Pseudorapidity distribution of multiplicity in Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV *

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Abstract Using the Glauber model, we discuss the number of binary nucleon-nucleon collisions in heavy-ion collisions. Based on the latter, after considering the effect of energy loss of the nucleons in multiple collisions, we derive the pseudorapidity distribution of the multiplicity as a function of the impact parameter in nucleus-nucleus collisions. Using this, we analyze the experimental measurements carried out by the BRAHMS Collaboration in Au + Au collisions at $\sqrt{s_{NN}}=200$ GeV. The results are in good agreement with the experimental observations.

Key words Glauber model, number of binary nucleon-nucleon collisions, pseudorapidity distribution

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1 Introduction

The pseudorapidity distribution of multiplicity in heavy-ion interactions at high-energy can be obtained directly in experiments. It then provides a feasible approach for studying the mechanism and process of particle production in nucleus-nucleus collisions. The heavy-ion collisions at high-energy are highly inelastic. Hence, as nuclei interpenetrate each other, the energy of both nuclei is degraded heavily. The lost energy is accumulated within a small space region in a short time, forming a domain with high-temperature and high-energy density. If the temperature or energy density in this domain exceed the critical value (T_c ∼200 MeV, ε_c ∼3.0 GeV/fm^3), the phase transition[1] predicted by QCD lattice gauge theory may occur, leading to the long expected quark-gluon plasma (QGP). Furthermore, the matter of such a high-temperature and high-energy density also offers a scenario for studying the origin of the universe, the search for a new kind of matter and, may be, the discovery of new physics. Thus, to obtain matter of high-temperature and high-energy density becomes one of the primary goals in high-energy experimental physics. The study of the rapidity distribution of the multiplicity may allow one to determine theoretically the energy density for the initial matter produced by two colliding nuclei. Therefore the rapidity distribution in heavy-ion interactions is one of the most important subjects in high-energy experimental and theoretical investigations.

The BRAHMS Collaboration at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven National Laboratory (BNL) has studied the pseudorapidity distribution of multiplicity in Au+Au collisions at $\sqrt{s_{NN}}$ = 200 GeV[2], which is the maximum center-of-mass (c.m.s.) energy so far reached. This experiment shows that, for different centrality cuts, the shapes and ranges of pseudorapidity distributions are approximately the same, but their heights decrease evidently (especially in the central pseudorapidity region) with the increase of centralities (or impact parameters). In Ref. [3] this observation has been discussed in the framework of the overlapping cylinder model, which becames now the basis of the thermalized cylinder model[4—8]. In this work, using the Glauber model, we first present the number of binary nucleon-nucleon collisions in heavy-ion interactions. Then based on this, after considering the effect of energy loss of nucleons in their multiple collisions, we give the pseudorapidity distribution as a function of the impact parameter for the multiplicities in nucleus-

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nucleus collisions by simple repeated additions of the corresponding distributions generated in effective binary nucleon-nucleon collisions. Finally, by making use of the constructed model, we analyze the experimental measurements carried out by the BRAHMS Collaboration on Au+Au collisions at $\sqrt{s_{\rm NN}}=200$ GeV. The theoretical results are in reasonable agreement with the experimental observations.

2 The number of binary nucleon-nucleon collisions in heavy-ion collisions

In high-energy nucleus-nucleus collisions the two nuclei will interpenetrate each other and the nucleons of one nucleus will collide with those of the other one. Using the Glauber model[9—11] we discuss the number of binary nucleon-nucleon collisions in heavy-ion collisions.

