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Next-to-leading-order calculation in k_T factorization*

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Abstract We explain the framework for calculating next-to-leading-order (NLO) corrections to exclusive processes in the k_T factorization theorem, taking $\pi\gamma^* \to \gamma$ as an example. Partons off-shell by k_T^2 are considered in both the quark diagrams from full QCD and the effective diagrams for the pion wave function. The gauge dependences in the above two sets of diagrams cancel, when deriving the k_T-dependent hard kernel as their difference. The light-cone singularities in the k_T-dependent pion wave function are regularized by rotating the Wilson lines away from the light cone. Both the large double logarithms $\ln^2 k_\mathrm{T}$ and $\ln^2 x$, x being a parton momentum fraction, arise from the loop correction to the virtual photon vertex, the former being absorbed into the pion wave function, and the latter into a jet function.

Key words k_T factorization, radiative correction, meson wave function

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1 Introduction

The k_T factorization theorem[1—6], as a fundamental tool of perturbative QCD (PQCD), has been widely applied to inclusive and exclusive processes. It has been pointed out that the k_T factorization theorem is appropriate for processes dominated by contributions from small parton momentum fractions x[7]. Its application to exclusive B meson decays has led to the PQCD approach[8—12], which is free of the singularities from the end-point regions of x that usually appear in collinear factorization theorem[13—18]. The current application of the k_T factorization theorem to exclusive processes is mainly made at leading order (LO) in the strong coupling constant α_s[19]: the important logarithms in the hadron wave functions have been organized to all orders, but hard kernels are still evaluated at tree level. To demonstrate that the k_T factorization theorem is a systematical tool, higher-order calculations of hard kernels are demanded.

In this talk we shall elucidate the framework for higher-order calculations, deriving the next-to-leading-order (NLO) hard kernel for the scattering process $\pi\gamma^* \to \gamma$ as an example. The point is that partons in both the quark diagrams from full QCD and the effective diagrams for the pion wave function are off mass shell by k_T^2. The difference between the two sets of diagrams defines the hard kernel in the k_T factorization theorem, a procedure similar to the derivation of Wilson coefficients in an effective field theory. This is the way to obtain a k_T-dependent hard kernel without breaking gauge invariance, since the gauge dependences cancel between the above two sets of diagrams. A physical quantity is expressed as a convolution of a hard kernel with model wave functions, which are determined by methods beyond a perturbation theory, such as lattice QCD and QCD sum rules, or extracted from experimental data. A gauge-invariant hard kernel then leads to gauge-invariant predictions from the k_T factorization theorem.

We stress that the light-cone singularities [20] in the naive definition for k_T-dependent hadron wave functions must be regularized. These singularities, not present in the quark diagrams, are not physical. If not regularized, higher-order hard kernels, computed as the difference of the quark diagrams and the effective diagrams, will be divergent. We shall adopt the modified definition, in which the Wilson lines involved in the nonlocal matrix elements for hadron wave functions are rotated away from the light cone. After the subtraction of the singularities, a hard kernel

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depends unavoidably on the regularization schemes, which can, nevertheless, be regarded as part of the factorization-scheme dependence. This dependence, usually minimized by adhering to a fixed prescription for deriving hard kernels, does not cause a trouble. The removal of the light-cone singularities from wave functions and the gauge invariance of hard kernels are the two essential ingredients for making physical predictions from the $k_{\rm T}$ factorization theorem.

2 $O(\alpha_{\rm s}) k_{\rm T}$ factorization

We first set up the framework for computing the hard kernel for the pion transition form factor in the $k_{\rm T}$ factorization theorem. The momentum P_1 of the pion and the momentum P_2 of the out-going on-shell photon are chosen as

$$P_1=(P_1^+,0,\mathbf{0}_{\rm T})\,,\quad P_2=(0,P_2^-,\mathbf{0}_{\rm T})\,. \tag{1}$$

