

# Theoretical description of octupole deformed band in neutron-rich even-even Ba isotopes<sup>\*</sup>

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**Abstract** The reflection asymmetric shell model (RASM) has been applied to describe the alternating-parity rotational bands in neutron-rich <sup>142,144,146,148</sup>Ba octupole nuclei. The calculated rotational bands are in good agreement with the available experimental data. The parity splitting and the parity inversion are also reproduced by the present calculation.

**Key words** octupole deformed band, shell model, parity splitting and inversion

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## 1 Introduction

It has been suggested<sup>[1]</sup> that in the neutron-rich nuclei Ba-Sm region around  $N \sim 88$  one may expect octupole correlations similar to those found in the actinide nuclei around  $Z \sim 88$ . This suggestion has been supported by experimental observations of octupole deformed bands in neutron-rich nuclei <sup>142–146,148</sup>Ba<sup>[2]</sup>, <sup>143,145,147</sup>La<sup>[3, 4]</sup>, <sup>144,146,148</sup>Ce<sup>[5, 6]</sup>, <sup>148,150</sup>Sm<sup>[7]</sup> and <sup>146,148</sup>Nd<sup>[8]</sup>. Experimentally, the most characteristic feature in these even-even octupole deformed nuclei is the appearance of alternating parity bands with  $I^P = 0^+, 1^-, 2^+, 3^-, \dots$ , with the negative-parity levels being shifted up in energy with respect to the positive-parity levels.

Following these experimental findings, several theoretical models and approaches were carried out in the past years to describe the octupole deformed nuclei in the Ba-Sm region, such as the Nilsson-Srtutinsky approach<sup>[9]</sup>, the self-consistent methods<sup>[10]</sup> as well as other approaches based on the algebraic model<sup>[11]</sup>. For other theoretical discussions on the

subject see also Ref. [12]. However, most of these models or methods are phenomenological, or they can only be used to calculate the intrinsic dipole moment or to determine equilibrium shapes, and the results of the most calculations can not be directly compared with the measured energy levels of rotational bands.

In the beginning of the 2000s, a microscopic model called the reflection asymmetric shell model (RASM) was developed to describe the high spin states in typical octupole deformed nuclei<sup>[13]</sup>. It follows the basic philosophy of the standard shell model and the octupole coupling force is included in the Hamiltonian. RASM is an extension of the projected shell model<sup>[14]</sup> by including the parity projection in addition to angular momentum projection, and one of its advantages is that the calculated rotational band states have good angular momentum and good parity, and can be directly compared with the experimental spectra. This model has successfully provided a good description of the properties of the octupole deformed bands in Ra-Th isotopes<sup>[13]</sup>. In this paper, this model was applied to describe the octupole deformed bands of

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the neutron-rich <sup>142,144,146,148</sup>Ba in Ba-Sm region, where the octupole deformation is somewhat soft.

## 2 Brief description of the reflection-asymmetric shell model

Let us briefly review the framework of the RASM. Considering a set of deformed BCS multi-quasiparticle states  $\{|\Phi_\kappa\rangle\}$ , where  $\kappa$  identifies the quasiparticle configurations. We may construct a wider class of states by forming a trial wave function

$$|\Psi\rangle = \sum_{IMK\kappa p} F_{MK\kappa}^{Ip} P^p P_{MK}^I |\Phi_\kappa\rangle. \quad (1)$$

where  $\{P^p P_{MK}^I |\Phi_\kappa\rangle\}$  is the set of simultaneously angular momentum- and parity-projected multi-quasiparticle states which forms the shell model space.  $P^p$  ( $P_{MK}^I$ ) is the parity (angular momentum) projection operator.  $F_{MK\kappa}^{Ip}$  is the coefficient to be determined by diagonalizing the shell-model Hamiltonian, namely solving the eigenvalue equation,

$$\sum_{K\kappa} \{ \langle \Phi_{\kappa'} | H P^p P_{K'K}^I | \Phi_\kappa \rangle - E_I^p \langle \Phi_{\kappa'} | P^p P_{K'K}^I | \Phi_\kappa \rangle \} F_{MK\kappa}^{Ip} = 0 \quad (2)$$

with the normalization condition,

$$\sum_{K'\kappa'K\kappa} F_{MK'\kappa'}^{Ip*} \langle \Phi_{\kappa'} | P^p P_{K'K}^I | \Phi_\kappa \rangle F_{MK\kappa}^{Ip} = 1. \quad (3)$$

