Impact of ultra-light particles on polarization of laser light in strong external fields*

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Abstract The recent results by the PVLAS group on possible changes of polarization of laser light in a transverse magnetic field are beyond the QED expectations by many orders of magnitude. If confirmed, they may indicate new physics associated with ultra-light particles. I describe here how the polarization of light is modified in an external magnetic field by interactions with a spin-zero particle of no definite parity. While the PVLAS-type experiments cannot tell such a particle from one with definite parity, the parity property could be studied in photon regeneration experiments if the polarization of the regenerated photons could be measured. This talk was based on my recent work.

Key words ultra-light particle, polarization, strong field

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1 Introduction

Photons can self-interact as a result of quantum effects. In QED this is summarized in the celebrated Euler-Heisenberg effective Lagrangian in the weak field and low frequency limit[1]. The self-interactions result in nonlinear optical phenomena in vacuum in the presence of an external electromagnetic field[2]. Due to the smallness and technical difficulties, the experimental observation of such effects is very challenging. After long search for years without success[3], the PVLAS group has claimed recently that they observed the rotation of the plane of polarization[4] and the ellipticity[5] of a linearly polarized laser beam after the latter traverses a transverse strong magnetic field. Very recently, the PVLAS group has updated its results[6] that are less stringent than its original ones, but the new upper bounds on the rotation and ellipticity are still many orders of magnitude higher than expected in QED[2, 7]. For the ellipticity, this is roughly a factor of 10^3, while for the rotation the QED result is so tiny that it cannot be observed in laboratory experiments in the foreseeable future.

If the PVLAS results are true, they would be caused by some new physics beyond QED or even the standard model. It is natural in this circumstance to ask whether it is possible to accommodate the results in terms of photon self-interactions alone but without restricting oneself to QED. It turns out that this is not possible at the first nontrivial order in the low energy effective theory of photons for reasonable values of low energy parameters[8]. This would mean that some ultra-light new particles with a mass of order the laser frequency or lower and interacting with the electromagnetic field have to be invoked. The virtual and real production of these particles by a laser in an external field induces respectively the ellipticity and rotation of the initial laser beam[9—11]. If the PVLAS results are confirmed by other groups, this would indicate new physics in a manner that is complementary to collider physics.

The best motivated candidate for such an ultra-light particle seems to be the axion in QCD, a pseudoscalar particle from spontaneous breakdown of the Peccei-Quinn symmetry. While it was originally suggested to solve the strong CP problem, it is also one of the most attractive candidates for dark matter. However, if the results of PVLAS are saturated by an axion-like particle, it is almost certain that it cannot be the QCD axion because the mass-coupling relation for the latter is disfavored by PVLAS and strongly excluded by helioscope experiments and astrophysi-

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cal observations. Leaving aside the motivation for a QCD axion, there is actually no theoretical prejudice for such a particle to be pseudoscalar or scalar. It could well be that such a spin-zero particle, originating from certain high energy theory, is neither pseudoscalar nor scalar, but is a mixture of the two. Then, in the effective field theory at the PVLAS energy scale, we should include all possible interactions of the particle with the electromagnetic field at leading order in the dimensional analysis. We mention in passing that there is no direct threat to measurements of parity and CP violation at much higher energy scales where our effective theory does not apply.

2 Analysis of light propagation

The effective theory at the lowest non-trivial order is defined by[12]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{1}{2}(\partial^{\mu}\varphi)^2-\frac{1}{2}m^2\varphi^2+\frac{1}{4}\lambda_+\varphi F^{\mu\nu}F_{\mu\nu}+\frac{1}{4}\lambda_-\varphi\tilde{F}^{\mu\nu}F_{\mu\nu}\,, \tag{1}$$

where φ is a spin-zero axion-like field, $F_{\mu\nu}$ the electromagnetic field tensor with dual $\tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ ($\epsilon^{0123}=+1$), and $\lambda_\pm$ are two coupling constants. The separate cases with either $\lambda_+=0$ (φ being a pseudoscalar) or $\lambda_-=0$ (a scalar) have been studied in the literature. The equations of motion (EoM's) are

$$\begin{aligned}(\partial^2+m^2)\varphi &= \frac{1}{4}\lambda_+F^{\mu\nu}F_{\mu\nu}+\frac{1}{4}\lambda_-\tilde{F}^{\mu\nu}F_{\mu\nu},\\ \partial_\mu F^{\mu\nu} &= \partial_\mu\left(\lambda_+\varphi F^{\mu\nu}+\lambda_-\varphi\tilde{F}^{\mu\nu}\right).\end{aligned} \tag{2}$$

