Octet baryon masses with two-gluon exchange effect *

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Abstract We report our investigation on the octet baryon spectrum in the nonrelativistic quark model by taking into account the two-gluon exchange effect. The calculated octet baryon masses agree better with the experimental data. It is also shown that the two-gluon exchange interactions bring a significant correction to the Gell-Mann–Okubo mass formula.

Key words octet baryon masses, two-gluon exchange, Gell-Mann–Okubo mass formula

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1 Introduction

There are many approaches to describe the baryon mass spectrum, such as the $SU(6)$ model, the quark model, the bag model, the large N_c baryon model and so on. These models incorporate partly the dynamics of quantum chromodynamics (QCD) and have arrived at remarkable success. Recent lattice calculations have also made significant progress on the spectrum of bound states[1]. However, the quark model still remains a basic and indispensable tool in understanding hadron spectroscopy[2, 3]. In the quark model with dynamics, the spectrum of hadrons is dominated by two ingredients: the long range forces to bound quarks as hadrons and the short range forces expected from gluon exchanges. The short range forces include a Coulomb term, hyperfine interactions and spin-orbit interactions. Actually the hyperfine interactions are the most important short range forces that are responsible for such prominent features such as the Δ-N and ρ-π mass splittings[2].

The hyperfine interactions employed in quark model are usually derived from one-gluon exchange. The next step for a further improvement of hyperfine interactions is to consider higher order terms such as two-gluon exchange. The potential with two-gluon exchange has already been calculated by Gupta and Radford[4, 5]. Quarkonium spectra was investigated with the higher order potential[6, 7] and the theoretical results agree excellently with experiments. Since the nonrelativistic quark model is the simplest and most economical quark potential model[8—13], it is meaningful to estimate the two-gluon exchange effect in the light flavor baryon spectrum. We report here briefly our main results and conclusions[14] on the octet baryon spectrum in the nonrelativistic quark model by taking into account α_s^2 order hyperfine interactions via two-gluon exchange.

We adopt the Isgur-Karl quark model with the hyperfine interactions include not only one-gluon exchange of α_s order but also two-gluon exchange of α_s^2 order. We find that the calculated ground baryon masses are in good agreement with the experimental masses and better than those with only one-gluon exchange. Meanwhile, we also find that the two-gluon exchange interactions bring a significant correction to the Gell-Mann–Okubo mass formula.

2 Model calculations and results

The Hamiltonian in the quark model[11] is

$$H = \sum_i m_i + H_0 + H_{\rm hyp}, \tag{1}$$

$$H_0 = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} V_{\rm conf}^{ij}, \tag{2}$$

$$H_{\rm hyp} = \sum_{i<j} H_{\rm hyp}^{ij}, \tag{3}$$

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where $V_{\rm conf}^{ij}$ is the spin-independent potential and $H_{\rm hyp}^{ij}$ is the hyperfine interaction. The potential $V_{\rm conf}^{ij}$ is $\frac{1}{2}Kr_{ij}^2+U(r_{ij})$, $U(r_{ij})$ is a function which only depends on r_{ij}. The hyperfine interaction $H_{\rm hyp}^{ij}$ is

$$H_{\rm hyp}^{ij} = H_{\rm hyp}^{ij}(\alpha_{\rm s})+H_{\rm hyp}^{ij}(\alpha_{\rm s}^2), \tag{4}$$

where we add the order $\alpha_{\rm s}^2$ interaction.

The hyperfine interaction of $\alpha_{\rm s}$ order derived from the one-gluon exchange process is

$$H_{\rm hyp}^{ij}(\alpha_{\rm s}) = \frac{2\alpha_{\rm s}}{3m_im_j}\left[\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij})+\frac{1}{r_{ij}^3}\left(\frac{3\vec{S}_i\cdot\vec{r}_{ij}\ \vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^2}-\vec{S}_i\cdot\vec{S}_j\right)\right], \tag{5}$$

where m_i and $\vec{S}_i$ are the mass and spin of the ith quark and $\vec{r}_{ij}$ is the relative position of the pair ij of quarks. The first and second term are called the Fermi contact term and the tensor term respectively.

