(2))\times (U_{sd}(6)\supset O_{sd}(6)\supset O_d(5)\supset O_d(3)),$$
$$U(18)\supset(U_c(3)\supset SU_T(2))\times (U_{sd}(6)\supset SU_{sd}(3)\supset O_d(3)).$$

The subgroups $U_d(5)$, $O_{sd}(6)$ and $SU_{sd}(3)$ describe vibrational, γ-unstable and rotational nuclei respectively.

In the lighter nuclei region where the protons and neutrons are in the same major shell, the IBM-3 can describe the low-energy levels of some nuclei well and explain their isospin and F-spin symmetry structure[20—23]. The ^{24}Mg is the even-even $N=Z$ nucleus which lies in the lighter nuclei region. By using the IBM-3 model, it is easy to distinguish the typical mixed symmetry states, which are caused by the relative motions between proton bosons and neutron bosons. The mixed symmetry states have important significance in the study of nuclear structure. This paper is principally aimed at calculating and discussing the isospin structure, and the mixed symmetry structure of ^{24}Mg at low spin in the IBM-3.

2 The IBM-3 Hamiltonian and the parameter

The isospin-invariant IBM-3 Hamiltonian can be written as[15]

$$H=\varepsilon_s\hat{n}_s+\varepsilon_d\hat{n}_d+H_2\,, \tag{1}$$

where

$$H_2=\frac{1}{2}\sum_{L_2T_2}C_{L_2T_2}((d^+d^+)^{L_2T_2}\cdot(\tilde{d}\tilde{d})^{L_2T_2})+\frac{1}{2}\sum_{T_2}B_{0T_2}((s^+s^+)^{0T_2}\cdot(\tilde{s}\tilde{s})^{0T_2})+\sum_{T_2}A_{2T_2}((s^+d^+)^{2T_2}\cdot(\tilde{d}\tilde{s})^{2T_2})+\frac{1}{\sqrt{2}}\sum_{T_2}D_{2T_2}((s^+d^+)^{2T_2}\cdot(\tilde{d}\tilde{d})^{2T_2})+\frac{1}{2}\sum_{T_2}G_{0T_2}((s^+s^+)^{0T_2}\cdot(\tilde{d}\tilde{d})^{0T_2}), \tag{2}$$

and

$$(b_1^+b_2^+)^{L_2T_2}\cdot(\tilde{b}_3\tilde{b}_4)^{L_2T_2}=(-1)^{(L_2+T_2)}\sqrt{(2L_2+1)(2T_2+1)}[(b_1^+b_2^+)^{L_2T_2}\times(\tilde{b}_3\tilde{b}_4)^{L_2T_2}]^{00}, \tag{3}$$

$$\tilde{b}_{lm,m_z}=(-1)^{(l+m+1+m_z)}b_{l-m-m_z}\,. \tag{4}$$

Where T_2 and L_2 represent the two-boson system isospin and angular momentum. The parameters A, B, C, D and G are the two-body matrix elements. $A_{T_2}=\langle sd20|H_2|sd20\rangle$, T_2=0, 1, 2; $B_{T_2}=\langle s^20T_2|H_2|s^20T_2\rangle$, $G_{T_2}=\langle s^20T_2|H_2|d^20T_2\rangle$, $D_{T_2}=\langle sd2T_2|H_2|d^22T_2\rangle$ and $C_{L_2T_2}=\langle d^2L_2T_2|H_2|d^2L_2T_2\rangle$, with T_2=0,2, L_2=0, 2, 4; $C_{L_21}=\langle d^2L_21|H_2|d^2L_21\rangle$ with L_2=1, 3. The parameters A_1, C_{11}, C_{31} are Majorana parameters which are similar to those in the IBM-2. By making use of IBM-3, we assume ^{16}O as the core, so there are four valence protons and four valence neutrons. In order to analyze the symmetry structures of ^{24}Mg nucleus, we have rewritten the Hamiltonian in terms of a linear combination of the corresponding Casimir operators. In the Casimir operator form, the Hamiltonian is

$$H_{\rm Casimir}=\lambda C_{2U_{sd}(6)}+a_TT(T+1)+a_1C_{1U_d(5)}+a_3C_{2SU_{sd}(3)}+a_2C_{2U_d(5)}+a_4C_{2O_d(5)}+a_5C_{O_d(3)}\,. \tag{5}$$