The LO quark diagram, in which the anti-quark $\bar{\rm q}$ carries the on-shell fractional momentum $k=(xP_1^+,0,\mathbf{0}_{\rm T})$ and the internal quark carries P_2-k, leads to the amplitude

$$G^{(0)}(x,Q^2) = \frac{{\rm tr}[\not\epsilon(\not P_2-\not k)\gamma_\mu \not P_1\gamma_5]}{(P_2-k)^2} = -\frac{{\rm tr}[\not\epsilon \not P_2\gamma_\mu \not P_1\gamma_5]}{xQ^2}\,, \tag{2}$$

with the leading spin structure $\not P_1\gamma_5$ of the pion and $Q^2\equiv 2P_1\cdot P_2$. We have suppressed other constant factors, such as the electric charge, the color number, and the pion decay constant, which are irrelevant in the following discussion.

The trivial factorization of Eq. (2) reads [7],

$$\begin{aligned} G^{(0)}(x,Q^2) &= \int {\rm d}x'{\rm d}^2k'_{\rm T}\varPhi^{(0)}(x;x',k'_{\rm T})\times H^{(0)}(x',Q^2,k'_{\rm T})\,,\\ \varPhi^{(0)}(x;x',k'_{\rm T}) &= \delta(x-x')\delta(\boldsymbol{k}'_{\rm T})\,,\\ H^{(0)}(x,Q^2,k_{\rm T}) &= -\frac{{\rm tr}[\not\epsilon \not P_2\gamma_\mu \not P_1\gamma^5]}{xQ^2+k_{\rm T}^2}\,. \end{aligned} \tag{3}$$

Once we concentrate on the small x region, the treatment of the parton $k_{\rm T}$ differs from that in the collinear factorization theorem: $k_{\rm T}^2$ in the denominator of Eq. (3) is not small compared to xQ^2, and the internal quark propagator should not be expanded into a power series in $k_{\rm T}^2$ [21, 22]. $k_{\rm T}$ in the numerator, being power-suppressed by $1/Q$, is combined with the three-parton meson wave functions to form a gauge-invariant set of higher-twist contributions as in the collinear factorization theorem. This special treatment of the parton $k_{\rm T}$ characterizes the distinction between $k_{\rm T}$ and collinear factorizations[19]. Because of the zeroth-order wave function $\varPhi^{(0)}\propto\delta(\boldsymbol{k}'_{\rm T})$, the LO hard kernel $H^{(0)}$ does actually not depend on the parton transverse momentum.

The $O(\alpha_{\rm s})$ quark diagrams corresponding to Eq. (2) from full QCD are displayed in Fig. 1, in which the upper line represents the q quark. The factorization of the collinear divergences from these radiative corrections is Ref. [7]:

$$\begin{aligned} G^{(1)}(x,Q^2) = \int & {\rm d}x'{\rm d}^2k'_{\rm T}\big[\varPhi^{(1)}(x;x',k'_{\rm T})\times\\ & H^{(0)}(x',Q^2,k'_{\rm T})+\\ & \varPhi^{(0)}(x;x',k'_{\rm T})H^{(1)}(x',Q^2,k'_{\rm T})\big]\,, \end{aligned} \tag{4}$$

where the $O(\alpha_{\rm s})$ effective diagrams $\varPhi^{(1)}$ are defined by the leading-twist quark-level wave function[7, 23]

$$\begin{aligned} \varPhi(x;x',k'_{\rm T}) = \int & \frac{{\rm d}y^-}{2\pi{\rm i}}\frac{{\rm d}^2y_{\rm T}}{(2\pi)^2}{\rm e}^{-{\rm i}x'P_1^+y^-+{\rm i}\boldsymbol{k}'_{\rm T}\cdot\boldsymbol{y}_{\rm T}}\times\\ & \langle 0|\bar q(y)W_y(n)^\dagger I_{n;y,0}W_0(n)\times\\ & \not n_-\gamma_5 q(0)|q(P_1-k)\bar q(k)\rangle\,, \end{aligned} \tag{5}$$

with $y=(0,y^-,\boldsymbol{y}_{\rm T})$ being the coordinate of the anti-quark field $\bar q$, $n_-=(0,1,\mathbf{0}_{\rm T})$ a null vector along P_2, and $|q(P_1-k)\bar q(k)\rangle$ the leading Fock state of the pion.