The nucleon distribution of a nucleus with mass number A is given by

$$\rho_{\rm P}(r)=\frac{\rho_0}{A\{1+\exp[(r-r_0)/a]\}}\,, \tag{1}$$

where r_0 and a are two constants, here taken to be[12, 13] $r_0=1.19A^{1/3}-1.61A^{-1/3}$, $a=0.54$ fm. The constant ρ_0 is determined by the normalization

$$\int\rho_{\rm P}(r){\rm d}V=1\,.$$

For Au we have A=197, r_0 = 6.65 fm and ρ_0 = 0.15 fm^3. In terms of $\rho_{\rm P}(r)$, we get the nuclear thickness function

$$T_{\rm P}(s)=\int\rho_{\rm P}(s,z){\rm d}z\,, \tag{2}$$

which is normalized according to

$$\int T_{\rm P}(s){\rm d}^2s=1\,.$$

Obviously, $T_{\rm P}(s)$ is the probability for finding a nucleon in the flux tube with unit bottom area, which is located at the displacement s with respect to the center of the nucleus, as shown in Fig. 1. Hence, in a A-B nuclear collision with impact parameter b, the probability for a nucleon-nucleon collision in a unit cross section is

$$T_{\rm P}(b)=\int T_{\rm PA}(s)T_{\rm PB}(s-b){\rm d}^2s\,,$$

which is normalized to one

$$\int T_{\rm P}(b){\rm d}^2b=1\,.$$

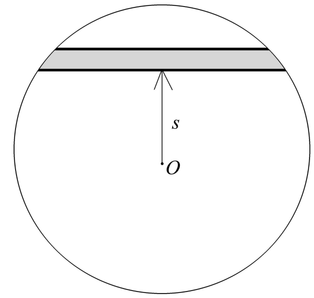

Fig. 1. The flux tube with unit bottom area located at the displacement s with respect to the center of the nucleus.

Then, the probability for a nucleon-nucleon collision is $T_{\rm P}(b)\sigma_{\rm NN}^{\rm in}$. $\sigma_{\rm NN}^{\rm in}$ is the total nucleon-nucleon inelastic cross section which is taken to be $\sigma_{\rm NN}^{\rm in}=42$ mb. Thus, the probability for having n times a nucleon-nucleon collision is given by

$$P(n,b)=\begin{pmatrix}AB\\ n\end{pmatrix}[T_{\rm P}(b)\sigma_{\rm NN}^{\rm in}]^n[1-T_{\rm P}(b)\sigma_{\rm NN}^{\rm in}]^{AB-n}, \tag{3}$$

where the first item is the number of combinations of n, out of total AB possible nucleon-nucleon collisions. The second term is the probability of having n times of nucleon-nucleon collision and the last term is the probability of missing $AB-n$ times of collisions. The above equation is normalized as

$$\sum_{n=0}^{AB}P(n,b)=1\,. \tag{4}$$

The number of binary nucleon-nucleon inelastic collisions in a A-B nuclear collision at impact parameter b can then be written as

$$N_{\rm NN}(b)=\sum_{n=1}^{AB}nP(n,b)\Big/\sum_{n=1}^{AB}P(n,b)\,, \tag{5}$$

where

$$\sum_{n=1}^{AB}P(n,b)=\sum_{n=0}^{AB}P(n,b)-P(0,b)=1-[1-T_{\rm P}(b)\sigma_{\rm NN}^{\rm in}]^{AB}$$

and

$$\sum_{n=1}^{AB}nP(n,b)=ABT_{\rm P}(b)\sigma_{\rm NN}^{\rm in}\,. \tag{6}$$

Inserting these expressions into Eq. (5), we get

$$N_{\rm NN}(b)=\frac{ABT_{\rm P}(b)\sigma_{\rm NN}^{\rm in}}{1-[1-T_{\rm P}(b)\sigma_{\rm NN}^{\rm in}]^{AB}}\,. \tag{7}$$

