Analysis of sextupole effects on β function beating in the SSRF storage ring*

TIAN Shun-Qiang(田顺强)[1,2] HOU Jie(后接)[1,2] CHEN Guang-Ling(陈光玲)[1,2] LIU Gui-Min(刘桂民)[1;1)]

1 (Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China)
2 (Graduate University of Chinese Academy of Sciences, Beijing 100049, China)

Abstract In a storage ring, asymmetry of the β function with momentum deviation is the main reason for asymmetry of the dynamic aperture. This paper applies simulation method based on AT code in Matlab to investigate sensitivity of the β function beating and the tune shift to quadrupole field error with the presence of bending field error in the Shanghai Synchrotron Radiation Facility (SSRF) storage ring. Sextupole effect on the variation trend is analyzed. Dynamics of the lattice for working points close to and away from the second order structural resonance stop-band are compared. These results show that the β function beating with momentum deviation doesn't lie in the influence of the second order structural resonance stop-bands completely, but it is relevant to lattice structure.

Key words SSRF, storage ring, sextupole, β function, structural resonance stop-band

PACS 29.20.dk, 41.85.-p, 42.65.-k

1 Introduction

Storage ring of the third generation light source require large dynamic acceptance and low sensitivity to a variety of errors, in order that high injection efficiency and long beam lifetime can be achieved in operating, especially for low emittance[1, 2]. Fine optimizations are necessary procedures, involving the choice of tunes, linear matching and nonlinear dynamics compensations in the presence of strong sextupoles. Moreover, analysis of sensitivity of the lattice to magnetic field errors is an indispensable course to check the optimum results.

Traditional stability diagram in the space of a focusing and a defocusing magnetic gradient can be extended to study more complicated super-periodic structure, in which the basic cell has more than two gradients as variables[3]. With this method, it is easy to understand the mechanism and the physics hidden behind the structural resonances, and easy to study the sensitivity of the lattice to quadrupole field errors. If a working point is close to the second order structural resonances, the β functions vary rapidly and asymmetrically with the quadrupole field errors.

Super-periodic structural resonance stop-bands will be generated in the following cases:

$$Q_{x,y}=\frac{M\times\mu}{2\pi}\quad(\mu=\pi,2\pi,3\pi,\cdots),\tag{1}$$

where M is the number of super-period, and μ is the phase advance in a super-period. The SSRF storage ring consists of 4 super-periods and 20 double bend achromatic (DBA) cells[4, 5]. The horizontal tune is 22.22, which is near the second order structural resonance stop-band Q_x=22. In the theory as shown above, with the sensitivity of the lattice to quadrupole field errors, the second order structural resonance results in asymmetry of the horizontal β functions on and off momentum, meanwhile it leads to asymmetry of the dynamic apertures on and off momentum. If integer of the tune changes to 23, these asymmetric effects should be improved[6], because Q_x=23 is not the structural resonance stop-band.

But this method only considers the sensitivity of the lattice to the quadrupole field errors; it ignores the effects of the sextupoles. Meanwhile, the magnetic field deviations of the off-momentum particles include not only the quadrupole field errors but also the bend and the sextupole field errors. The results obtained by the method only including quadrupole

Received 16 August 2007
* Supported by SSRF Project
1) E-mail: liugm@sinap.ac.cn

Table 1. Working points of the storage rings in three light sources and corresponding structural resonance stop-bands.

name of the light source	SPEAR3	DIAMOND	SOLEIL
cell/super-period/structural model	18/2/DBA	24/6/DBA	16/4/DBA
tunes Q_x/Q_y	14.13/6.22	27.23/12.36	18.2/10.3
integer structure resonance stop-bands	$Q_x=14, Q_y=6$	$Q_y=12$	
half-integer structure resonance stop-bands		$Q_x=27$	$Q_x=18, Q_y=10$

field errors can't deduce to the off-momentum case. So, conclusions resulting from this method are insufficient to understand the global dynamical properties of the storage ring. However, working points of the storage ring in some third generation light sources are near to integer or half-integer structural resonance stop-bands, shown in Table 1, these machines are still being operated successfully, with the available compensation of sextupoles[7—9].