The factor $W_y(n)$ with $n^2\neq 0$ denotes the Wilson line operator,

$$W_y(n)=P\exp\left[-{\rm i}g\int_0^\infty{\rm d}\lambda n\cdot A(y+\lambda n)\right]\,. \tag{6}$$

The two Wilson lines $W_y(n)$ and $W_0(n)$ are connected by a link $I_{n;y,0}$ at infinity in this case[7, 24]. Eq. (5) contains additional collinear divergences from the region with a loop momentum parallel to n_-, as the Wilson line direction approaches the light cone, ie., as $n\to n_-$[20]. It will be shown that n^2 serves as an infrared regulator for the light-cone singularities, and that the wave function depends on the additional scale $\zeta^2\equiv 4(n\cdot P_1)^2/|n^2|$, ie., on the external kinematic variable. Besides, $\varPhi$ also depends on the factorization scale $\mu_{\rm f}$, which is not shown explicitly. Note that Eq. (5) does not directly reduce to the distribution amplitude in the collinear factorization theorem, when integrated over $k_{\rm T}$, but to a convolution of a hard kernel with the distribution amplitude[25].

With one-gluon exchange, the outgoing partons from $\varPhi^{(1)}$, ie., the partons participating in the hard scattering, carry the transverse momenta, so that $H^{(0)}$ in Eq. (4) depends on $k'_{\rm T}$ nontrivially in the first-order factorization. Being convoluted with $\varPhi^{(0)}$, the partons entering the NLO hard kernel $H^{(1)}$ are still on-shell. To acquire the nontrivialc $k_{\rm T}$ dependence, $H^{(1)}$ must be convoluted with the higher-order wave functions $\Phi^{(i)}$, $i \geqslant 1$: the gluon exchanges in $\Phi^{(i)}$ render the incoming partons of $H^{(1)}$, ie., the incoming partons of the quark diagrams $G^{(1)}$ and the effective diagrams $\Phi^{(1)}$ off-shell by $k_{\rm T}^2$[7]. We thus derive $H^{(1)}(x,Q^2,k_{\rm T})$ according to the formula

$$H^{(1)}(x,Q^2,k_{\rm T}) = G^{(1)}(x,Q^2,k_{\rm T}) - \int {\rm d}x'{\rm d}^2k'_{\rm T}\Phi^{(1)} \times (x,k_{\rm T};x',k'_{\rm T})H^{(0)}(x',Q^2,k'_{\rm T}), \quad (7)$$

where $\Phi^{(1)}(x,k_{\rm T};x',k'_{\rm T})$ is defined by Eq. (5) but with the $\bar{\rm q}$ quark momentum $k=(xP_1^+,0,\boldsymbol{k}_{\rm T})$. As stated in the Introduction, the gauge dependences of $G^{(1)}$ and $\Phi^{(1)}$ cancel in the above expression, such that $H^{(1)}(x,Q^2,k_{\rm T})$ turns out to be gauge-invariant.

3 $O(\alpha_{\rm s})$ quark diagrams

The loop integrals associated with the $O(\alpha_{\rm s})$ quark diagrams in Figs. 1(a)—(f) in the Feynman gauge, where the $\bar{\rm q}$ quark carries the momentum $k=(xP_1^+,0,\boldsymbol{k}_{\rm T})$, are Ref. [26]. The results for the self-energy corrections are

$$G_{\rm a}^{(1)}(x,Q^2,k_{\rm T}) = -\frac{\alpha_{\rm s}}{8\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2}{k_{\rm T}^2{\rm e}^{\gamma_E}}+2\right)\times H^{(0)}(x,Q^2,k_{\rm T}), \quad (8)$$