The average number of binary nucleon-nucleon collisions within a certain range of impact parameters (or centrality cut) is given by

$$\bar{N}_{\rm NN}=\int N_{\rm NN}(b){\rm d}^2b\Big/\int{\rm d}^2b\,. \tag{8}$$

Table 1 presents the impact parameter and the mean number of binary nucleon-nucleon collisions in each centrality cut in Au+Au collisions at $\sqrt{s_{\rm NN}}=200$ GeV as given by Eq. (8). Inside the brackets the experimental measurements of the BRAHMS Collaboration are shown. The centrality cut is defined as the percentage of the total Au+Au inelastic cross section $\sigma^{\rm in}_{\rm Au+Au}=6.9$b. Fig. 2 is a graphical representation of the results of Table 1. The solid circles are the results from Eq. (8). The open circles are the experimental measurements performed by the BRAHMS Collaboration. As can be seen from Table 1 or Fig. 2, Eq. (8) gives a good description of the experimental observations.

Table 1. The impact parameter b and the mean number of binary nucleon-nucleon collisions $\bar{N}_{\rm NN}$ in each centrality cut in Au+Au collisions at $\sqrt{s_{\rm NN}}=200$ GeV.

centrality cuts	b/fm	$\overline{N}_{\rm NN}$
0—5%	0—3.31	1016.8(1000±125)
5%—10%	3.31—4.69	809.9(785±115)
10%—20%	4.69—6.63	586.0(552±100)
20%—30%	6.63—8.12	373.0(335±58)
30%—40%	8.12—9.37	228.6(192±43)
40%—50%	9.37—10.48	132.1(103±31)

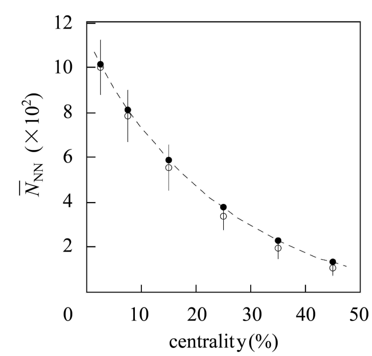

Fig. 2. The centrality dependence of the mean number of binary nucleon-nucleon collisions in Au+Au collisions at $\sqrt{s_{\rm NN}}$=200 GeV. The solid circles are the results from Eq. (8), the open circles the results given by the BRAHMS collaboration.

3 Pseudorapidity distribution of multiplicity in nucleus-nucleus collisions

In nucleus-nucleus collisions the energy of a nucleon will decrease step by step with each collision with other nucleons. The lost energy is finally distributed among the measurable particles in the final state. It is obvious that the total number of particles produced in a nucleus-nucleus collision is the sum of those produced in each nucleon-nucleon collision. Therefore, the rapidity distribution of multiplicity in a nucleus-nucleus collision at impact parameter b can be expressed as

$$\frac{{\rm d}N_{AB}(b)}{{\rm d}y}=\sum_{n=1}^{AB}P(n,b)\sum_{i=1}^{n}\frac{{\rm d}N_{\rm NN}(\sqrt{s^i_{\rm NN}},b)}{{\rm d}y}\,, \qquad (9)$$

where $P(n,b)$ is given by Eq. (3). It is the probability for the occurrence of n nucleon-nucleon collisions. $\sqrt{s^i_{\rm NN}}$ is the c.m.s. energy of a nucleon in the ith nucleon-nucleon collision, and ${\rm d}N_{\rm NN}(\sqrt{s^i_{\rm NN}},b)/{\rm d}y$ the corresponding rapidity distribution, which, in this paper, is taken to be[14]

$$\frac{{\rm d}N_{\rm NN}(\sqrt{s_{\rm NN}},b)}{{\rm d}y}=\frac{C(\sqrt{s_{\rm NN}})}{1+\exp\left[\dfrac{|y(b)|-y_0(\sqrt{s_{\rm NN}})}{\varDelta}\right]}\,, \qquad (10)$$

where

$$\varDelta=0.60\,,$$

$$C(\sqrt{s_{\rm NN}})=0.52\ln(\sqrt{s_{\rm NN}})+0.02\,,$$

and

$$y_0(\sqrt{s_{\rm NN}})=0.45\ln(\sqrt{s_{\rm NN}})+1.4\,.$$

The relation between rapidity and pseudorapidity is

$$y(b)=\frac{1}{2}\ln\left[\frac{\sqrt{p^2_{\rm T}(b)\cosh^2\eta+m^2}+p_{\rm T}(b)\sinh\eta}{\sqrt{p^2_{\rm T}(b)\cosh^2\eta+m^2}-p_{\rm T}(b)\sinh\eta}\right] \qquad (11)$$