To study the propagation of a laser beam in the magnetized vacuum, we split the electromagnetic field into a quantum (laser) and a classical (external) part. Since the external field will be much larger in strength than the quantum ones, the leading effects to quantum fields can be approximated by linearizing the EoM's with respect to them:

$$\begin{aligned}(\partial_t^2-\nabla^2+m^2)\varphi &= \lambda_+(\nabla\times\boldsymbol{a})\cdot\boldsymbol{B}+\lambda_-(\partial_t\boldsymbol{a})\cdot\boldsymbol{B},\\ (\partial_t^2-\nabla^2)\boldsymbol{a} &= \lambda_+(\nabla\varphi)\times\boldsymbol{B}-\lambda_-(\partial_t\varphi)\boldsymbol{B},\end{aligned} \tag{3}$$

where $\boldsymbol{a}$ is the vector potential for the laser light and $\boldsymbol{B}$ the external magnetic field.

We follow Ref. [11] to derive the evolution equation for a plane-wave light beam propagating in the z direction and perpendicularly to $\boldsymbol{B}$. We can choose $a^3=0$, and a^1,a^2,φ depend only on (t,z). Ignoring terms that are higher order in small parameters not to be considered here, the evolution equation in matrix form is

$$(1+\mathrm{i}\omega^{-1}\partial_z+\varOmega)\begin{pmatrix}a^1\\ a^2\\ \varphi\end{pmatrix}=0,$$

$$\varOmega=\begin{pmatrix}0 & 0 & +\mathrm{i}\delta_-\\ 0 & 0 & +\mathrm{i}\delta_+\\ -\mathrm{i}\delta_- & -\mathrm{i}\delta_+ & -\delta_0\end{pmatrix}, \tag{4}$$

where

$$\delta_-=\frac{1}{2\omega}(\lambda_-B_1-\lambda_+B_2),$$
$$\delta_+=\frac{1}{2\omega}(\lambda_-B_2+\lambda_+B_1),\ \delta_0=\frac{m^2}{2\omega^2}. \tag{5}$$

Without loss of generality, we choose $\boldsymbol{B}=|\boldsymbol{B}|\hat{x}$, so that $\delta_\pm=(2\omega)^{-1}\lambda_\pm|\boldsymbol{B}|$. Then $a^{1,2}$ are respectively the components parallel and perpendicular to $\boldsymbol{B}$, denoted below as $a_{\parallel,\perp}$.

The evolution equation is solved by diagonalizing the matrix $\varOmega$:

$$\varOmega_{\text{diag}}=U^{-1}\varOmega U=\delta_0\ \text{diag}\,(0,\epsilon^2,-(1+\epsilon^2))+\delta_0 O(\epsilon^4), \tag{6}$$

$$U=ER_a(\theta_\lambda)R_{a\varphi}(\epsilon)=E\begin{pmatrix}c_\lambda & s_\lambda c_\epsilon & -s_\lambda s_\epsilon\\ -s_\lambda & c_\lambda c_\epsilon & -c_\lambda s_\epsilon\\ 0 & s_\epsilon & c_\epsilon\end{pmatrix}, \tag{7}$$

where $E=\text{diag}(i,i,1)$ removes the $\pm i$ factors in $\varOmega$, $R_a(\theta_\lambda)$ rotates the two components $\delta_\pm$ into one,

$$\delta=\sqrt{\delta_+^2+\delta_-^2}=\frac{|\boldsymbol{B}|}{2\omega}\sqrt{\lambda_+^2+\lambda_-^2}\,, \tag{8}$$

which in turn mixes with φ. This last mixing is then diagonalized by $R_{a\varphi}(\epsilon)$. The following shortcuts have been used:

$$s_\lambda=\sin\theta_\lambda=\frac{\lambda_-}{\sqrt{\lambda_+^2+\lambda_-^2}},\ c_\lambda=\cos\theta_\lambda=\frac{\lambda_+}{\sqrt{\lambda_+^2+\lambda_-^2}}\,, \tag{9}$$

$$s_\epsilon=\sin\epsilon,\ c_\epsilon=\cos\epsilon,\ \tan 2\epsilon=\frac{2\delta}{\delta_0}=\frac{2\omega|\boldsymbol{B}|}{m^2}\sqrt{\lambda_+^2+\lambda_-^2}\,. \tag{10}$$

