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Meson spectra governed by the Fermi-Breit potential*

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Abstract We calculate the meson mass spectra in a quark potential model by using the complete Fermi-Breit potential including the terms of orbit-orbit interaction, spin-orbit coupling, and tensor force interaction. We find that these terms give nontrivial contributions to the calculated meson spectra. The orbit-orbit coupling term may lead to an instability of the solution of the Schrödinger equation and should be regularized.

Key words meson spectra, Breit potential, quark model

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1 Introduction

In the description of low-energy hadronic phenomena simplifications arise due to the effect of spontaneous chiral symmetry breaking which generates non-perturbatively constituent quark masses of a few hundred MeV[1], even though the current quark mass in quantum chromodynamics is of the order of only a few MeV. Once the generation of the constituent quark mass is taken into account, the other non-perturbative effects such as confinement and gluon exchanges can be treated as a relatively weak interaction if one investigates low-energy phenomena. On the basis of relatively large constituent quark masses and a physically inspired phenomenological potential, non-relativistic and relativity-corrected potential quark models have been successfully applied to describe many properties of low-lying hadronic states[2—12]. Progress has been made in studying the hadron bound states and the fine structure of the mass spectra[1—7]. The success of the non-relativistic potential model also promotes its application to scattering problems[2, 7—10, 13—15].

In a non-relativistic model for a quark-antiquark state on which we focus our attention, the potential consists of two main parts. The confining part of the potential has been traditionally included by a linearly-rising potential with a string tension coefficient b. The other part is described by a one-gluon exchange potential, whose coupling constant α_s is taken phenomenologically as a parameter to describe the meson spectra. The earliest version of a complete non-relativistic one-gluon exchange potential up to the second order in the relative velocity v is the Fermi-Breit potential (or just the Breit potential)[16].

In the studies of meson bound states various improvements of the Fermi-Breit potential have been applied[2—5, 8, 9, 17, 18]. However, all of these improvements neglected the momentum-dependent orbit-orbit coupling term, $\dfrac{C_{ij}\alpha_s}{2m_im_j}\Big(\dfrac{\boldsymbol{p}^2}{r}+\dfrac{\boldsymbol{r}\cdot(\boldsymbol{r}\cdot\boldsymbol{p})\boldsymbol{p}}{r^3}\Big)$. In Ref. [19] the contributions of the retardation effect and the one-loop radiative corrections of the quark-antiquark potential were investigated using a quasipotential approach. With this potential which includes momentum-dependent coupling terms, the authors calculated the charmonium and bottomonium spectra. In the present paper we would like to consider a complete Fermi-Breit potential form which includes the orbit-orbit coupling term as well as the other terms. We calculate the mass spectra of 28

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mesons (some of them in their ground states and some of them in excited states). We find that the contributions from the orbit-orbit interaction, spin-orbit coupling, and tensor force term in the potential cannot be omitted. The orbit-orbit coupling term may lead to an instability of the solution of the Schrödinger equation and should be regularized.

The paper is organized as follows. In section 2 we briefly describe the quark potential model. In section 3, we discuss the matrix formulation of the bound state Schrödinger equation and introduce the meson wave functions which will be used in our calculations. The matrix elements of the Schrödinger equation are presented in section 4. Finally our results, the discussion and a summary is given in section 5.

2 Quark potential model

Mesons are quark-antiquark bound states. The nonlocal one-gluon exchange potential (OGEP) for these states in coordinate space, the Fermi-Breit potential[20], is given by

$$\begin{aligned} V^{\rm Breit} = \boldsymbol{C}_{ij}\,\alpha_s \Bigg\{ & \frac{1}{r} - \frac{1}{2m_i m_j}\Big(\frac{\boldsymbol{p}_i\cdot\boldsymbol{p}_j}{r} + \frac{\boldsymbol{r}\cdot(\boldsymbol{r}\cdot\boldsymbol{p}_i)\,\boldsymbol{p}_j}{r^3}\Big) - \\ & \frac{\pi}{2}\delta(\boldsymbol{r})\Big(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16\,\boldsymbol{s}_i\cdot\boldsymbol{s}_j}{3m_i m_j}\Big) - \\ & \frac{1}{2\,r^3}\Bigg[\frac{1}{m_i^2}(\boldsymbol{r}\times\boldsymbol{p}_i)\cdot\boldsymbol{s}_i - \frac{1}{m_j^2}(\boldsymbol{r}\times\boldsymbol{p}_j)\cdot\boldsymbol{s}_j + \\ & \frac{1}{m_i m_j}\Big(2\,(\boldsymbol{r}\times\boldsymbol{p}_i)\cdot\boldsymbol{s}_j - 2\,(\boldsymbol{r}\times\boldsymbol{p}_j)\cdot\boldsymbol{s}_i - \\ & 2\,\boldsymbol{s}_i\cdot\boldsymbol{s}_j + 6\frac{(\boldsymbol{s}_i\cdot\boldsymbol{r})(\boldsymbol{s}_j\cdot\boldsymbol{r})}{r^2}\Big)\Bigg]\Bigg\}, \end{aligned} \tag{1}$$