$$G_{\rm b}^{(1)}(x,Q^2,k_{\rm T}) = -\frac{\alpha_{\rm s}}{8\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2}{k_{\rm T}^2{\rm e}^{\gamma_E}}+2\right)\times H^{(0)}(x,Q^2,k_{\rm T}), \quad (9)$$

$$G_{\rm c}^{(1)}(x,Q^2,k_{\rm T}) = -\frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2{\rm e}^{-\gamma_E}}{xQ^2+k_{\rm T}^2}+2\right)\times H^{(0)}(x,Q^2,k_{\rm T}), \quad (10)$$

where $1/\epsilon$ denotes the ultraviolet pole, $C_{\rm F}$ is a color factor, μ the renormalization scale, and γ_E the Euler constant. Since the external partons are off-shell by $k_{\rm T}^2$, the collinear divergences in Figs. 1(a) and 1(b) are represented by the infrared logarithms $\ln k_{\rm T}^2$ in Eqs. (8) and (9), respectively. The internal quark in Fig. 1(c) is off-shell by the invariant mass squared $xQ^2+k_{\rm T}^2$, which then replaces the argument $k_{\rm T}^2$ in the infrared logarithm.

In the small x region we drop terms suppressed by powers of x or $k_{\rm T}^2/Q^2$. The loop correction to the virtual photon vertex gives

$$G_{\rm d}^{(1)}(x,Q^2,k_{\rm T}) = \frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2}{k_{\rm T}^2{\rm e}^{\gamma_E}} - 2\ln\frac{Q^2}{k_{\rm T}^2}\ln\frac{Q^2}{xQ^2+k_{\rm T}^2}+2\ln\frac{Q^2}{xQ^2+k_{\rm T}^2} + \ln\frac{Q^2}{k_{\rm T}^2}-\frac{2\pi^2}{3}+\frac{3}{2}\right)H^{(0)}(x,Q^2,k_{\rm T}). \quad (11)$$

At small x the q quark in Fig. 1(d) is energetic, implying the existence of the collinear logarithmic enhancement $\ln(Q^2/k_{\rm T}^2)$, and the internal quark is close to the mass shell, implying the soft enhancement $\ln[Q^2/(xQ^2+k_{\rm T}^2)]$. Their overlap then leads to the double logarithm $\ln(Q^2/k_{\rm T}^2)\ln[Q^2/(xQ^2+k_{\rm T}^2)]$ in Eq. (11). This double logarithm can be reexpressed as

$$-2\ln\frac{Q^2}{k_{\rm T}^2}\ln\frac{Q^2}{xQ^2+k_{\rm T}^2} = -\ln^2\frac{Q^2}{k_{\rm T}^2}-\ln^2\frac{Q^2}{xQ^2+k_{\rm T}^2}+\ln^2\frac{xQ^2+k_{\rm T}^2}{k_{\rm T}^2}. \quad (12)$$

The first term is known as the Sudakov logarithm[4, 27], which will be absorbed into the pion wave function. The second term exists even in the collinear factorization theorem without taking into account $k_{\rm T}$[28, 29], $\ln[Q^2/(xQ^2+k_{\rm T}^2)] \sim \ln^2 x$, which can be factorized into a jet function associated with the internal quark[30].

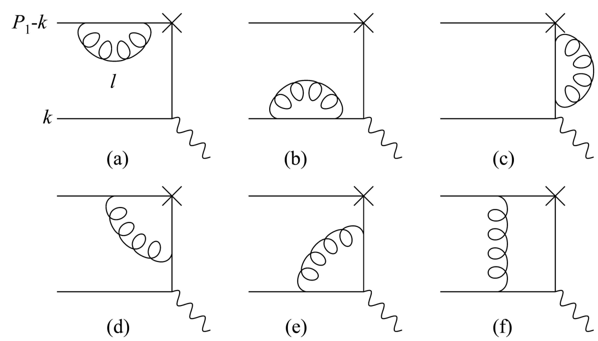

Fig. 1. $O(\alpha_{\rm s})$ quark diagrams for $\pi\gamma^* \to \gamma$ with $\times$ representing the virtual photon vertex.