The Fermi contact term and the tensor term of $\alpha_{\rm s}^2$ order[4, 6, 7] derived from two-gluon exchange are

$$\begin{aligned}H_{\rm conact}^{ij}(\alpha_{\rm s}^2) = &\frac{16\pi\alpha_{\rm s}}{9m_im_j}\vec{S}_i\cdot\vec{S}_j\left\{\left[\frac{\alpha_{\rm s}}{12\pi}(26+9\ln 2)\right]\delta^3(\vec{r}_{ij})-\right.\\ &\frac{\alpha_{\rm s}}{24\pi^2}(33-2n_{\rm f})\vec{\nabla}^2\left[\frac{\ln(\mu_{\rm GR}r_{ij})+\gamma_{\rm E}}{r_{ij}}\right]+\\ &\left.\frac{21\alpha_{\rm s}}{16\pi^2}\vec{\nabla}^2\left[\frac{\ln(\sqrt{m_im_j}r_{ij})+\gamma_{\rm E}}{r_{ij}}\right]\right\},\end{aligned} \tag{6}$$

$$\begin{aligned}H_{\rm tensor}^{ij}(\alpha_{\rm s}^2) = &\frac{2\alpha_{\rm s}}{3m_im_j}\frac{1}{r_{ij}^3}\left[\frac{3(\vec{S}_i\cdot\vec{r}_{ij})(\vec{S}_j\cdot\vec{r}_{ij})}{r_{ij}^2}-\vec{S}_i\cdot\vec{S}_j\right]\cdot\\ &\left\{\frac{4\alpha_{\rm s}}{3\pi}+\frac{\alpha_{\rm s}}{6\pi}(33-2n_{\rm f})\left[\ln(\mu_{\rm GR}r_{ij})+\gamma_{\rm E}-\frac{4}{3}\right]-\right.\\ &\left.\frac{3\alpha_{\rm s}}{\pi}\left[\ln(\sqrt{m_im_j}r_{ij})+\gamma_{\rm E}-\frac{4}{3}\right]\right\},\end{aligned} \tag{7}$$

where $\mu_{\rm GR}$ is a renormalization scale[15] and $n_{\rm f}$ is the number of effective quark flavors.

We then write approximate solutions[11, 12] by perturbation theory in U and $H_{\rm hyp}$. In the $U=H_{\rm hyp}=0$ limit, H_0 becomes

$$H_0\rightarrow H_0'=\frac{p_\rho^2}{2m_\rho}+\frac{p_\lambda^2}{2m_\lambda}+\frac{3K}{2}(\rho^2+\lambda^2), \tag{8}$$

where

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2), \qquad \vec{p}_\rho=m_\rho\frac{{\rm d}\vec{\rho}}{{\rm d}t}, \tag{9}$$

$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3), \qquad \vec{p}_\lambda=m_\lambda\frac{{\rm d}\vec{\lambda}}{{\rm d}t}, \tag{10}$$

$$m_\rho \equiv m, \quad m_\lambda\equiv\frac{3mm'}{2m+m'}, \quad x\equiv m_d/m_{\rm s}. \tag{11}$$

In the $S=0$ sector, $m=m'=m_d$; in the $S=-1$ sector, $m=m_d$, $m'=m_{\rm s}$. The solutions to H_0' are wave functions as follows. The wave function with N=0 is

$$\psi_{00} = \frac{\alpha_\rho^{3/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}\exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right]. \tag{12}$$