For the original PVLAS experiment, one has $\omega=\frac{2\pi}{\lambda}\approx 1.17$ eV, $|\boldsymbol{B}|=5$ T ≈ 976.68 eV^2. PVLAS found that one would want $\lambda_\pm\sim(4\times10^5\ \text{GeV})^{-1}$, $m\sim10^{-3}$ eV to explain the results; therefore, $\epsilon\sim 3\times10^{-6}$ is very small.

Using the above solution, we can follow the development of a state when it passes through a magnetic field. Assume the initial state is $\varPsi(z_0)=(a_\parallel(z_0),a_\perp(z_0),\varphi(z_0))^{\mathrm{T}}$ on entering the $\boldsymbol{B}$ field at z_0. It will evolve into the state, $\varPsi(z)=(a_\parallel(z),a_\perp(z),\varphi(z))^{\mathrm{T}}$, after traversing a distance $z-$

$z_0 = L \geqslant 0$ in the $+z$ direction:

$$\Psi(z_0+L) = e^{i\omega L} V_+(L)\Psi(z_0),\ V_+(L) = U e^{i\omega L \Omega_{\rm diag}} U^{-1}\,. \tag{11}$$

It is sufficient for our purpose to expand in ϵ to $O(\epsilon^2)$:

$$V_+(L) = V_0(L) + \epsilon V_1(L) + \epsilon^2 V_2(L) + O(\epsilon^3), \tag{12}$$

where

$$V_0(L) = \mathrm{diag}(1,1,e^{-i\zeta}),\ \zeta = \delta_0 \omega L\,, \tag{13}$$

$$V_1(L) = (1-e^{-i\zeta})\begin{pmatrix} & & is_\lambda \\ & & ic_\lambda \\ -is_\lambda & -ic_\lambda & 0 \end{pmatrix}, \tag{14}$$

$$V_2(L) = (-1+i\zeta+e^{-i\zeta})\begin{pmatrix} s_\lambda^2 & c_\lambda s_\lambda & \\ c_\lambda s_\lambda & c_\lambda^2 & \\ & & 0 \end{pmatrix} + (1-e^{-i\zeta}(1+i\zeta))\mathrm{diag}(0,0,1). \tag{15}$$

For $\lambda_+ = 0$, we have $c_\lambda = 0$, $s_\lambda = \mathrm{sign}(\lambda_-)$ and $s_\lambda \epsilon = \omega |\boldsymbol{B}| \lambda_- m^{-2}$. The case elaborated upon in Ref. [11] is then recovered.

For propagation in the $-z$ direction, the linearized EoM's can be obtained from Eq. (4) by $\partial_z \to -\partial_z$ and $\lambda_+ \to -\lambda_+$. Thus, an initial state, $\Psi(z_0)$, entering the $\boldsymbol{B}$ field at z_0 will evolve into the following one, after traversing a distance $z_0 - z = L$ in the $-z$ direction:

$$\Psi(z_0-L) = e^{i\omega L} V_-(L)\Psi(z_0),\ V_-(L) = V_+(L)|_{c_\lambda \to -c_\lambda}\,. \tag{16}$$

Now we study the optical implications of the above results. Suppose a beam of light propagates in the $+z$ direction and enters a transverse $\boldsymbol{B} = \boldsymbol{B}\hat{x}$ field. If the initial beam is linearly polarized at an angle θ measured counter-clockwise in the (xy) plane with respect to the field, it will evolve into the following state after traversing a distance L in the field:

$$\begin{pmatrix} a_\parallel \\ a_\perp \\ \varphi \end{pmatrix}(L) = e^{i\omega L} V_+(L)\begin{pmatrix} c_\theta \\ s_\theta \\ 0 \end{pmatrix} \equiv e^{i\omega L}\begin{pmatrix} \eta\cos(\theta+\Delta\theta)e^{i\phi_\parallel} \\ \eta\sin(\theta+\Delta\theta)e^{i\phi_\perp} \\ \rho e^{i\sigma} \end{pmatrix}, \tag{17}$$