It is seen that Eq. (2) is valid for any nuclear shape, while in the present study we consider the case of axial symmetry. The eigenvalues of  $H$  together with the wave functions can be calculated from Eq. (2).

The Hamiltonian is written as

$$\hat{H} = \hat{H}_0 - \frac{1}{2} \sum_{\lambda=2}^4 \chi_\lambda \sum_{\mu=-\lambda}^{\lambda} \hat{Q}_{\lambda\mu}^+ \hat{Q}_{\lambda\mu} - G_M \hat{P}_{00}^+ \hat{P}_{00} - G_Q \sum_{\mu=-2}^2 \hat{P}_{2\mu}^+ \hat{P}_{2\mu}. \quad (4)$$

where  $\hat{H}_0$  is the spherical single particle shell model Hamiltonian; and the second term includes quadrupole ( $\lambda=2$ ), octupole ( $\lambda=3$ ) and hexadecapole ( $\lambda=4$ ) interactions, which leads to the quadrupole, octupole and hexadecapole deformations, respectively; the third and fourth terms represent monopole and quadrupole pairing interactions, respectively. The coupling constants,  $\chi_\lambda$ , can be determined to be consistent with the deformations<sup>[13]</sup>. The details about the RASM were given in Ref. [13].

The Hamiltonian (4) is then diagonalized within the shell model space spanned by a selected set of the simultaneously parity- and angular momentum-projected BCS multi-quasiparticle states. The quasiparticle configurations employed in the present calculations for even-even nuclei include the vacuum, two neutrons and two protons quasiparticle states,  $||0\rangle, \alpha_{\nu_1}^\dagger \alpha_{\nu_2}^\dagger ||0\rangle, \alpha_{\pi_1}^\dagger \alpha_{\pi_2}^\dagger ||0\rangle$ , where  $\nu$ 's ( $\pi$ 's) denote the neutron (proton) Nilsson quantum numbers which run over the properly chosen low-lying quasiparticle states.

The monopole pairing strength constant  $G_M$  in Eq. (4) may be calculated by  $G_M = 16.0/A$  for neutron and proton. The quadrupole pairing strength  $G_Q$  is set as  $G_Q = f_Q G_M$ . In the present calculation, we set  $f_Q$  as 0.16. The three major shells of  $N = 4, 5$  and 6 for neutrons and 3, 4 and 5 for protons are included to calculate the Nilsson single particle states. The Nilsson parameters  $\kappa$  and  $\mu$  are taken from Ref. [15]. The deformation parameters including the quadrupole  $\varepsilon_2$ , the octupole  $\varepsilon_3$  and the hexadecapole  $\varepsilon_4$  are chosen to have reasonable values, and these values for Ba isotopes are list in Table 1.

Table 1. The deformation parameters used for the calculations for the Ba isotopes.

nucleus	$\varepsilon_2$	$\varepsilon_3$	$\varepsilon_4$
<sup>142</sup> Ba	0.12	0.07	-0.06
<sup>144</sup> Ba	0.13	0.07	-0.05
<sup>146</sup> Ba	0.14	0.07	-0.06
<sup>148</sup> Ba	0.16	0.06	-0.07

## 3 Results and discussions

The calculated results of alternating-parity rotational bands of <sup>142,144,146,148</sup>Ba along with the experimental spectra are shown in Fig.1. It is seen that theoretical and experimental level energies are in a good agreement for all nuclei considered, particularly at low spins. The calculated results predict the positions of the 1<sup>-</sup> states for the isotopes <sup>142</sup>Ba and <sup>148</sup>Ba, for which the corresponding experimental levels are still not measured. In addition, the interesting features of parity splitting observed experimentally have been reproduced, namely, the large parity splitting at low spins and the fast quenching of the parity splitting with increasing spin.