where $\boldsymbol{C}_{ij}$ is the scattering channel color matrix, α_s is the QCD coupling constant, $r=|\boldsymbol{r}|=|\boldsymbol{r}_1-\boldsymbol{r}_2|$ is the distance between quark i and quark j, m_i and m_j are the masses of the constituent quarks, $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are the quark momenta, $\boldsymbol{s}_i=\frac{1}{2}\boldsymbol{\sigma}_i$ is the spin of the quark i and $\boldsymbol{\sigma}_i$ its corresponding Pauli spin vector. The operator form of the color matrix is given by

$$\boldsymbol{C}_{ij} = -\frac{1}{4}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j^*\,, \tag{2}$$

where $\boldsymbol{\lambda}$ are the Gell-Mann matrices. In the center of mass frame we have $\boldsymbol{p}_i=-\boldsymbol{p}_j$. By introducing the notation $\boldsymbol{p}\equiv\boldsymbol{p}_i=-\boldsymbol{p}_j$, Eq. (1) can be expressed as

$$\begin{aligned} V^{\rm Breit} = \boldsymbol{C}_{ij}\,\alpha_s \Bigg\{ & \frac{1}{r} - \frac{\pi}{2}\,\delta(\boldsymbol{r})\frac{(m_i^2+m_j^2)}{m_i^2 m_j^2} + \\ & \frac{1}{2m_i m_j}\Big(\frac{\boldsymbol{p}^2}{r} + \frac{\boldsymbol{r}\cdot(\boldsymbol{r}\cdot\boldsymbol{p})\,\boldsymbol{p}}{r^3}\Big) - \\ & \frac{2\pi}{3m_i m_j}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\,\delta(\boldsymbol{r}) - \frac{1}{4m_i m_j r^3}(\boldsymbol{r}\times\boldsymbol{p})\cdot \\ & \Big[\Big(2+\frac{m_j}{m_i}\Big)\boldsymbol{\sigma}_i + \Big(2+\frac{m_i}{m_j}\Big)\boldsymbol{\sigma}_j\Big] - \\ & \frac{3}{4m_i m_j r^3}S_{ij}^{r}\Bigg\} + \boldsymbol{C}_{ij}(-V_0)\,. \end{aligned} \tag{3}$$

Here the last term is a constant potential used to adjust the meson mass in solving the Schrödinger equation, and

$$S_{ij}^{r} = \frac{(\boldsymbol{r}\cdot\boldsymbol{\sigma}_i)(\boldsymbol{r}\cdot\boldsymbol{\sigma}_j)}{r^2} - \frac{1}{3}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j), \tag{4}$$

is the tensor-force operator varying with $\boldsymbol{r}$.

In addition to the one-gluon exchange potential, the interaction between the quark and the antiquark also includes a confining potential which is taken traditionally to be proportional to the separation r[9, 21, 22]:

$$V^{\rm c} = -\boldsymbol{C}_{ij}\Big(\frac{3}{4}b\Big)\,r\,, \tag{5}$$

where $\boldsymbol{C}_{ij}$ is the same color matrix as in the Fermi-Breit potential and b is a string tension coefficient. Because free mesons are color singlet states, the color factor is $\boldsymbol{C}_{ij}=-4/3$.

3 Bound state Schrödinger equation and meson wave functions

A conventional method to solve the Schrödinger equation is the matrix approach which can be used easily in numerical calculations[9, 22, 23]. Using this approach to calculate the bound states of a meson we need a set of basis wave functions. In this section we discuss the matrix formulation of the bound state Schrödinger equation and introduce the meson wave functions used in our calculations.

The Schrödinger equation for a meson bound state in coordinate space is

$$\frac{\boldsymbol{p}^2}{2\mu_r}\varPhi(\boldsymbol{r}) + V(\boldsymbol{r})\varPhi(\boldsymbol{r}) = E\varPhi(\boldsymbol{r}), \tag{6}$$

where $\mu_r=m_i m_j/(m_i+m_j)$, $\boldsymbol{p}$, and $\boldsymbol{r}$ are the reduced mass, center-of-mass momentum, and relative coordinate between the quark and antiquark, $\boldsymbol{V}=\boldsymbol{V}^{\rm Breit}+\boldsymbol{V}^{c}$ is the total potential, and $\varPhi(\boldsymbol{r})$ is the meson wave function. The energy E and the total mass M of the meson satisfy $E=M-m_i-m_j$.