where the real parameters η, $\Delta\theta$, ρ, σ, $\phi_{\parallel,\perp}$ are worked out to $O(\epsilon^2)$. The polarization of the initial beam has been rotated by,

$$\Delta\theta = -\epsilon^2 \sin^2\frac{\zeta}{2}\sin 2(\theta_\lambda+\theta), \tag{18}$$

while the relative phase shift of the parallel and perpendicular components of the light beam results in the ellipticity, $\tan\chi$, with $\sin 2\chi = \sin 2(\theta + \Delta\theta)\sin(\phi_\perp - \phi_\parallel)$ and $\chi \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$,

$$\tan\chi \approx \chi \approx \frac{1}{2}\epsilon^2(\zeta - \sin\zeta)\sin 2(\theta_\lambda+\theta). \tag{19}$$

The production probability of φ is

$$P[(\gamma \text{ at } \theta) \to \varphi] = |\rho|^2 = 4\epsilon^2 \sin^2(\theta_\lambda+\theta)\sin^2\frac{\zeta}{2}\,. \tag{20}$$

The familiar results for a pure pseudoscalar or scalar are recovered by putting $c_\lambda = 0$, $|s_\lambda| = 1$ or $s_\lambda = 0$, $|c_\lambda| = 1$ respectively.

In a photon regeneration[13] or shining-light-through-walls experiment like ALPS[14], φ particles would be produced by a laser in the production zone, then penetrate a wall that blocks the laser, and enter into the detection zone. This beam of φ particles will evolve into the state

$$e^{i\omega L}\Big(i\epsilon s_\lambda(1-e^{-i\zeta}),\ \ i\epsilon c_\lambda(1-e^{-i\zeta}),\ \ e^{-i\zeta} + \epsilon^2[1-e^{-i\zeta}(1+i\zeta)]\Big)^{\rm T}\,. \tag{21}$$

The probability to produce a photon is thus

$$P[\varphi \to \text{photon}] = 4\epsilon^2 \sin^2\frac{\zeta}{2}\,. \tag{22}$$

The produced photon is linearly polarized at an angle θ to the magnetic field, determined by $\tan\theta\tan\theta_\lambda = 1$. It would thus be possible to extract the parity property of φ if the photon's polarization could be measured.

Consider two interesting specific configurations. When $\sin(\theta_\lambda + \theta) = 0$, i.e., $\tan\theta = -\lambda_-/\lambda_+$, there are no optical effects and a photon with this particular polarization relative to $\boldsymbol{B}$ cannot be converted into a φ, although an existing φ can still be converted into a photon with an orthogonal polarization. When $\cos(\theta_\lambda+\theta) = 0$, i.e., $\tan\theta = \lambda_+/\lambda_-$, there are no optical effects either due to equal attenuation and phase retardation of the two orthogonal polarizations, but the photon to φ transition probability reaches its maximum which is equal to that of the inverse transition.

While the above results are relevant for astrophysical sources of φ particles, they are insufficient for laboratory optical experiments where the laser light is reflected back and forth in a magnetic field for a larger gain of the signal. Due to parity violating interactions, the changes of polarization in opposite directions are not equal, and can cancel partially for a parity-symmetric round trip. Suppose the initial state $\Psi(0)$ propagates first for a distance L in the $+z$ direction and perpendicularly to $\boldsymbol{B}$, gets reflected by a high-reflectivity mirror, then propagates in the $-z$

direction for the same distance and is reflected back to its starting point. The state becomes

$$\Psi_{\text{round}} = (R\mathrm{e}^{\mathrm{i}\omega L}V_-)(R\mathrm{e}^{\mathrm{i}\omega L}V_+)\Psi(0)\,, \qquad (23)$$

where $R=\mathrm{diag}(1,1,0)$ expresses the fact that the produced φ particles are not reflected but penetrate the mirror to disappear. The evolution is thus not coherent at the mirrors.