The loop correction to the out-going on-shell photon vertex is written as

$$G_{\rm e}^{(1)}(x,Q^2,k_{\rm T}) = \frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2}{k_{\rm T}^2{\rm e}^{\gamma_E}} + \ln\frac{xQ^2+k_{\rm T}^2}{k_{\rm T}^2}+\frac{3}{2}\right)H^{(0)}(x,Q^2,k_{\rm T}), \quad (13)$$

which does not contain a double logarithm for the following reason. In the large x region the internal quark is off-shell by $O(Q^2)$, and the soft enhancement disappears. In the small x region the $\bar{q}$ quark becomes soft, and the associated collinear enhancement is diminished by the limited phase space for the loop momentum. Therefore, there is a lack of overlap of the collinear and soft enhancements, and only the $O(1)$ single logarithm exists.

At last, the evaluation of the box diagram Fig. 1(f) is simple, giving a power-suppressed contribution at small x. In the region with $x \sim O(1)$, ie., $k^+ \sim O(Q)$, the internal quark in Fig. 1(f) is off-shell by $1/[P_2\cdot(k-l)] \sim 1/Q^2$ for either a collinear loop momentum $l^+ \sim O(Q)$ or an ultraviolet loop momentum $l^\mu \sim O(Q)$, the same as $1/(P_2\cdot k) \sim 1/Q^2$ in the LO amplitude. Namely, the radiative correction from the box diagram does not change the LO power-law behavior, and its contribution is finite. In the region with small $x \sim O(\Lambda)$, Λ being a hadronic scale, the LO amplitude scales like $1/(P_2\cdot k) \sim 1/(Q\Lambda)$, while the internal quark in Fig. 1(f) remains off-shell by $1/[P_2\cdot(k-l)] \sim 1/Q^2$ for either collinear or ultraviolet l. Thus the contribution from the box diagram becomes power-suppressed and negligible, and we have $G_f^{(1)}(x,Q^2,k_{\rm T})=0$ at leading power.

The sum of the radiative corrections from the quark diagrams Figs. 1(a)—(f) gives

$$G^{(1)}(x,Q^2,k_{\rm T})=\sum_{\rm i=a}^{\rm f}G_{\rm i}^{(1)}(x,Q^2,k_{\rm T})=$$
$$-\frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(2\ln\frac{Q^2}{k_{\rm T}^2}\ln\frac{Q^2}{xQ^2+k_{\rm T}^2}-3\ln\frac{Q^2}{k_{\rm T}^2}+1+\frac{2\pi^2}{3}\right)\times$$
$$H^{(0)}(x,Q^2,k_{\rm T})\,. \tag{14}$$

It is observed that all the ultraviolet poles cancel and the μ dependence disappears completely, a consequence of the conservation of the current that defines the pion transition form factor. It will be demonstrated in the next section that the effective diagrams for the pion wave function generate the same infrared logarithms $\ln k_{\rm T}^2$.

4 $O(\alpha_{\rm s})$ effective diagrams

The explicit expressions for the $O(\alpha_{\rm s})$ effective diagrams displayed in Fig. 2(a)—(g) are also Ref. [26]. We compute the convolution of $\Phi^{(1)}$ with the LO hard kernel $H^{(0)}$ in Eq. (4) over the integration variables x' and $k'_{\rm T}$, denoted by $\otimes$ below:

$$\Phi_{\rm i}^{(1)}\otimes H^{(0)} \equiv \int{\rm d}x'{\rm d}^2k'_{\rm T}\Phi_{\rm i}^{(1)}(x,k_{\rm T};x',k'_{\rm T})\times H^{(0)}(x',Q^2,k'_{\rm T})\,. \tag{15}$$