We expand the meson wave function $\Phi(\boldsymbol{r})$ into a set of basis functions $\phi_{nl}(\boldsymbol{r})$:

$$\Phi(\boldsymbol{r})=\sum_{n,\,l}a_{nl}\phi_{nl}(\boldsymbol{r}). \tag{7}$$

Inserting this expansion into Eq. (6) and multiplying with $\phi^{\dagger}_{ml'}(\boldsymbol{r})$ from the left we get

$$\sum_{n,\,l}a_{nl}\left[\phi^{\dagger}_{ml'}(\boldsymbol{r})\frac{\boldsymbol{p}^2}{2\mu_r}\phi_{nl}(\boldsymbol{r})+\phi^{\dagger}_{ml'}(\boldsymbol{r})V(\boldsymbol{r})\phi_{nl}(\boldsymbol{r})\right]=$$

$$E\sum_{n,\,l}a_{nl}\phi^{\dagger}_{ml'}(\boldsymbol{r})\phi_{nl}(\boldsymbol{r})\,. \tag{8}$$

By integrating this equation over the whole coordinate space, we obtain

$$\sum_{n,\,l}a_{nl}\left[\boldsymbol{T}_{mn}+\boldsymbol{V}_{mn}\right]=E\sum_{n,\,l}a_{nl}\boldsymbol{B}_{mn}\,, \tag{9}$$

where

$$\boldsymbol{T}_{mn}=\langle ml'|\boldsymbol{T}|nl\rangle=(2\pi)^3\int\mathrm{d}\boldsymbol{r}\,\phi^{\dagger}_{ml'}(\boldsymbol{r})\,\frac{\boldsymbol{p}^2}{2\mu_r}\phi_{nl}(\boldsymbol{r})\,, \tag{10}$$

$$\boldsymbol{V}_{mn}=\langle ml'|\boldsymbol{V}|nl\rangle=(2\pi)^3\int\mathrm{d}\boldsymbol{r}\phi^{\dagger}_{ml'}(\boldsymbol{r})\boldsymbol{V}(\boldsymbol{r})\phi_{nl}(\boldsymbol{r})\,, \tag{11}$$

$$\boldsymbol{B}_{mn}=\langle ml'|nl\rangle=(2\pi)^3\int\mathrm{d}\boldsymbol{r}\,\phi^{\dagger}_{ml'}(\boldsymbol{r})\phi_{nl}(\boldsymbol{r})\,. \tag{12}$$

In the present paper we use the same basis functions as in Ref. [9]. In coordinate space they are given by

$$\phi_{nl}(\boldsymbol{r})=R_{nl}(r)\,Y_{lm_l}(\hat{\boldsymbol{r}})=N_{nl}\,r^l\exp\Big(-\frac{n\beta^2}{2}r^2\Big)Y_{lm_l}(\hat{\boldsymbol{r}})\,, \tag{13}$$

where

$$N_{nl}=\frac{(\sqrt{2}\,i)^l}{4\pi}\sqrt{\frac{(2/\sqrt{\pi})^3}{(2l+1)!!}}\;(n\beta^2)^{\frac{1}{2}(l+\frac{3}{2})}\,. \tag{14}$$

The radial basis wave functions $R_{nl}(r)$ are products of r^l with Gaussian functions of different widths. The advantage of using Gaussian basis functions is that one gets analytical expression for the matrix elements. Then, the expansion coefficients can be determined by solving the Schrödinger equation numerically. In our calculations, we take six Gaussian basis functions as in Refs. [9, 22], $n=1$, 2, 3, 4, 5, 6. For the total angular momentum $J=0$ and 1, the values of the orbital angular momentum l are 0, 1, and 2.

For $l'=0$, $l'=1$, and $l'=2$, Eq. (9) becomes

$$\sum_{n=1}^{6}a_{n_0}\left[\boldsymbol{T}_{mn}+\langle m_0|\boldsymbol{V}|n_0\rangle\right]+$$

$$\sum_{n=1}^{6}a_{n_2}\langle m_0|\boldsymbol{V}|n_2\rangle=E\sum_{n=1}^{6}a_{n_0}\boldsymbol{B}_{mn}\,, \tag{15}$$

$$\sum_{n=1}^{6}a_{n_1}\left[\boldsymbol{T}_{mn}+\langle m_1|\boldsymbol{V}|n_1\rangle\right]=E\sum_{n=1}^{6}a_{n_1}\boldsymbol{B}_{mn}\,, \tag{16}$$

and

$$\sum_{n=1}^{6}a_{n_2}\left[\boldsymbol{T}_{mn}+\langle m_2|\boldsymbol{V}|n_2\rangle\right]+$$

$$\sum_{n=1}^{6}a_{n_0}\langle m_2|\boldsymbol{V}|n_0\rangle=E\sum_{n=1}^{6}a_{n_2}\boldsymbol{B}_{mn}\,. \tag{17}$$