Experimental and theoretical investigations showed[15, 16] that the transverse momentum of the final state particles increases with decreasing impact parameter b. Thus, in Eq. (10), the rapidity distribution in nucleon-nucleon collisions depends on the impact parameter b of the nucleus-nucleus collisions. The relation between pseudorapidity and rapidity distribution is (m and $m_{\rm T}$ are the mass and the transverse mass of primary pion, respectively.)

$$\frac{{\rm d}N_{AB}(b)}{{\rm d}\eta}=\sqrt{1-\frac{m^2}{m^2_{\rm T}\cosh^2 y}}\frac{{\rm d}N_{AB}(b)}{{\rm d}y}\,. \qquad (12)$$

In Fig. 3, the solid curve shows the pseudorapidity distribution in p+p collision at $\sqrt{s}=200$ GeV, the dashed and dash-dotted curves represent the corresponding distributions at $\sqrt{s}=62.8$ GeV and $\sqrt{s}=20$ GeV, respectively. The stars are the experimental measurements[17] of the UA5 Collaboration. In our calculation, we assume that the particles in the final state are all pions with $p_{\rm T}=0.35$ GeV/c. As can be seen from Fig. 3, Eq. (10) reproduces the experimental date quite well. It can also be seen that the heights of the distributions become lower with decreasing c.m.s. energy. However, the general shapes and ranges of the distributions are almost independent of the c.m.s. energy.

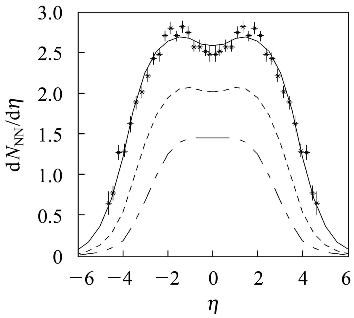

Fig. 3. Pseudorapidity distribution in p+p collisions at $\sqrt{s}$=200 GeV. The stars are the results from the UA5 collaboration. The solid line is the result from Eq. (10). The dashed and dash-dotted lines are the results for p+p collisions at $\sqrt{s}$=62.8 GeV and $\sqrt{s}$=20 GeV, respectively.

In order to get the rapidity distribution from Eq. (9), we need to know the c.m.s. energy in each nucleon-nucleon collision. It is evident that this is scarcely possible. Therefore we introduce here a simplified model to deal with this problem.

As a first step of simplification we may ignore the difference in c.m.s. energy for different nucleon-nucleon collisions. Such as, in Au+Au collisions at $\sqrt{s_{\rm NN}}=200$ GeV, we may just take $\sqrt{s^i_{\rm NN}}=200$ GeV for all nucleon-nucleon collisions. Then, from Eq. (6) and Eq. (7), Eq. (9) becomes

$$\frac{{\rm d}N_{AB}(b)}{{\rm d}y} \approx N_{\rm NN}(b)\{1-[1-T_{\rm P}(b)\sigma^{\rm in}_{\rm NN}]^{AB}\}\times \frac{{\rm d}N_{\rm NN}(\sqrt{s_{\rm NN}},b)}{{\rm d}y} . \quad (13)$$