Where λ parameter can be used to determine the position of the mixed symmetry states. The parameters in the Hamiltonian can be determined by fitting to the experimental spectra. The low-lying levels of ^{24}Mg can be described by the following Hamiltonians,

$$H_{\rm Casimir}=0.224C_{2U_{sd}(6)}+1.06T(T+1)+0.04C_{1U_d(5)}-0.145C_{2SU_{sd}(3)}+0.545C_{2U_d(5)}+0.053C_{2O_d(5)}-0.01C_{O_d(3)}\,. \tag{6}$$

The energy levels and wave function are given by the computation program written by Van Isacker[24]. The parameters of the calculation are listed in Table 1.

Table 1. The parameters of Hamiltonian of ^{24}Mg.

$\varepsilon_{s_\nu}=\varepsilon_{s_\pi}$	2.014		
$\varepsilon_{d_\nu}=\varepsilon_{d_\pi}$	4.931		
$A_i(i=0,1,2)$	−4.372	−1.988	1.988
$C_{i0}(i=0,2,4)$	−2.716	−1.666	−3.846
$C_{i2}(i=0,2,4)$	3.644	4.694	2.524
$C_{i1}(i=1,3)$	−3.084	−3.184	
$B_i(i=0,2)$	−3.792	2.568	
$D_i(i=0,2)$	1.085	1.085	
$G_i(i=0,2)$	−1.297	−1.297	

The calculated and experimental energy levels[25] are exhibited in Fig. 1. From Fig. 1 we can see that when the spin value is below 8^+, the theoretical calculations are in agreement with experimental data. ^{24}Mg nucleus has large deformations. The ratio $E(4_1^+)/E(2_1^+)=3.01$ is close to 3.3. The collective motion of the nuclei shows rotation character strongly. In the calculations, the ground-state band shows the obvious rotation property and the result agrees with experiments. It is noticed that the isotope exhibits backbanding in the ground band, which can be explained by the collective backbanding proposed in Ref. [26].

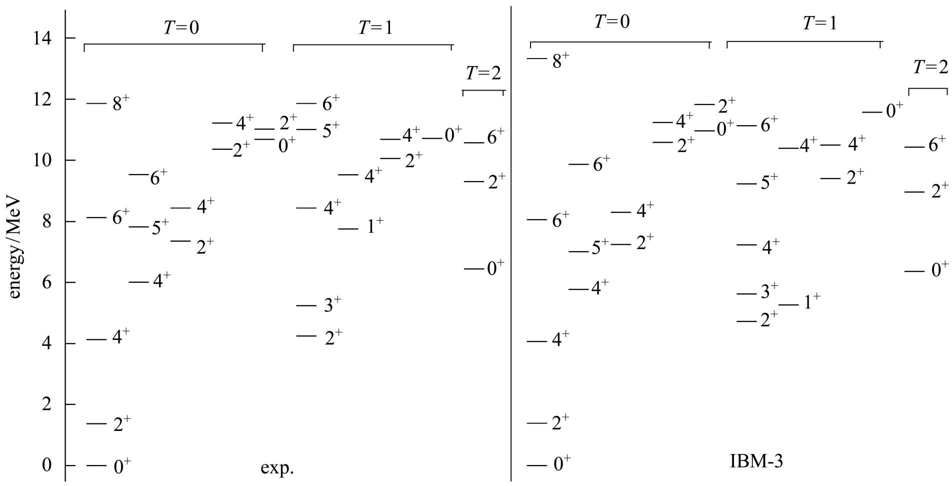

Fig. 1. Comparison between lowest excitation energy bands ($T=T_z$, T_z+1, T_z+2) of the IBM-3 calculation and experimental excitation energies of ^{24}Mg.