In Eqs. (15) and (17), the matrix elements coupling the states with $l=0$ and $l=2$ are not zero for a meson spin $S=1$. These nonzero elements are arising from the tensor force interaction in the Breit potential [the sixth term in Eq. (3)]. Because of the coupling between the states of $l=0$ and $l=2$, Eqs. (15) and (17) must be combined in the calculations as shown below, although only mesons with angular momentum $l=0$ and 1 are considered in this paper.

We introduce a twelve-dimensional vector

$$a^{\dagger}=\big(a_1,a_2,a_3,a_4,a_5,a_6,b_1,b_2,b_3,b_4,b_5,b_6\big), \tag{18}$$

where $a_i=a_{il}\,(i=1,2,3,4,5,6)$ for a fixed l (=0, 1, or 2), $b_i=a_{i2}$ $(i=1,2,3,4,5,6)$ for $l=0$ and $S=1$ (with nonvanishing coupling matrix elements in this case), $b_i=a_{i0}$ $(i=1,2,3,4,5,6)$ for $l=2$ and $S=1$, and $b_i=0$ $(i=1,2,3,4,5,6)$ for all other quantum numbers. In this twelve-dimension space, Eqs. (15), (16) and (17) can be expressed as one matrix equation

$$\boldsymbol{H}a=E\boldsymbol{B}a\,, \tag{19}$$

where $\boldsymbol{H}=\boldsymbol{T}+\boldsymbol{V}$ is the Hamiltonian matrix in the twelve-dimension space and $\boldsymbol{B}$ is a 12×12 matrix constructed by the direct product of the same two 6×6 matrices $\big(\boldsymbol{B}_{mn}\big)$ $(m,n=1,2,3,4,5,6)$.

In next section we will present the matrix elements of $\boldsymbol{H}$ and $\boldsymbol{B}$. The eigenvalues of E can then be obtained by solving Eq. (19) numerically. This will finally allow us to obtain the meson masses.

4 Matrix elements

In this section, we calculate the matrix elements needed in Eqs. (15)—(17). From Eqs. (10), (12), and (13), we have

$$\boldsymbol{T}_{mn}=(2l+3)\frac{mn}{m+n}\boldsymbol{B}_{mn}\frac{\beta^2}{2\mu_r}\,, \tag{20}$$

and

$$\boldsymbol{B}_{mn}=\left(\frac{2\sqrt{mn}}{m+n}\right)^{l+3/2}\delta_{l'l}\,. \tag{21}$$

In general the matrix elements of interaction potential can be expressed as

$$\boldsymbol{V}_{mn} = \sum_{m_l m_s}\sum_{m_l' m_s'} \langle l\, m_l\, s\, m_s | J m_J\rangle \langle l'\, m_l'\, s'\, m_s' | J m_J'\rangle \times (2\pi)^3 \int \mathrm{d}\boldsymbol{r} \phi^{\dagger}_{ml'}(\boldsymbol{r})\, \chi^{\dagger}_{sm_s'}\, c^{\dagger}(ij) \times V(\boldsymbol{r}) \phi_{nl}(\boldsymbol{r})\, \chi_{sm_s}\, c(ij)\,, \tag{22}$$

where l, S, and J are the quantum numbers of the orbital angular momentum, the spin, and total angular momentum of the meson. m_l, m_s and m_J are the corresponding magnetic quantum numbers, χ_{sm_s} is the spin wave function, and $c(ij)$ is the color wave function of the meson.

Below we give explicitly the matrix elements of the interaction potential Eq. (3). Since all of them possess the same general forms there is no need to specify the quantum numbers. Because deriving the final results is a rather cumbersome procedure, we give here only the results.

The matrix elements for the first and second terms of the interaction potential are

$$(\boldsymbol{V}_1^{\mathrm{Breit}})_{mn} = \boldsymbol{C}_{\mathrm{f}} \frac{4\pi\alpha_s}{(2\pi)^{3/2}} \beta \frac{2^l l!}{(2l+1)!!} \sqrt{m+n}\, \boldsymbol{B}_{mn}, \tag{23}$$

$$(\boldsymbol{V}_2^{\mathrm{Breit}})_{mn} = -\boldsymbol{C}_{\mathrm{f}} (4\pi\alpha_s) \frac{(m_i^2+m_j^2)}{8m_i^2 m_j^2} \times (mn)^{3/4} \beta^3 \pi^{-3/2} \delta_{l0} \delta_{l'l}, \tag{24}$$

where $\boldsymbol{C}_{\mathrm{f}} = -\frac{4}{3}$ is the color matrix element.