Eq. (13) overestimates the multiplicity in nucleus-nucleus collisions. On physical grounds we may argue as follows. From Fig. 2 or Table 1 we see that with decreasing impact parameter the number of binary nucleon-nucleon collisions $N_{\rm NN}(b)$increases. However, more such collisions mean more energy loss and therefore a lower mean c.m.s. energy for each NN collision and a smaller contribution to the multiplicities in the final state. Accordingly, ${\rm d}N_{AB}(b)/{\rm d}y$ should decrease as $N_{\rm NN}(b)$ increases. To take into account this effect of energy loss we introduces a factor of energy loss $\beta(b)$, which is defined as

$$\beta(b)=1+\alpha(b)(N_{\rm NN}(b)-1) , \quad (14)$$

where $\alpha(b)$ is a parameter, which can be obtained from a fit to the experimental data. Noticing the peculiarity of the c.m.s. energy dependence of the pseudorapidity distribution in p+p collisions (see Fig. 3), when the effect of energy loss is taken into account, the rapidity distribution in nucleus-nucleus collisions should possess the form

$$\frac{{\rm d}N_{AB}(b)}{{\rm d}y} = \frac{N_{\rm NN}(b)\{1-[1-T_{\rm P}(b)\sigma^{\rm in}_{\rm NN}]^{AB}\}}{1+\alpha(b)[N_{\rm NN}(b)-1]}\times \frac{{\rm d}N_{\rm NN}(\sqrt{s_{\rm NN}},b)}{{\rm d}y} .$$

For p+p collisions, $N_{\rm NN}(b)$=1, and also $T_{\rm P}(b)\sigma^{\rm in}_{\rm NN}$ has the value 1 [cf. Eq. (7)]. In this way, the above equation will reduce to the rapidity distribution in p+p collisions. This is just what we expect, and the primary reason for choosing the form Eq. (14) for $\beta(b)$. Furthermore, we define

$$N^{\rm eff}_{\rm NN}(b)=\frac{N_{\rm NN}(b)}{1+\alpha(b)[N_{\rm NN}(b)-1]} , \quad (15)$$

the effective number of binary nucleon-nucleon collisions in heavy-ion collisions. Then

$$\frac{{\rm d}N_{AB}(b)}{{\rm d}y} = N^{\rm eff}_{\rm NN}(b)\{1-[1-T_{\rm P}(b)\sigma^{\rm in}_{\rm NN}]^{AB}\}\times \frac{{\rm d}N_{\rm NN}(\sqrt{s_{\rm NN}},b)}{{\rm d}y} . \quad (16)$$

Since the dependence of $p_{\rm T}(b)$ in Eq. (11) and $\alpha(b)$ in Eq. (14) on the impact parameter b is not well known, the mean multiplicity in each centrality cut cannot be obtained by integrating over the impact parameter as we did in Eq. (8). Here we adopt a common method, namely using the multiplicity at mid impact parameter in each centrality cut (cf. Table 1) to approximate the mean multiplicity. Substituting Eq. (16) into Eq. (12), we can then discuss the pseudorapidity distribution in nucleus-nucleus collisions. For Au+Au collisions at $\sqrt{s_{\rm NN}}=200$ GeV, the results are shown in Fig. 4. The triangles, circles, squares and stars in this figure are the experimental data of the BRAHMS Collaborations. The corresponding centrality cuts of the curves are 0—5%, 5%—10%, 10%—20%, 20%—30%, 30%—40%, 40%—50%, counting from top to bottom, respectively. It can be seen from this figure that the theoretical results are in good agreement with the experimental data for the various centrality cuts in the whole pseudorapidity region, and Eq. (16) is a successful description of the rapidity distribution of multiplicities in nucleus-nucleus collisions.

In our calculations, proceeding from small to large centrality cuts, the transverse momenta are taken to be $p_{\rm T}(b)=0.75$, 0.60, 0.45, 0.43, 0.38 and 0.30 GeV/c, that is, $p_{\rm T}(b)$ decreases with increasing centrality. This is consistent with the experimental and theoretical conclusion, as mentioned above, about the relation between $p_{\rm T}(b)$ and impact parameter b. The experimental data show that the parameter $\alpha(b)$ = 0.0034, 0.0041, 0.0052, 0.0073, 0.0102, 0.0150 increases with centrality. Inserting the values of $\alpha(b)$ into (14), we get the dependence of $\beta(b)$ on centrality, as shown in Fig. 5. It can be seen that the factor of energy loss decreases with the increase of centrality, which is consistent with the analysis we gave above.