From Fig. 1, we see that the 0_2^+ state is the band head of an isospin excitation with $T=2$, and it is quite close to the experimental level. The 1_1^+ and 3_1^+ states are identified in isospin $T=1$ excited states in our calculations. The calculated energy value of 3_1^+state is 5.621 MeV, and close to the experimental one of 5.235 MeV. It is found that the calculated 1_1^+ state at 5.256 MeV is lower than the experimental one at 7.747 MeV. In the IBM-3 Hamiltonian one can fit this level by changing the Majorana interaction with L_2=1.

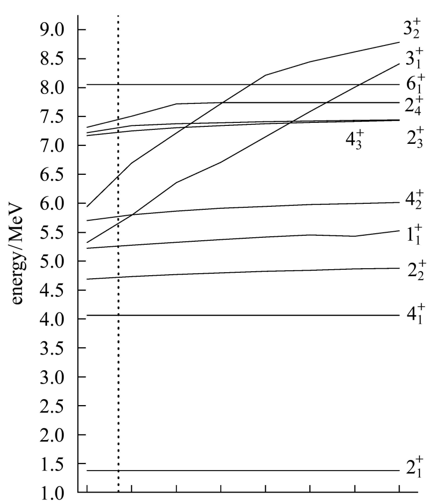

Fig. 2. Variation in level energy of ^{24}Mg as a function of C_{31}; all other parameters were kept at their best-fit values.

The C_{11}, C_{31} and A_1 are Majorana parameters, the variation of which greatly affects the mixed symmetry states. Fig. 2 and Fig. 3 show that the 1_1^+, 3_1^+, 3_2^+ and 2_4^+ states are the mixed symmetry states. The 3_1^+ state is the lowest mixed symmetry state, whose experimental energy level is 5.235 MeV. The main components of the wave function for 1_1^+, 3_1^+, 3_2^+ and 2_4^+ states are

$$\begin{aligned}
|1_1^+\rangle \;=\;& 0.8239|\mathrm{d}_\upsilon\mathrm{s}_\upsilon\mathrm{d}_\pi\mathrm{s}_\pi\rangle - 0.218|\mathrm{d}_\upsilon\mathrm{s}_\upsilon(\mathrm{d}_\pi^2)_2\rangle - \\
& 0.218|(\mathrm{d}_\upsilon^2)_2\mathrm{d}_\pi\mathrm{s}_\pi\rangle + 0.147|(\mathrm{d}_\upsilon^2)_4(\mathrm{d}_\pi^2)_4\rangle + \\
& 0.206|\mathrm{s}_\upsilon\mathrm{d}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle - 0.2931|\mathrm{d}_\upsilon\mathrm{d}_\pi(\mathrm{s}_\delta^2)_0\rangle - \\
& 0.206|\mathrm{d}_\upsilon\mathrm{s}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle + \cdots,
\end{aligned}$$

$$\begin{aligned}
|3_1^+\rangle \;=\;& -0.8312|\mathrm{d}_\upsilon\mathrm{s}_\upsilon\mathrm{d}_\pi\mathrm{s}_\pi\rangle - 0.1054|\mathrm{d}_\upsilon\mathrm{s}_\upsilon(\mathrm{d}_\pi^2)_2\rangle - \\
& 0.1054|(\mathrm{d}_\upsilon^2)_2\mathrm{d}_\pi\mathrm{s}_\pi\rangle - 0.1772|(\mathrm{d}_\upsilon^2)_2(\mathrm{d}_\pi^2)_4\rangle + \\
& 0.2078|\mathrm{s}_\upsilon\mathrm{d}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle - 0.2078|\mathrm{d}_\upsilon\mathrm{s}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle + \cdots,
\end{aligned}$$