The matrix element for the l-l, s-s, and l-s coupling terms in the potential Eq. (3) are given by

$$(\boldsymbol{V}_3^{\mathrm{Breit}})_{mn} = \boldsymbol{C}_{\mathrm{f}} \frac{4\pi\alpha_s}{m_i m_j} \beta^3 \frac{2mn}{\sqrt{m+n}} \frac{\boldsymbol{B}_{mn}}{(2\pi)^{3/2}}\,, \quad (l=0)\,, \tag{25}$$

$$(\boldsymbol{V}_3^{\mathrm{Breit}})_{mn} = \boldsymbol{C}_{\mathrm{f}} \frac{4\pi\alpha_s}{m_i m_j} \beta^3 \frac{2mn}{\sqrt{m+n}} \frac{\boldsymbol{B}_{mn}}{(2\pi)^{3/2}} \frac{2^l (l+1)!}{(2l+1)!!} - \boldsymbol{C}_{\mathrm{f}} \frac{\alpha_s (2\pi)^3}{2m_i m_j} l(l-1) \int \mathrm{d}\boldsymbol{r} \phi^{\dagger}_{ml'}(\boldsymbol{r})\, r^{-3} \phi_{nl}(\boldsymbol{r})\,, \quad (l>0)\,, \tag{26}$$

$$(\boldsymbol{V}_4^{\mathrm{Breit}})_{mn} = -\boldsymbol{C}_{\mathrm{f}} \frac{2}{3} \frac{4\pi\alpha_s}{4m_i m_j} (mn)^{3/4} \beta^3 \pi^{-3/2} \times 2\left[s(s+1)-3/2\right] \delta_{l0} \delta_{l'l}\,, \tag{27}$$

$$(\boldsymbol{V}_5^{\mathrm{Breit}})_{mn} = 0\,, \qquad (l=0)\,, \tag{28}$$

$$(\boldsymbol{V}_5^{\mathrm{Breit}})_{mn} = \boldsymbol{C}_{\mathrm{f}} \frac{\sqrt{6}\alpha_s}{4m_i m_j} \left(4 + \frac{m_i}{m_j} + \frac{m_j}{m_i}\right) \hat{s}^2 \hat{l} \sqrt{l(l+1)} \times (-1)^l (2\pi)^3 \int \mathrm{d}\boldsymbol{r} \phi^{\dagger}_{ml'}(\boldsymbol{r})\, r^{-3} \phi_{nl}(\boldsymbol{r}) \times (-1)^{1+J} \begin{Bmatrix} s & s & 1 \\ l & l & J \end{Bmatrix} \begin{Bmatrix} s & s & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}\,, \quad (l>0)\,, \tag{29}$$

where the symbol $\hat{A}$ denotes $\sqrt{2A+1}$.

The matrix element for the tensor force term is

$$(\boldsymbol{V}_6^{\mathrm{Breit}})_{mn} = 0\,, \qquad (l=0)\,, \tag{30}$$

$$(\boldsymbol{V}_6^{\mathrm{Breit}})_{mn} = \boldsymbol{C}_{\mathrm{f}} \frac{3\alpha_{\mathrm{s}}}{m_i m_j} \sqrt{\frac{5}{6}} \delta_{s1} (\hat{l})^2 \begin{pmatrix} l' & l & 2 \\ 0 & 0 & 0 \end{pmatrix} (-1)^J \times \begin{Bmatrix} s & l & J \\ l' & s & 2 \end{Bmatrix} \frac{\hat{l}'}{\hat{l}} (-1)^{(l'+l)} \times (2\pi)^3 \int_0^{\infty} \mathrm{d}\, r\, r^2 R^*_{ml'}(r) r^{-3} R_{nl}(r), (l>0), \tag{31}$$

where

$$(2\pi)^3 \int_0^{\infty} \mathrm{d}\, r\, r^2 R^*_{ml'}(r) r^{-3} R_{nl}(r) = A_{l'l} \frac{1}{2} \left(\frac{l'+l}{2} - 1\right)! \,\nu^{-(l'+l)/2}\,, \tag{32}$$

$$A_{l'l} = \frac{(-1)^{l'} i^{(l'+l)} 2^{(\frac{l'+l}{2}+2)}}{\sqrt{(2l'+1)!!(2l+1)!!}} \frac{1}{\sqrt{\pi}} \beta^{(l'+l+3)} (mn)^{3/4} \times \frac{1}{2} (m^{l'/2} n^{l/2} + n^{l'/2} m^{l/2})\,, \tag{33}$$

$$\nu = \frac{1}{2} \beta^2 (m+n)\,. \tag{34}$$

The matrix element for the constant term is

$$(\boldsymbol{V}_7^{\mathrm{Breit}})_{mn} = \boldsymbol{C}_{\mathrm{f}} (-V_0) \boldsymbol{B}_{mn}\,. \tag{35}$$