The wave functions with $N=2$ are

$$\begin{aligned}\psi_{00}^{\lambda\lambda} &= \sqrt{\frac{2}{3}}\frac{\alpha_\rho^{3/2}\alpha_\lambda^{7/2}}{\pi^{3/2}}(\lambda^2-\frac{3}{2}\alpha_\lambda^{-2})\times \exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right],\\
\psi_{00}^{\rho\lambda} &= \frac{2}{\sqrt{3}}\frac{\alpha_\rho^{5/2}\alpha_\lambda^{5/2}}{\pi^{3/2}}\vec{\rho}\cdot\vec{\lambda}\times \exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right],\\
\psi_{00}^{\rho\rho} &= \sqrt{\frac{2}{3}}\frac{\alpha_\rho^{7/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}(\rho^2-\frac{3}{2}\alpha_\rho^{-2})\times \exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right],\\
\psi_{22}^{\lambda\lambda} &= \frac{1}{\sqrt{2}}\frac{\alpha_\rho^{3/2}\alpha_\lambda^{7/2}}{\pi^{3/2}}\lambda_+\lambda_+\times \exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right],\\
\psi_{22}^{\rho\lambda} &= \frac{\alpha_\rho^{5/2}\alpha_\lambda^{5/2}}{\pi^{3/2}}\rho_+\lambda_+\times \exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right],\\
\psi_{22}^{\rho\rho} &= \frac{1}{\sqrt{2}}\frac{\alpha_\rho^{7/2}\alpha_\lambda^{3/2}}{\pi^{3/2}}\rho_+\rho_+\times \exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right],\\
\psi_{11}^{\rho\lambda} &= \frac{\alpha_\rho^{5/2}\alpha_\lambda^{5/2}}{\pi^{3/2}}(\rho_+\lambda_3-\rho_3\lambda_+)\times \exp\left[-\frac{1}{2}\alpha_\rho^2\rho^2-\frac{1}{2}\alpha_\lambda^2\lambda^2\right],\end{aligned} \tag{13}$$

where $\alpha_\rho=(3Km_\rho)^{1/4}$, $\alpha_\lambda=(3Km_\lambda)^{1/4}$.

When U differs from zero, it can be shown that in the $S=0$ sector, the confinement energies can be determined by three constants E_0, a, and b. To break $SU(3)$, ρ and λ excitation energies are decreased by $(m_d/m_\rho)^{1/2}$ and $(m_d/m_\lambda)^{1/2}$, respectively. We use the confinement energies obtained by considering U effects in Ref. [12, 13].

We then calculate the hyperfine matrix elements. In the nonstrange sector, the states are completely symmetric and it follows that

$$\langle\alpha|H_{\rm hyp}|\beta\rangle = 3\langle\alpha|H^{12}_{\rm hyp}|\beta\rangle, \qquad (14)$$

where $|\alpha\rangle$ and $|\beta\rangle$ are any two states. In the $S=-1$ sector, since the states are always symmetric under exchanger of quarks one and two, then

$$\langle\alpha|H^{13}_{\rm hyp}|\beta\rangle = \langle\alpha|H^{23}_{\rm hyp}|\beta\rangle, \qquad (15)$$

and the hyperfine matrix elements become

$$\langle\alpha|H_{\rm hyp}|\beta\rangle = \langle\alpha|H^{12}_{\rm hyp}+2H^{13}_{\rm hyp}|\beta\rangle. \qquad (16)$$

The hyperfine matrix elements of the $S=-2$ and $S=-3$ sectors can be obtained from the $S=0$ and $S=-1$ sectors by making the interchange $m_{\rm u}\leftrightarrow m_{\rm s}$ everywhere. Since the hyperfine matrix elements of $\alpha_{\rm s}$ order have been calculated[10—12], we only calculate the hyperfine matrix elements of $\alpha_{\rm s}^2$ order derived from two-gluon exchange. We also express the hyperfine matrix elements with order $\alpha_{\rm s}^2$ here in the unit of δ, which is $4\alpha_{\rm s}\alpha^3/3\sqrt{2\pi}m_d^2$. We assume that this value differs slightly[11, 12], which is 270 MeV for considering the results of order $\alpha_{\rm s}^2$ effects and mixing between the ground states with $N=0$ and the exited states with $N=2$. The other parameters chosen are $\alpha_{\rm s}=0.7$, $x=0.616$, $\alpha=0.38$, $m_d=0.275$ GeV and $\mu_{d\rm GR}=0.333$ GeV. It shows that the values of matrix elements of higher order are in the range $(0.0014–0.0334)\delta$, which are one order less than those obtained by order $\alpha_{\rm s}$.