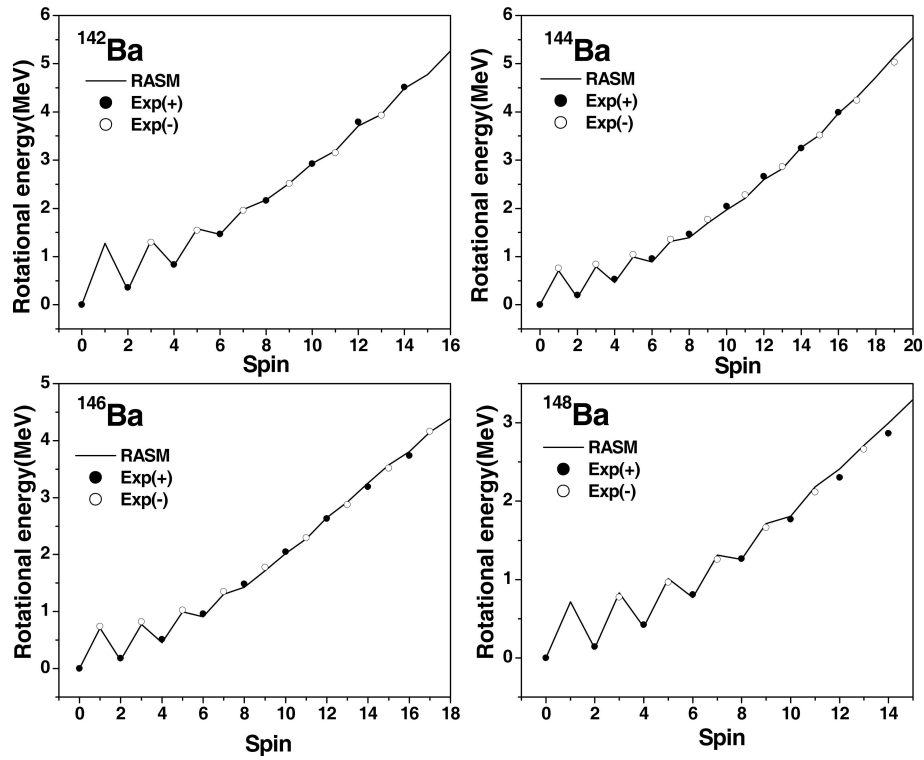


Fig. 1. Comparison of theoretical (solid line) and experimental alternating parity bands (solid circle stands for positive parity and open circle stands for negative parity) of the  $^{142,144,146,148}\text{Ba}$  nuclei. Experimental data are taken from Ref. [2].