The self-energy corrections in Figs. 2(a) and 2(b) are similar to the quark diagrams in Figs. 1(a) and 1(b), respectively, and the results are

$$\Phi_{\rm a}^{(1)}\otimes H^{(0)} = -\frac{\alpha_{\rm s}}{8\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu_{\rm f}^2}{k_{\rm T}^2{\rm e}^{\gamma_E}}+2\right)\times H^{(0)}(x,Q^2,k_{\rm T})\,, \tag{16}$$

$$\Phi_{\rm b}^{(1)}\otimes H^{(0)} = -\frac{\alpha_{\rm s}}{8\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu_{\rm f}^2}{k_{\rm T}^2{\rm e}^{\gamma_E}}+2\right)\times H^{(0)}(x,Q^2,k_{\rm T})\,. \tag{17}$$

The contribution from the box diagram Fig. 2(c) is power-suppressed in the small x region as explained before, and we have $\Phi_{\rm c}^{(1)}\otimes H^{(0)}=0$.

Choosing $n^+<0$, ie., $n^2<0$ as in [5, 8, 11, 12], Fig. 2(d) leads, in the small x region, to

$$\Phi_{\rm d}^{(1)}\otimes H^{(0)} = \frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu_{\rm f}^2}{k_{\rm T}^2{\rm e}^{\gamma_E}}-\ln^2\frac{\zeta^2}{k_{\rm T}^2}+\ln\frac{\zeta^2}{k_{\rm T}^2}+2-\frac{\pi^2}{3}\right)H^{(0)}(x,Q^2,k_{\rm T})\,, \tag{18}$$

which reproduces the Sudakov logarithm $\ln^2(Q^2/k_{\rm T}^2)$ from Fig. 1(d) in Eq. (12), noticing the scale $\zeta^2=|n^-/n^+|Q^2$. The light-cone divergences are regularized at the price that the universality of the wave function is lost, for it depends on the external kinematic variable through ζ^2. This problem can be alleviated by extracting the evolution in ζ^2 from Eq. (5)[20], ie., by resumming $\ln^2(\zeta^2/k_{\rm T}^2)$ in Eq. (18) into the Sudakov factor[27, 31]. The initial condition of the evolution is universal, like a distribution amplitude in the collinear factorization theorem. We stress that the Sudakov resummation, accurate up to fixed loops, does not remove the ζ^2 dependence of a wave function completely. That is, nonfactorizability may occur at subleading level in the $k_{\rm T}$ factorization of the pion transition form factor.

The hard kernel associated with $\Phi_{\rm e}^{(1)}$ demands the physical range of l^+ to be $-\bar{k}^+ \leqslant l^+ \leqslant k^+$, which corresponds to the range of the parton momentum fraction $1\geqslant x'\geqslant 0$. When computing the convolution of $\Phi_{\rm e}^{(1)}$ with $H^{(0)}$, this fact should be taken into account. Moreover, we assume $\zeta^2\sim Q^2$ by choosing $|n^+|\sim n^-$ to avoid creating the additional large logarithm $\ln(\zeta^2/Q^2)$. The leading-power expression for

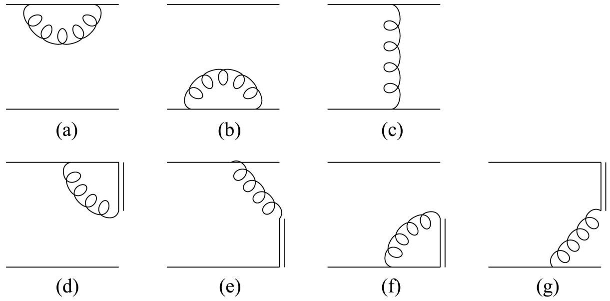

Fig. 2. $O(\alpha_{\rm s})$ effective diagrams for the pion wave function.

Fig. 2(e) is then in the small x region given by

$$\Phi_{\rm e}^{(1)}\otimes H^{(0)} = \frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\ln^2\frac{\zeta^2(xQ^2+k_{\rm T}^2)}{Q^2k_{\rm T}^2}\times H^{(0)}(x,Q^2,k_{\rm T}), \quad (19)$$

where terms vanishing with $k_{\rm T}^2\to 0$ have been dropped. It is found that Fig. 2(e) does not generate a large double logarithm with $\zeta^2\sim Q^2$.