In this paper a linear confining potential is used. Its matrix element is given by

$$(\boldsymbol{V}^c)_{mn} = \boldsymbol{C}_{\mathrm{f}} \left(-\frac{3}{4}\right) \frac{b}{\beta} \frac{8\pi}{(2\pi)^{3/2}} \frac{2^l (l+1)!}{(2l+1)!!} \frac{\boldsymbol{B}_{mn}}{\sqrt{m+n}}\,. \tag{36}$$

5 Results and conclusions

We focus our attention first on the calculated results of the meson spectra given in Table 1. In the second column we list the experimental meson masses from Ref. [10]. In the third and fourth columns we list our theoretical meson masses M and the root-mean-squared radii calculated with the Fermi-Breit potential. For comparison, we also present in columns 5 to 7 the results from previous papers[9, 10]. Finally we present the values of the Gaussian parameter β used in the basis wave functions in the eighth column. Fig. 1 shows the experimental data and our results of the meson spectra. The expansion coefficients of the wave function in Eq. (19) are listed in Table 2.

In our model, the adjustable parameters are the string tension coefficient b, the constant potential term V_0, five quark masses $m_u = m_d$, m_s, m_c, m_b, and the QCD coupling constant α_s. In our calculations they are taken as: $b = 0.228$ GeV^2, $V_0 = -0.798$ GeV, $m_u = m_d = 0.380$ GeV, $m_s = 0.532$ GeV, $m_c = 1.795$ GeV, $m_b = 5.161$ GeV, and $\alpha_s = 0.185$. The parameter β in Table 1 is a parameter which describes the width of the wave function. The values of β are different for different meson groups. For instance, the values of β for the six D mesons are the same, and likewise for the four B mesons. The values of β have been chosen in such a way as to improve the accuracy. The final values turned out to be almost equal to those of a previous work[9]. In the

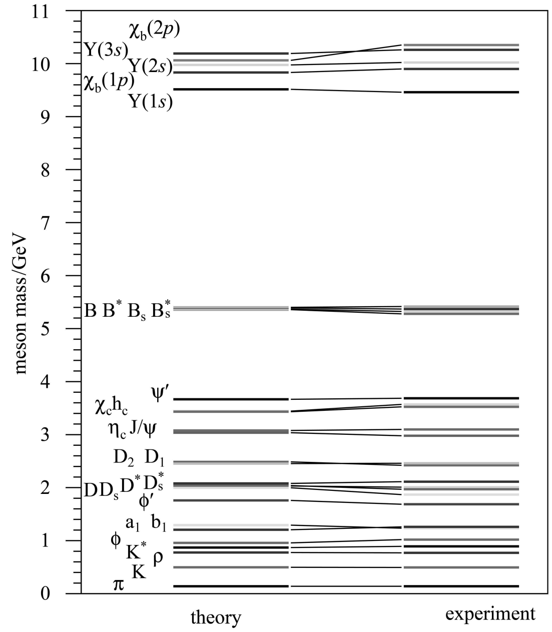

Fig. 1. Comparison of the theoretical meson masses and experimental data.

Table 1. Results of meson masses and root-mean-square radii.