Since the sextupoles will add focusing or defocusing quadrupole fields on the particles, whose transversal displacements are not zero when transiting the magnets, and must influence the β functions. We think that the β function beating with respect of the momentum deviation is relevant to the chromaticity correctional effects of the sextupoles, the structural resonance stop-bands are not the dominant reason, and the asymmetric effect of the β functions and/or dynamic aperture does not correspond to the working point completely. In the following sections, analysis of the sextupole chromaticity correctional effect and comparison of two tunes will prove this idea.

2 Chromaticity correctional effect of sextupole

Commonly, the tune shift and the β function beating with momentum deviation are used to evaluate the chromaticity correctional effect, and validate the nonlinear optimization. In this paper, we apply analysis of the tune shift and the β function beating with respect to the magnetic field error to study the correctional effects, and use the relationship between the momentum deviation and the field error to investigate the dynamic properties of the particles.

In a storage ring, there is a deviation between the equivalent magnetic field strength of the off momentum particle and the one of on momentum particle. In other words, the momentum deviation generates field error. So the variation of dynamic property induced by magnetic field error can be used to analyze the case with momentum deviation approximately.

Formula (2) shows the additional quadrupole fields of sextupoles with momentum deviation.

$$\lambda_j(1-\delta)\Delta x_j \approx \lambda_j \Delta x_j \ , \tag{2}$$

where λ_j is the strength of sextupole, δ is the momentum deviation, and Δx_j is the transversal displacement of particle. Because the additional quadrupole field errors with momentum deviations added by sextupoles are high order small quantities (as $\lambda_j\delta\Delta x_j$) compared with δ, whose contribution to the change of β functions or tunes is very small. So the sextupole field error can be ignored, only the bending field error and the quadrupole field error are considered.

In the simulative process, the same errors $\Delta B/B_0$ are added to all the bending magnets, and $\Delta K_i/K_{i0}$ to all the quadrupoles, where the tag i denotes different magnets. In the $\Delta B/B_0 \approx \Delta K_i/K_{i0}$ space, the relationship between the field errors and the momentum deviations is as formula (3).

$$\frac{\Delta K_i}{K_{i0}} = \frac{\Delta B}{B_0} \approx -\delta \ . \tag{3}$$

On this line, namely the bisectrix of the first quadrant and the third one, variations of the parameters due to field errors are equivalent to the cases of momentum deviations approximately. The simulation results with Accelerator Toolbox (AT)[10] code in Matlab are laid out in the following sections.

2.1 Lattice parameters of the SSRF storage ring

Matching parameters of the SSRF storage ring are showed in Table 2, and the magnet parameters in Table 3, including bending magnets quadrupoles and sextupoles, whose strengths are normalized.

Table 2. Matching parameters of the storage ring.

parameters	values
tune Q_x/Q_y	22.22/11.32
the center of the long straight section $\beta_x \backslash \beta_y \backslash \eta_x$/m	10/6/0.15
the center of the short straight section $\beta_x \backslash \beta_y \backslash \eta_x$/m	3.6/2.5/0.11
natural emittance/nm·rad	3.91
natural chromaticity ξ_x/ξ_y	−55.68/−17.91

2.2 Linear lattice

When the lattice only contains bending magnets and quadrupoles, not considering sextupoles or other nonlinear elements, the storage ring is a linear system. The tune close to the super-periodic structural resonances will drop into the stop-bands easily,

Table 3. Magnet parameters of the storage ring.