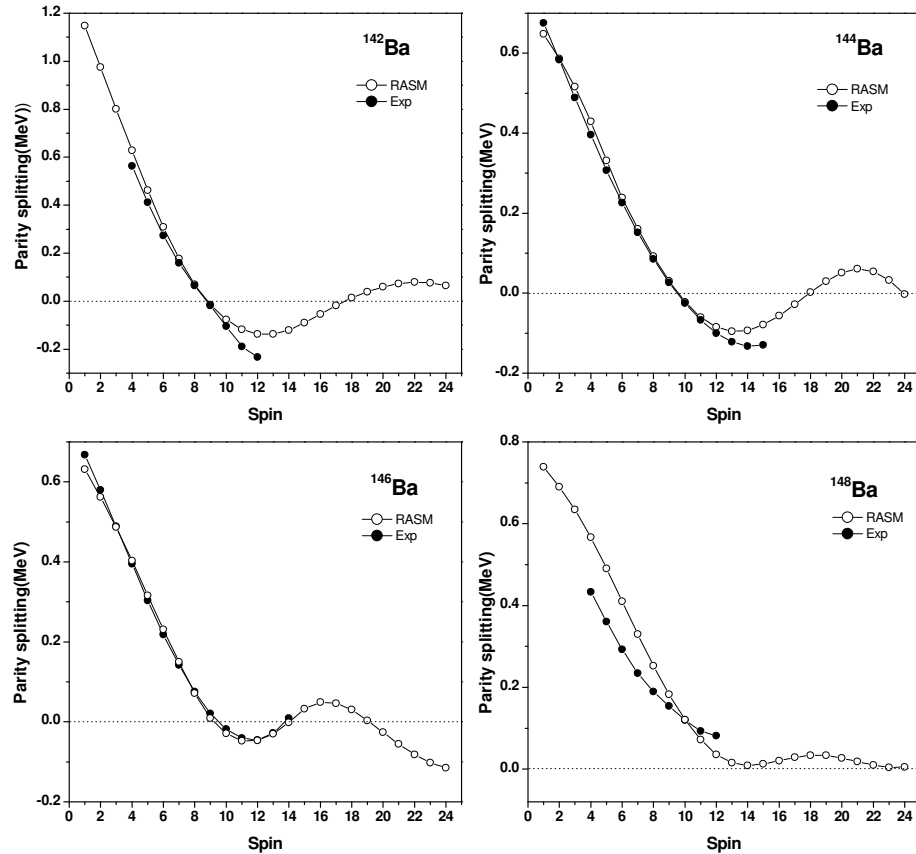


Fig. 2. Comparison of the experimental (solid circle) and calculated (open circle) values of the parity splitting for  $^{142,144,146,148}\text{Ba}$  nuclei.

In order to investigate the parity splitting dependence on the angular momentum clearly, we can extract the parity splitting from the rotational band levels. We use the following formula to calculate the parity splitting  $\Delta E(I)$ <sup>[16]</sup>:

$$\Delta E(I) = (-1)^I \left( \frac{1}{2}[E(I+1) - 2E(I) + E(I-1)] - \frac{1}{8}[E(I+2) - 2E(I) + E(I-2)] \right) \quad (5)$$

if  $I \geq 2$ , and

$$\Delta E(1) = - \left( \frac{1}{2}[E(2) - 2E(1)] - \frac{1}{8}[E(4) - 2E(2)] \right) \quad (6)$$

for  $I=1$ . Fig. 2 shows the results of calculations of parity splitting comparing with experimental data. From this figure, one can see that with increasing angular momentum, the parity splitting of all considered nuclei decreases gradually and disappears (i.e.  $\Delta E = 0$ ) around  $I = 9\hbar$  except  $^{148}\text{Ba}$ . After decreasing to zero, the parity splitting of  $^{142,144,146}\text{Ba}$  nuclei inverts its sign (i.e.  $\Delta E < 0$ ) for the first time and increases again in absolute value, but the size of this inverted parity splitting is smaller than at low spin. More interestingly, the sign of experimental parity splitting in  $^{146}\text{Ba}$  changes for the second time around  $I = 14\hbar$  (i.e.  $\Delta E > 0$  again). All these features are reproduced well by the present RASM calcula-

tions, especially the critical values of spin at which the sign of parity splitting changes for the first time in  $^{142,144,146}\text{Ba}$  nuclei and changes for the second time in  $^{146}\text{Ba}$  nucleus. For  $^{142,144}\text{Ba}$  nuclei we also expect that the theoretical parity splitting has a tendency to invert its sign for the second time at higher spin, but the experimental data of higher spin states are still not available up to now. The reason for this parity inversion is not clear, and its origin is still an interesting question. Some theoretical investigations are carried out for an understanding about the parity inversion, based on phenomenological models, for example, see Ref. [16]. The study of the parity inversion based on the microscopic model RASM is in progress and the results will be addressed in the forthcoming paper.

## 4 Summary

In summary, we have applied the reflection asymmetric shell model for description of the octupole bands in neutron-rich nuclei  $^{142,144,146,148}\text{Ba}$ , and the calculated results reproduced the experimental rotational bands, the parity splitting and the parity inversion very well. It has been demonstrated by the calculations that the RASM is a useful model to investigate the presence and properties of octupole deformation in nuclear structure.

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