meson	M(exp)/GeV	M/GeV	$\sqrt{\langle r^2\rangle}$/fm	M/GeV[10]	M/GeV[9]	$\sqrt{\langle r^2\rangle}$/fm[9]	β/GeV
$\pi(1^1S_0)$	0.140	0.139	0.337	0.143	0.140	0.512	0.3444
$K(1^1S_0)$	0.494	0.497	0.387	0.494	0.495	0.521	0.3936
$K^*(1^3S_1)$	0.892	0.870	0.663	0.907	0.904	0.674	0.3936
$\rho(1^3S_1)$	0.770	0.775	0.706	0.788	0.774	0.769	0.3444
$\phi(1^3S_1)$	1.020	0.958	0.640	1.031	0.992	0.647	0.3444
$b_1(1^1P_1)$	1.235	1.293	0.940	1.397	1.330	0.978	0.3444
$a_1(1^3P_1)$	1.260	1.206	0.856	1.573	1.353	0.993	0.3444
$\phi(2^3S_1)$	1.686	1.759	0.926	1.852	1.870	0.983	0.3444
$D(1^1S_0)$	1.869	1.998	0.591	1.865	1.913	0.585	0.3936
$D^*(1^3S_1)$	2.010	2.031	0.618	1.998	1.998	0.626	0.3936
$D_s(1^1S_0)$	1.969	2.038	0.526	1.976	2.000	0.508	0.3936
$D_s^*(1^3S_1)$	2.112	2.079	0.559	2.121	2.072	0.546	0.3936
$D_1(1^1P_1)$	2.422	2.486	0.831	2.408	2.506	0.840	0.3936
$D_2(1^3P_2)$	2.460	2.454	0.807	2.381	2.514	0.845	0.3936
$\eta_c(1^1S_0)$	2.979	3.036	0.378	2.978	3.033	0.388	0.5136
$J/\psi(1^3S_1)$	3.097	3.079	0.408	3.128	3.069	0.404	0.5136
$h_c(1^1P_1)$	3.570	3.444	0.584	3.520	3.462	0.602	0.5136
$\chi_c(1^3P_1)$	3.525	3.433	0.575	3.507	3.466	0.606	0.5136
$\psi'(2^3S_1)$	3.686	3.667	0.694	3.689	3.693	0.666	0.5136
$B(1^1S_0)$	5.279	5.358	0.584	5.272	5.322	0.574	0.4500
$B^*(1^3S_1)$	5.324	5.367	0.592	5.319	5.342	0.583	0.4500
$B_s(1^1S_0)$	5.369	5.390	0.524	5.368	5.379	0.503	0.4500
$B_s^*(1^3S_1)$	5.416	5.402	0.533	5.426	5.396	0.513	0.4500
$\Upsilon(1^3S_1)$	9.460	9.514	0.257	9.453	9.495	0.255	0.6356
$\chi_b(1^3P_1)$	9.899	9.832	0.396	9.889	9.830	0.423	0.6356
$\Upsilon(2^3S_1)$	10.020	9.974	0.501	10.023	9.944	0.519	0.6356
$\chi_b(2^3P_1)$	10.260	10.190	0.598	10.257	10.166	0.604	0.6356
$\Upsilon(3^3S_1)$	10.350	10.061	0.145	10.359	10.340	0.573	0.6356

Table 2. Expansion coefficients of the meson wave functions.

meson	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4	b_5	b_6
$\pi(1^1S_0)$	−1.744	21.700	−95.776	199.557	−194.520	71.904	0.000	0.000	0.000	0.000	0.000	0.000
$K(1^1S_0)$	−0.872	14.899	−65.616	136.548	−132.858	49.069	0.000	0.000	0.000	0.000	0.000	0.000
$K^*(1^3S_1)$	1.919	−5.884	18.927	−32.709	28.322	−9.586	−0.009	0.210	−0.894	1.932	−1.988	0.792
$\rho(1^3S_1)$	1.513	−4.449	16.206	−29.867	27.025	−9.435	−0.016	0.276	−1.172	2.535	−2.611	1.042
$\phi(1^3S_1)$	0.960	−2.001	9.210	−18.110	17.024	−6.073	−0.009	0.184	−0.777	1.686	−1.735	0.692
$b_1(1^1P_1)$	1.420	−2.069	4.943	−6.582	4.563	−1.242	0.000	0.000	0.000	0.000	0.000	0.000
$a_1(1^3P_1)$	−0.993	−0.140	2.958	−8.915	10.318	−4.346	0.000	0.000	0.000	0.000	0.000	0.000
$\phi(2^3S_1)$	3.846	−12.399	24.629	−30.326	19.051	−4.766	−0.440	−0.071	0.615	−1.937	2.296	−1.006
$D(1^1S_0)$	1.144	−1.441	4.051	−5.665	4.075	−1.130	0.000	0.000	0.000	0.000	0.000	0.000
$D^*(1^3S_1)$	−1.402	3.211	−11.031	19.473	−17.093	5.839	0.001	−0.066	0.279	−0.603	0.618	−0.245
$D_s(1^1S_0)$	0.620	0.702	−2.193	5.102	−5.244	2.064	0.000	0.000	0.000	0.000	0.000	0.000
$D_s^*(1^3S_1)$	−0.905	1.374	−6.506	12.591	−11.707	4.137	0.003	−0.077	0.321	−0.696	0.714	−0.284
$D_1(1^1P_1)$	1.440	−2.070	4.946	−6.713	4.771	−1.353	0.000	0.000	0.000	0.000	0.000	0.000
$D_2(1^3P_2)$	1.305	−1.493	3.151	−3.397	1.713	−0.232	0.000	0.000	0.000	0.000	0.000	0.000
$\eta_c(1^1S_0)$	0.398	1.946	−7.215	15.286	−14.991	5.648	0.000	0.000	0.000	0.000	0.000	0.000
$J/\psi(1^3S_1)$	0.720	−0.439	2.918	−5.420	4.901	−1.644	−0.003	0.075	−0.315	0.682	−0.699	0.278
$h_c(1^1P_1)$	1.007	−0.572	1.704	−2.207	1.483	−0.376	0.000	0.000	0.000	0.000	0.000	0.000
$\chi_c(1^3P_1)$	0.950	−0.347	0.964	−0.776	0.126	0.133	0.000	0.000	0.000	0.000	0.000	0.000
$\psi'(2^3S_1)$	4.439	−14.705	31.069	−41.084	28.196	−7.845	−0.097	−0.034	0.218	−0.600	0.678	−0.287
$B(1^1S_0)$	1.848	−4.605	12.945	−20.242	16.209	−5.153	0.000	0.000	0.000	0.000	0.000	0.000
$B^*(1^3S_1)$	−1.930	5.135	−14.902	23.980	−19.655	6.380	0.000	−0.021	0.089	−0.194	0.199	−0.079
$B_s(1^1S_0)$	1.208	−1.933	6.172	−10.023	8.270	−2.672	0.000	0.000	0.000	0.000	0.000	0.000
$B_s^*(1^3S_1)$	−1.309	2.605	−8.794	15.185	−13.117	4.420	0.000	−0.027	0.113	−0.245	0.251	−0.099
$\Upsilon(1^3S_1)$	−0.092	−1.508	3.932	−8.314	8.137	−3.214	0.003	−0.044	0.189	−0.414	0.428	−0.171
$\chi_b(1^3P_1)$	0.375	1.057	−1.692	2.950	−2.588	0.958	0.000	0.000	0.000	0.000	0.000	0.000
$\Upsilon(2^3S_1)$	2.621	−4.938	10.436	−17.542	14.756	−5.159	−0.009	−0.064	0.207	−0.433	0.435	−0.172
$\chi_b(2^3P_1)$	2.583	−5.884	10.904	−15.551	11.686	−3.655	0.000	0.000	0.000	0.000	0.000	0.000
$\Upsilon(3^3S_1)$	0.102	−0.099	−0.337	1.072	−1.218	0.486	0.613	0.643	−0.693	0.990	−0.764	0.263