magnet	strength	length/m
bending	1.2726 T	1.44
QL1	−12.2022 T/m	0.32
QL2	15.9187 T/m	0.58
QL3	−14.3114 T/m	0.32
QL4	−12.5098 T/m	0.26
QL5	16.3064 T/m	0.32
QM1	−17.9836 T/m	0.32
QM2	17.8700 T/m	0.58
QM3	−12.2201 T/m	0.32
QM4	−15.8072 T/m	0.26
QM5	17.0361 T/m	0.32
S1	166.1022 T/m^2	0.20
S2	−257.6654 T/m^2	0.24
S3	321.8210 T/m^2	0.20
S4	−282.0661 T/m^2	0.24
S5	425.2103 T/m^2	0.20
S6	−455.1651 T/m^2	0.24
SD	−296.0824 T/m^2	0.20
SF	352.0162 T/m^2	0.24

because of the sensitivity of the lattice to the quadrupole field errors, and the phenomena must induce aberration of the β function, leading to an instable structure. Meanwhile, the bending field error can not change the trend of β function beating or tune shift with respect to the quadrupole field errors, because there is no additional quadrupole field that can affect the particle with non-zero displacement, resulting from the bending field error. Fig. 1 and Fig. 2 display the simulation results. As the above discussion, bisectrix of the first quadrant and the third one in field errors space can be considered as the cases of particles with momentum deviations. When the momentum deviation is slightly positive, the horizontal working point drops into the second order structural resonance stop-band Q_x=22, leading the horizontal β function to break. As a rule, it must be compensated by introducing sextupoles.

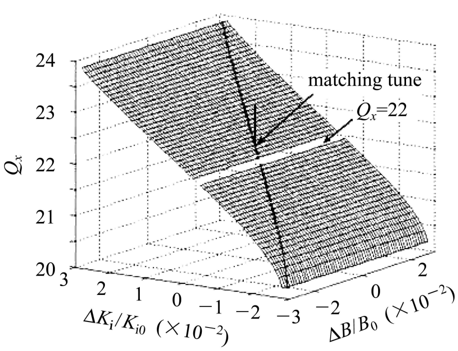

Fig. 1. Variation of the horizontal working point generated by field errors with all the sextupoles off in the SSRF storage ring.

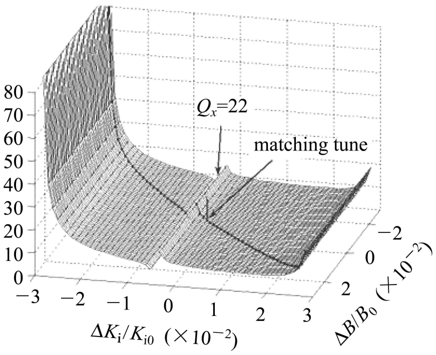

Fig. 2. Variation of the horizontal β function in the center of long straight section generated by field errors with all the sextupoles off in the SSRF storage ring.

2.3 Nonlinear lattice

When the sextupoles are introduced into lattice for compensation, the simulation results of horizontal β function and working point with different field errors are showed in Fig. 3 and Fig. 4.

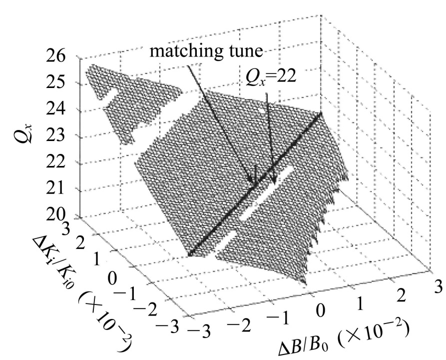

Fig. 3. Variation of the horizontal working point generated by field errors with all the sextupoles on in the SSRF storage ring.

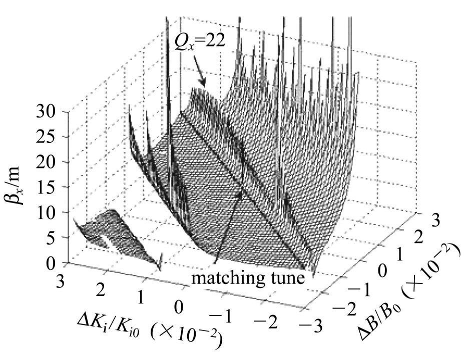

Fig. 4. Variation of the horizontal β function in the center of long straight section generated by field errors with all the sextupoles on in the SSRF storage ring.