At last we diagonalize the complete Hamiltonian matrices to get the ground-state baryon masses. We can find that the calculated ground-state masses agree good with the experimental values[16]. In addition, we compare our results of the $\alpha_{\rm s}^2$ order with the results using the $\alpha_{\rm s}$ order in Table 1. It shows that the results of the $\alpha_{\rm s}^2$ order are more accord with the experimental values, since most ΔM are somewhat smaller and the value of $\sum(\Delta M)^2=33$ MeV^2 is much smaller than that of 193 MeV^2 in Ref. [12].

We further study the corrections to the Gell-Mann–Okubo mass formula (GMO) and Gell-Mann's equal spacing rule (GME). The corrections to GMO for the baryon-octet and GME for the baryon-decuplet are

$$\frac{M_{\rm N}+M_\Xi}{2} = \frac{3M_\Lambda+M_\Sigma}{4}+\delta_{\rm GMO}, \qquad (17)$$

$$M_{\Sigma^*}-M_\Delta = M_{\Xi^*}-M_{\Sigma^*}+\delta_{\rm GME1} = M_\Omega-M_{\Xi^*}+\delta_{\rm GME2}, \qquad (18)$$

Here as $\delta_{\rm GMO}=\delta_{\rm GME1}=\delta_{\rm GME2}=0$, Eqs. (17) and (18) are the standard mass formulas which do not consider the $H_{\rm hyp}$ term. However, in the real world, the corrections are not zero. The deviations can be explained by a quark-mass dependent hyperfine interaction, the wave function size and mixing with excited states. In Table 2, it shows that the correction to the GMO is distinctly improved and the correction to the GME is also getting better slightly. Since the calculated Ξ^* mass is not very good, there is only little improvement on the corrections to the GME. However, the octet ground baryons are more close to reality and the corrections to the GMO improve significantly. It implies that the correction to the GMO is mainly due to the hyperfine interactions of $\alpha_{\rm s}^2$ order.

Table 1. The corrections to GMO and GME.

state (J^P)	mass	mass[12]	$M_{\rm exp}$	ΔM	$\Delta M^{[12]}$
$\rm N\frac{1}{2}^+$	939	940	939	0	−1
$\Lambda\frac{1}{2}^+$	1113	1110	1116	3	6
$\Sigma\frac{1}{2}^+$	1192	1190	1193	1	3
$\Xi\frac{1}{2}^+$	1317	1325	1318	−1	−7
$\Delta\frac{3}{2}^+$	1231	1240	1232	1	−8
$\Sigma^*\frac{3}{2}^+$	1383	1390	1385	2	−5
$\Xi^*\frac{3}{2}^+$	1526	1530	1530	4	0
$\Omega\frac{3}{2}^+$	1673	1675	1672	−1	−3

Table 2. The corrections to GMO and GME.

	$\delta_{\rm GMO}$	$\delta_{\rm GME1}$	$\delta_{\rm GME2}$
$\delta_{\rm exp}$	−6.75	8	11
$\delta^{[12]}$	2.5	10	5
$\delta_{\rm our}$	−4.75	9	5

From our above study, we find that the values of the matrix elements of hyperfine interactions with higher order are in the range 0.4—10 MeV, which are one order less than those matrix elements 2—170 MeV obtained by leading order $O(\alpha_{\rm s})$. After diagonalizing the complete Hamiltonian matrices, the calculated ground state masses are in good agreement with the experimental masses. In addition, the correction to the GMO for the baryon-octet is distinctly improved and the correction to the GME for the baryon-decuplet is also slightly improved.

It is noted that $\alpha_{\rm s}$ becomes 0.7 for considering the higher order hyperfine interactions, and it is smaller than the value $\pi/2$ in Ref. [12]. This implies that the applicability of perturbative QCD calculations becomes more reasonable in the quark-model framework by taking into account higher order effects.

3 Conclusion

Therefore we conclude that the effects from higher order hyperfine interaction should be considered in the quark model. Taking into account the hyperfine interactions of order $O(\alpha_s^2)$ due to two-gluon exchange, the masses of ground states baryons agree better with the experimental data. The higher order hyperfine interactions bring also more realistic correction to the Gell-Mann–Okubo mass formula.

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