The result from Fig. 2(f) is the same as that of Fig. 2(d), but with the replacement of $P_1-k\approx P_1$ by k, ie., ζ by $x\zeta$. Keeping terms which do not vanish with $k_{\rm T}^2\to 0$, we have

$$\Phi_{\rm f}^{(1)}\otimes H^{(0)} = \frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu_{\rm f}^2}{k_{\rm T}^2{\rm e}^{\gamma_E}}-\ln^2\frac{x^2\zeta^2}{k_{\rm T}^2}+\ln\frac{x^2\zeta^2}{k_{\rm T}^2}+2-\frac{\pi^2}{3}\right)H^{(0)}(x,Q^2,k_{\rm T}), \quad (20)$$

where the double logarithm, being large in the region of $x\sim O(1)$, attenuates with the decrease of x. It should disappear, after combined with the contribution from Fig. 2(g), since such a double logarithm is absent in the corresponding quark diagram Fig. 1(e) in any region of x. The same variable transformation relating $\Phi_{\rm f}^{(1)}$ to $\Phi_{\rm d}^{(1)}$ is not applicable to $\Phi_{\rm g}^{(1)}$, because the latter involves the nontrivial convolution with $H^{(0)}$. Hence, $\Phi_{\rm g}^{(1)}\otimes H^{(0)}$ is expected to have an expression different from $\Phi_{\rm e}^{(1)}\otimes H^{(0)}$. Retaining terms which are finite as $k_{\rm T}\to 0$, Fig. 2(g) leads in the small x region with $xQ^2\gg x^2\zeta^2$ to

$$\Phi_{\rm g}^{(1)}\otimes H^{(0)} = \frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\ln^2\frac{x^2\zeta^2}{k_{\rm T}^2}H^{(0)}(x,Q^2,k_{\rm T}). \quad (21)$$

The cancellation of the double logarithms in the summation of Eqs. (20) and (21) is obvious.

Summing all the above $O(\alpha_{\rm s})$ quark-level wave functions, we derive

$$\Phi^{(1)}\otimes H^{(0)} = \sum_{i=a}^{g}\Phi_i^{(1)}\otimes H^{(0)}=\frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(\frac{1}{\epsilon}+\ln\frac{4\pi\mu_{\rm f}^2}{k_{\rm T}^2{\rm e}^{\gamma_E}}-\ln^2\frac{\zeta^2}{k_{\rm T}^2}+\ln^2\frac{\zeta^2(xQ^2+k_{\rm T}^2)}{Q^2k_{\rm T}^2}+\ln\frac{\zeta^2}{k_{\rm T}^2}+\ln\frac{x^2\zeta^2}{k_{\rm T}^2}+2-\frac{2\pi^2}{3}\right)H^{(0)}(x,Q^2,k_{\rm T}). \quad (22)$$

In contrast to Eq. (14), which is independent of the renormalization scale μ, the above expression depends on the factorizations scale $\mu_{\rm f}$. The Sudakov resummation and the renormalization-group method can be applied to organize the logarithms $\ln^2(\zeta^2/k_{\rm T}^2)$ and $\ln(\mu_{\rm f}^2/k_{\rm T}^2)$ to all orders, respectively[8].