numerical calculations it has been noted that in case the momentum-dependent orbit-orbit coupling term was involved in the potential, the mass of the π-meson heavily depended on the parameter β_π. For instance, the solution for π shows a fluctuations when we change the parameter β_π perturbatively, e.g. by 0.1 GeV around $\beta_\pi = 0.3444$ GeV. This instability can be improved by regularizing the orbit-orbit coupling interaction in the potential[24]. As one can see from Table 1 and Fig. 1, the Fermi-Breit interaction with a confining potential can describe the gross features of the low-lying meson states. The splitting between π and ρ is well described. The splitting between J/ψ and η_c is however too small and needs further investigation in the future.

In our calculations the 28 experimental meson masses (listed in the second column of Table 1) are input quantities. The values of the adjustable parameters b, V_0, $m_{u,d}$, m_s, m_c, m_b, and α_s are determined by minimize the χ^2 between the 28 experimental meson masses and the corresponding calculated meson masses. We find that the determined values of the quark masses are in reasonable regions. The value of b is between the values 0.18[9] and 0.35[22, 25]. The value of α_s is a little bit small but acceptable. We also find that if we omit the momentum-related orbit-orbit coupling term in the Fermi-Breit potential, the masses of π and K, for instance, will change by about 5% and 1%, respectively. The root-mean-square radii of the mesons will change if we omit the orbit-orbit coupling term. As the root-mean-square radii are related to the meson reaction cross sections and decay properties, further investigations of the effect of the momentum dependent potential by combining the bound state mass and the experimental cross section data will be of great interest. On the other hand, we only consider the velocity-dependent terms in the Fermi-Breit potential and use a fixed strong coupling constant in this paper. It would be also interesting to consider velocity-dependent effects in the confining potential and use a running coupling constant in future investigations.

In summary, we calculated the meson masses in a quark potential model using a complete Fermi-

Breit potential. The momentum-dependent orbit-orbit coupling term, $\frac{C_{ij}\alpha_s}{2m_im_j}\Big(\frac{\boldsymbol{p}^2}{r}+\frac{\boldsymbol{r}\cdot(\boldsymbol{r}\cdot\boldsymbol{p})\boldsymbol{p}}{r^3}\Big)$, is taken into account. It turned out that the contributions from the orbit-orbit coupling interaction, the spin-orbit coupling, and the tensor interaction cannot be omitted. A high degree of accuracy of the masses is essential since they contribute to all considered terms. The orbit-orbit coupling term may lead to an instability of the solution of the Schrödinger equation and should be regularized. Further investigations on the meson cross sections and the velocity-dependent effect in the confining potential will be of great interest.

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