$$\begin{aligned}
|3_2^+\rangle \;=\;& -0.47|\mathrm{s}_\upsilon\mathrm{d}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle + 0.47|\mathrm{d}_\upsilon\mathrm{s}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle + \\
& 0.1239|\mathrm{d}_\upsilon\mathrm{d}_\pi(\mathrm{d}_\delta^2)_4\rangle - 0.6646|\mathrm{d}_\upsilon\mathrm{d}_\pi(\mathrm{s}_\delta^2)_0\rangle + \\
& -0.21|\mathrm{d}_\upsilon\mathrm{d}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle + \cdots,
\end{aligned}$$

$$\begin{aligned}
|2_4^+\rangle \;=\;& -0.3377|\mathrm{d}_\upsilon\mathrm{d}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle + 0.3257|\mathrm{d}_\upsilon\mathrm{d}_\pi\mathrm{d}_\delta\mathrm{s}_\delta\rangle - \\
& 0.2388\{|\mathrm{d}_\upsilon\mathrm{s}_\pi(\mathrm{d}_\delta^2)_4\rangle + |\mathrm{s}_\upsilon\mathrm{d}_\pi(\mathrm{d}_\delta^2)_4\rangle + \\
& |\mathrm{d}_\upsilon\mathrm{s}_\nu(\mathrm{d}_\pi^2)_4\rangle\} + 0.2303\{|(\mathrm{d}_\nu^2)_0\mathrm{d}_\pi\mathrm{s}_\pi\rangle + \\
& |\mathrm{s}_\upsilon\mathrm{d}_\nu(\mathrm{d}_\pi^2)_0\rangle + |\mathrm{s}_\upsilon\mathrm{d}_\pi(\mathrm{d}_\delta^2)_0\rangle + |\mathrm{s}_\upsilon\mathrm{d}_\nu(\mathrm{d}_\delta^2)_0\rangle\}\cdots.
\end{aligned}$$

Where $(\mathrm{d}_\upsilon^2)_2$ means that two (d_υ) bosons couple to $L=2$. The composition of the 1_1^+, 3_1^+, 3_2^+ and 2_4^+ state is four bosons and each state contains a δ boson component. "$\cdots$" represents some smaller component.

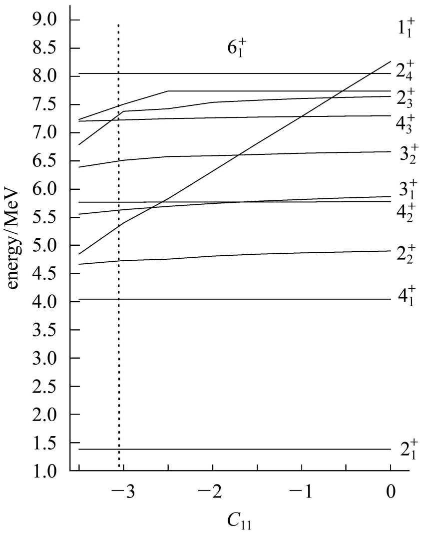

Fig. 3. Variation in level energy of ^{24}Mg as a function of C_{11}; all other parameters were kept at their best-fit values.

3 Electromagnetic transition

In the IBM-3 model, the quadrupole operator is expressed as[27]:

$$Q = Q^0 + Q^1 , \tag{7}$$

where

$$Q^0 = \alpha_0\sqrt{3}[(s^+\hat{d})^{20} + (d^+\hat{s})^{20}] + \beta_0\sqrt{3}[(d^+\hat{d})]^{20} , \tag{8}$$

$$Q^1 = \alpha_1\sqrt{2}[(s^+\hat{d})^{21} + (d^+\hat{s})^{21}] + \beta_1\sqrt{2}[(d^+\hat{d})^{21}] . \tag{9}$$