5 $O(\alpha_{\rm s})$ hard kernel

We renormalize Eq. (22) in the modified minimal subtraction scheme, and then take the difference of Eqs. (14) and (22) to obtain the $O(\alpha_{\rm s})$ hard kernel for the pion transition form factor. It is easy to find that the hard kernels $H_{\rm a,b}^{(1)}\equiv G_{\rm a,b}^{(1)}-\Phi_{\rm a,b}^{(1)}\otimes H^{(0)}$, $H_{\rm c}^{(1)}\equiv G_{\rm c}^{(1)}$, $H_{\rm d}^{(1)}\equiv G_{\rm d}^{(1)}-(\Phi_{\rm d}^{(1)}+\Phi_{\rm e}^{(1)})\otimes H^{(0)}$, $H_{\rm e}^{(1)}\equiv G_{\rm e}^{(1)}-(\Phi_{\rm f}^{(1)}+\Phi_{\rm g}^{(1)})\otimes H^{(0)}$, and $H_{\rm f}^{(1)}\equiv G_{\rm f}^{(1)}-\Phi_{\rm c}^{(1)}\otimes H^{(0)}=0$ associated with Figs. 1(a)—(f) are all free of the infrared logarithms $\ln k_{\rm T}^2$ as claimed before. Compared to Ref. [32], we do not need the additional soft function S to achieve this cancellation. The difference is that the self-energy corrections to the Wilson lines have been included into the set of effective diagrams for the pion wave function in Ref. [32]. Hence, S must be introduced to remove these artificially included infrared divergences. We stress that the self-energy corrections to the Wilson lines do not exist, because such diagrams are not generated in the derivation of the factorization theorem using the diagrammatic approach[7]. This observation is consistent with the postulation that the gauge fields appearing in the Wilson lines in Eq. (6) are regarded as bare fields[20].

After subtracting the effective diagrams from the quark diagrams, the resultant hard kernel depends on the factorization scheme that defines the renormalization of Eq. (22). The quark diagrams do not have such a scheme dependence as shown in Eq. (14). When making a physical prediction from the factorization theorem, one convolutes the hard kernel with a model for the pion wave function (not with the effective diagrams), so that the scheme dependence in the hard kernel remains. As stated in the Introduction, the scheme dependence of physical predictions is usually minimized by adhering to a fixed prescription for deriving hard kernels. The sum of the $O(\alpha_{\rm s})$ hard kernels is written as

$$H^{(1)}(x,Q^2,k_{\rm T})=\sum_{i=a}^{f}H_i^{(1)}(x,Q^2,k_{\rm T})=\frac{\alpha_{\rm s}}{4\pi}C_{\rm F}\left(-\ln\frac{\mu_{\rm f}^2}{xQ^2+k_{\rm T}^2}+2\ln\frac{\zeta^2}{Q^2}\ln\frac{Q^2}{xQ^2+k_{\rm T}^2}-\ln^2\frac{Q^2}{xQ^2+k_{\rm T}^2}+2\ln\frac{Q^2}{x\zeta^2}+\ln\frac{Q^2}{xQ^2+k_{\rm T}^2}-3\right)\times H^{(0)}(x,Q^2,k_{\rm T}). \quad (23)$$

The Sudakov logarithm $\ln^2(Q^2/k_{\rm T}^2)$ in Eq. (12) for $G_{\rm d}^{(1)}$ has been cancelled by that in Eq. (18) for $\Phi_{\rm d}^{(1)}\otimes H^{(0)}$, but the threshold logarithm $\ln^2[Q^2/(xQ^2+k_{\rm T}^2)]$ remains in $H^{(1)}$. The large threshold logarithm can be absorbed into a jet function[30], so that the pertubative expansion of the hard kernel is further improved.

6 Conclusion

In this talk we have elucidated the framework for the higher-order calculations in the $k_{\rm T}$ factorization theorem, which is appropriate for QCD processes dominated by contributions from small momentum fractions. The gauge invariance of a hard kernel and the removal of the light-cone singularities are the two essential ingredients for making physical predictions from the $k_{\rm T}$ factorization theorem. We have calculated the NLO $k_{\rm T}$-dependent hard kernel for $\pi\gamma^* \rightarrow \gamma$ in the region with a large momentum transfer Q^2 and a small momentum fraction x. We have demonstrated that the infrared logarithms $\ln k_{\rm T}^2$, reflecting the collinear divergences, cancel between the quark diagrams and the effective diagrams exactly. The quark diagrams generate the double logarithms $\ln^2(Q^2/k_{\rm T}^2)$ and $\ln^2 x$ from the loop correction to the virtual photon vertex. It has been shown that the former is absorbed into the pion wave function, and the latter into the jet function, confirming the observations made in our previous works[8, 30].

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