Chromaticity both are corrected to zero in two transverse planes. Computation results show that the half-integer structural resonance Q_x=22 is parallel with the bisectrix of the first quadrant and the third one in $\Delta B/B_0 \approx \Delta K_i/K_{i0}$ space. Horizontal oscillation frequencies of the particles with larger momentum deviation (up to 3% or more) do not drop into the stop-band, and the horizontal β functions vary smoothly. The two figures prove that the variation of β functions with momentum deviations is independent of the half-integer structural resonances. It can be predicted that the trend of β function beating is the same as the tune, whose integer part changes to 23. In Section 3, simulation results can prove this.

3 Comparisons between two tunes

Figure 5 provides horizontal β function beating in the center of long straight section of two working points 22.22/11.32 and 23.275/11.317 in three cases, including (a) only quadrupole field errors and without momentum deviation, (b) adding quadrupole field errors as well as the same value of bending field errors and without momentum deviation, (c) off momentum cases without any field error. In Fig. 5(a), because there is no closed orbit deviation, the sextupoles can't affect the β functions. When the working point is 22.22, within the vicinity of $\Delta K_i/K_{i0}$=0, β_x depends on the quadrupole field errors very sensitively due to the half-integer structural resonance stop-band Q_x=22, whereas, β_x of the tune 23.275 varies smoothly. In Fig. 5(b), because the bending field errors generate closed orbit deviations, additional quadrupole fields of the sextupoles modulate β_x, and β function beatings have the same trend between the two working points. In Fig. 5(c), for the storage ring has dispersion, the momentum deviation induces closed orbit deviation, the sextupoles affect the β function like the Fig. 5(b), and the β function beating of the two working points has the same asymmetry approximately. Therefore, the β function beating is independent of the working point or the structural resonance stop-bands almost, with the appropriate compensation of sextupoles.

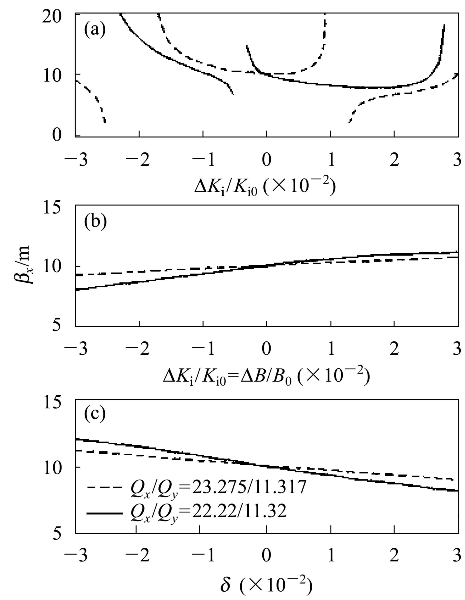

Fig. 5. The β function beating in the center of the long straight section of two working.

4 Conclusions

Correctional effects of the sextupoles have been investigated, and the mechanism that sextupoles enlarge energy acceptance and improve sensitivity of the lattice to errors has been studied. It is clarified that the half-integer structural resonance stop-bands are not the dominate reason for the β function beating with respect to the momentum deviation. In the SSRF storage ring, the tunes will not drop into the stop-bands though it is close to Q_x=22 and the beams will not lose.

References

1 Smith S L. Proc. of the EPAC. 2002. 44—48
2 Senichev Yu. Proc. of the PAC. 1999. 2442—2444
3 FANG Shou-Xian, QIN Qing. HEP & NP, 2006, **30**(9): 880 (in Chinese)
4 ZHAO Z T, XU H J. Proc. of the EPAC. 2004. 2368—2370
5 LIU G M et al. HEP & NP, 2006, **30**(1): 144 (in Chinese)
6 JIAO Y, FANG S X et al. NIM A, 2006, **566**: 270
7 Corbett J et al. Proc. of the EPAC. 2006. 3472—3474
8 Bartolini R, Jones J K, Martin I P S et al. Proc. of the EPAC. 2006. 2089—2091
9 Filhol J M et al. Proc. of the EPAC. 2006. 2723—2727
10 Terebilo A. Accelerator Toolbox for MATLAB, SLAC-PUB-8732, 2001