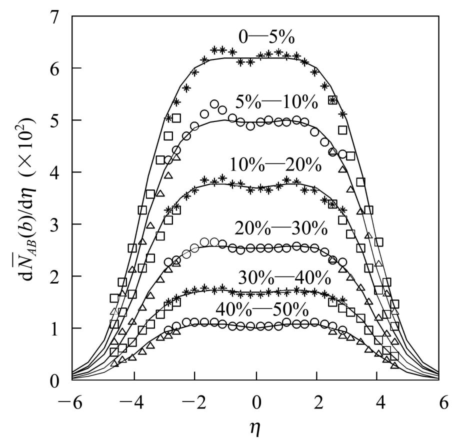

Fig. 4. Pseudorapidity distribution of multiplicity in Au+Au collision at $\sqrt{s}$=200 GeV. The triangles, circles, squares and stars are the experimental data of the BRAHMS Collaboration. The solid lines are the results from Eq. (12).

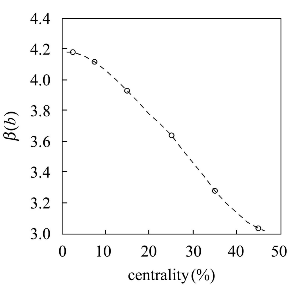

Fig. 5. The centrality dependence of the factor of energy loss.

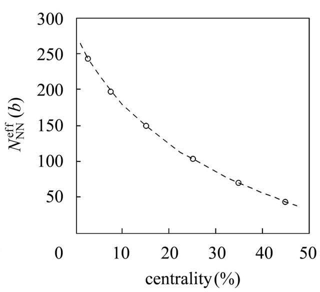

Fig. 6. The centrality dependence of effective number of binary nucleon-nucleon collisions.

Figure 6 shows the dependence of $N_{\rm NN}^{\rm eff}(b)$ on the centrality. Comparing it with Fig. 2, we see that $N_{\rm NN}^{\rm eff}(b)$ is much smaller than $N_{\rm NN}(b)$. This means that the nucleon will lose a large part of its energy during its multiple collisions with other nucleons. The lost energy is concentrated in a small space region. Then, what is the temperature or energy density in this region? Has the critical temperature or energy density for QGP phase transition been achieved? These problems will be discussed in detail in a forthcoming paper.

4 Conclusions

By using the Glauber model, we have calculated the number of binary nucleon-nucleon collisions in heavy-ion collisions, and achieve good agreement with the experimental data of the BRAHMS Collaboration. We present the pseudorapidity distribution for multiplicities in nucleus-nucleus collisions as a function of the impact parameter. Due to the energy loss of the nucleons in their multiple collisions, the contribution to the multiplicity in the final state from binary nucleon-nucleon collisions becomes less and less accompanied by an increase of $N_{\rm NN}(b)$, which is one of the geometric quantities describing the global properties of hadrons produced in hard processes. In order to describe such an effect of energy loss, we used an effective number $N_{\rm NN}^{\rm eff}(b)$ of binary nucleon-nucleon collisions, replacing the ture $N_{\rm NN}(b)$ and, as a result, the multiplicity in nucleus-nucleus collisions could simply be expressed as the sum of those produced in effective binary nucleon-nucleon collisions. The main characteristics of our model are: few free parameters, an intuitional physical picture and a simple mathematical treatment. By employing it to analyze the experimental measurements of the BRAHMS Collaboration in Au+Au collisions at $\sqrt{s_{\rm NN}}$ = 200 GeV, a good agreement with the experimental observations has been obtained.

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