In all optical experiments the light is reflected many times. The final state can be well approximated by that after N times of round trips:

$$\Psi_{N,\text{round}} = \mathrm{e}^{\mathrm{i}2\omega NL}(RV_-RV_+)^N\Psi(0)\,, \qquad (24)$$

where N is roughly half the total number of passage of light in the field. An initial laser beam of $\Psi(0)=(c_\theta,s_\theta,0)^{\mathrm{T}}$ will evolve into the one with the induced rotation and ellipticity being

$$\Delta\theta = -2N\epsilon^2\sin^2\frac{\zeta}{2}\sin 2\theta\cos 2\theta_\lambda\,, \qquad (25)$$

$$\chi = N\epsilon^2(\zeta-\sin\zeta)\sin 2\theta\cos 2\theta_\lambda\,. \qquad (26)$$

Thus the standard results for a particle of definite parity are modified by a factor $\cos 2\theta_\lambda$. When the two interactions are of the same strength, i.e., $\cos 2\theta_\lambda=0$, no net optical effects remain as intuitively expected, although the photon to φ transition generally occurs. It is also clear that a reflection symmetric experiment like PVLAS cannot tell a pseudoscalar or scalar particle from one of no definite parity.

3 Conclusion

We have considered the effects of a spin-zero particle of no definite parity on the evolution of a light beam propagating in a transverse magnetic field. We calculated the modifications to the light polarization for two experimental set-ups. For a parity asymmetric set-up, interesting phenomena can occur for special polarization configurations where neither rotation nor ellipticity is induced although particle transitions still take place. Those configurations are determined by the relative strength of the interactions. In a parity symmetric set-up like PVLAS, however, optical changes are simply modified by a common factor, compared to the pure-parity case. The factor is again fixed by the relative strength of the interactions. Thus, if the results in such an experiment can be saturated by a particle with definite parity, they can be equally well fitted with a particle of no definite parity by adjusting the factor. Nevertheless, the parity property could be studied in a photon regeneration experiment if the polarization of regenerated photons could be measured.

As a final remark, we mention that there are alternatives other than axion-like particles to interpret the PVLAS results. Especially interesting are the millicharged particles with mass of order 0.1 eV and an electric charge of order 10^{-6}e that interact with photons as in QED[15]. The next generation of experiments in the near future could be capable of deciding whether any of these possibilities is realistic.

References

1 Heisenberg W, Euler H. Z. Phys., 1936, **98**: 714; Weisskopf V F. Mat. Fys. Medd.-K. Dan Vidensk. Selsk., 1936, **14**: 6; Schwinger J. Phys. Rev., 1951, **82**: 664

2 Adler S L, Bahcall J N, Callan C G et al. Phys. Rev. Lett., 1970, **25**: 1061; Adler S L. Ann. Phys. (N.Y.), 1971, **67**: 599; Iacopini E, Zavattini E. Phys. Lett., 1979, **85B**: 151

3 Cameron R et al (BFRT Collaboration). Phys. Rev.D, 1993, **47**: 3707

4 Zavattini E et al (PVLAS Collaboration). Phys. Rev. Lett., 2006, **96**: 110406

5 Cantatore G (PVLAS Collaboration). Talk at IDM 2006, Rhodos, Greece, Sept. 11-16th, 2006

6 Zavattini E et al (PVLAS Collaboration). arXiv:0706.3419 [hep-ex]

7 Adler S L. J. Phys. A, 2007, **40**: F143; Biswas S, Melnikov K. Phys. Rev. D, 2007, **75**: 053003

8 HU X P, LIAO Y. Eur. Phys. J. C, 2008, **53**: 635

9 Sikivie P. Phys. Rev. Lett., 1983, **51**: 1415; Phys. Rev. Lett., 1984, **52**: 695

10 Maiani L, Petronzio R, Zavattini E. Phys. Lett. B, 1986, **175**: 359

11 Raffelt G, Stodolsky L. Phys. Rev. D, 1988, **37**: 1237

12 LIAO Y. Phys. Lett. B, 2007, **650**: 257

13 Bibber K Van et al. Phys. Rev. Lett., 1987, **59**: 759; Ruoso G et al. Z. Phys. C, 1992, **56**: 505

14 Ehret K et al. hep-ex/0702023

15 Gies H, Jaeckel J, Ringwald A. Phys. Rev. Lett., 2006, **97**: 140402; Ahlers M, Gies H, Jaeckel J et al. Phys. Rev. D, 2007, **75**: 035011