The M1 transition is also a one-boson operator with an isoscalar part and an isovector part

$$M = M^0 + M^1 , \tag{10}$$

where

$$M^0 = g_0\sqrt{3}(d^+\hat{d})^{10} = g_0 L/\sqrt{10} , \tag{11}$$

$$M^1 = g_1\sqrt{2}(d^+\hat{d})^{11} , \tag{12}$$

where g_0 and g_1 are the isoscalar and isovector g-factors respectively, and L is the angular momentum operator. For the ^{24}Mg, the parameters in the electromagnetic transitions are determined by fitting the experimental data, where $\alpha_0 = \beta_0$=0.0441, α_1=0.0332, β_1=0.0452, g_0=0, g_1=0.2335 respectively. Table 2 gives the electromagnetic transition rate calculated by IBM-3.

The study shows that there is a deviation of electromagnetic transitions in high isospin and medium and heavy nuclei. The g boson needs to be introduced[28]. ^{24}Mg lies in the lighter nuclei region. The calculation results are shown in Table 2 without g boson. Table 2 shows that the calculated B(E2) values are quite close to the experimental ones[25]. The results show that g boson may not be needed in the lighter nuclei region. As to the mixed symmetry states J_{ms}^+, their values of $B(\mathrm{M1};\ J_{ms}^+ \to J_s^+)$ are larger, while the corresponding $B(\mathrm{E2};\ J_{ms}^+ \to J)$ values are small[29]. Table 2 shows that the 1_1^+ and 3_1^+ states are the mixed symmetry states as their values of $B(\mathrm{E2};\ 1_1^+ \to J_s^+)$ and $B(\mathrm{E2};\ 3_1^+ \to J_s^+)$ are smaller but the corresponding B(M1) values are larger.

Table 2. The experimental and calculated $B(\mathrm{E2})(\mathrm{e}^2\mathrm{b}^2)$ and $B(\mathrm{M1})(\mu_N^2)$ for ^{24}Mg.

$J_i^+ \to J_f^+$	B(E2)		B(M1)	
	exp.	cal.	exp.	cal.
$2_1^+ \to 0_1^+$	0.0087	0.0087		
$2_2^+ \to 0_1^+$	0.0006	0.0017		
$2_2^+ \to 0_2^+$		0.0006		
$2_2^+ \to 2_1^+$	0.0013(1)	0.0012		0.00528
$2_3^+ \to 0_1^+$	0.0002	0.0000		
$0_2^+ \to 2_1^+$	0.0002	0.0000		
$0_2^+ \to 2_2^+$	0.0029	0.0029		
$1_1^+ \to 0_1^+$			0.00139	0.00139
$1_1^+ \to 0_2^+$				0.00048
$1_1^+ \to 2_1^+$		0.0015		0.00135
$1_1^+ \to 2_2^+$		0.0103		0.00000
$3_1^+ \to 2_1^+$	0.0009	0.0035	0.00003	0.00083
$3_1^+ \to 2_2^+$		0.0079		0.00000
$3_1^+ \to 4_1^+$		0.0007		0.00351
$4_1^+ \to 2_1^+$	0.0148(21)	0.0103		
$4_1^+ \to 2_2^+$		0.0008		
$4_2^+ \to 2_1^+$	0.0004	0.0000		
$4_2^+ \to 2_2^+$	0.0052(4)	0.0014		

4 Conclusion

By using the interacting boson model with isospin (IBM-3), we have calculated the isospin excitation bands at low spin, electromagnetic transitions and mixed symmetry structure of ^{24}Mg. The IBM-3 calculated results agree very well with the available experimental data in low energy level. The results conclude that the IBM-3 description of the low-lying levels in the ^{24}Mg nucleus is satisfactory. The present calculations also give the structures of the isospin and mixed symmetry states for ^{24}Mg nucleus. The 1_1^+, 3_1^+ and 3_2^+ states are the mixed symmetry states, and these states are the isospin excitation states, whose isospin value is T=1. The 3_1^+ state is the lowest mixed symmetry state.

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