

Characterizing variable for the critical point in momentum space^{*}

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Abstract The possible experimentally observable signal in momentum space for the critical point, which is free from the contamination of statistical fluctuations, is discussed. It is shown that the higher order scaled moment of transverse momentum can serve as an appropriate signal for the critical point, provided the transverse momentum distribution has a sudden change when energy increases passing through this point. A 2-D percolation model with a linear temperature gradient is constructed to check this suggestion. A sudden change of third order scaled moment of transverse momentum is observed.

Key words relative heavy ion collision, phase transition, critical end point, transverse momentum, scaled moments

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1 Introduction

Heavy-ion collision experiments at the relativistic heavy ion collider BNL-RHIC have found evidence that a phase transition from hadron gas to quark gluon plasma (QGP) has occurred^[1]. Theoretical studies predict the existence of a critical point, or critical end point (CEP) — a point which separates the first order phase transition at high baryon chemical potential, low temperature from the smooth crossover at low baryon chemical potential, high temperature^[2]. Theoretical estimation strongly indicates that, if the critical end point does exist, it is within the region of the phase diagram probed by the heavy ion collision experiments. Most probably it will appear on the quark-gluon phase transition boundary in the range of baryon chemical potential of 100—500 MeV, which corresponds to heavy ion collisions at $\sqrt{s_{NN}} = 5—50$ GeV^[3]. The RHIC experiments have already be-

gun their procedure of low energy scan to observe the critical point^[4].

In order to locate the critical point we must have some variable as an appropriate signal. This variable should be determined by the thermodynamic properties of the critical point. Two distinct properties of the critical point are:

(A) Some quantities have large fluctuations at the critical point.

A well-known example is the critical opalescence, which appears at the critical point of a liquid-gas system, indicating the existence of large density fluctuations.

(B) Some quantities change from zero to non-zero at the critical point.

For example, in a ferro-magnetic system the global magnetization changes from zero to non-zero at the critical point.

These two properties are closely related in thermo-

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dynamics, but phenomenologically they have different representations and lead to different experimental observables.

In the study of relativistic heavy ion collision the above-mentioned property A has been widely utilized by many authors to propose signals for the critical point^[5]. It is expected that large fluctuations exist at the critical point for many variables, such as transverse momentum, K/π ratio, etc. What is important is that these variables have to be measured event by event. Due to the limited number of particles in a single event, they have the disadvantage of being inevitably contaminated by the statistical fluctuations^[6–8]. There is no reliable method to eliminate the statistical fluctuations and it is unclear whether any fluctuation signal for the critical point can survive after eliminating the statistical fluctuations.

In the present letter we will discuss the possibility of finding some signal for the critical point, based on its property B. Such a signal, if it exists, will be measured in the whole event sample instead of event by event. Therefore, it is free from the troublesome problem of statistical fluctuations. The statistical errors can be reduced by enlarging the statistics, i.e. increasing the number of events in the sample.

The layout of the paper is as follows: In Sec. 2, we discuss the moments of final particle momentum distribution, which may be candidates for the characterizing variable of property B of the critical point mentioned above. Then, in Sec. 3, a 2-dimensional site-percolation model with temperature gradient, which can be used to exhibit this characterizing variable, is presented and discussed. Finally, the summary and conclusion are presented in Sec. 4.

2 Moments of final particle momentum distribution

In trying to find a measure basing on property B of the critical point it should be noticed that the only quantities that are observable in relativistic heavy ion experiments are the momentum, mass, charge, strangeness etc. of final state particles. On the contrary the variables that change from zero to non-zero at the critical point are usually in the coordinate space. The aim of this letter is to find a variable in momentum space that can characterize the critical behavior in coordinate space.

Let us ask a question: what information can be extracted from the transverse momentum of final-state particles? The answer is: all the available informa-

tion about transverse momentum is contained in its probability distribution, or equivalently, in the moments of the distribution of all orders. Among them are:

The first order moment $\langle p_t \rangle$ which gives the average transverse momentum, where $\langle \dots \rangle$ means average over the whole event sample.

The second order moment $\langle p_t^2 \rangle$ which describes the width of the distribution. Measuring in each event it is used in the various definitions of event-by-event p_t -fluctuation^[6–8].

The third order moment $\langle p_t^3 \rangle$ which indicates the peak of the distribution.

The fourth and other higher order moments which describe the detailed shape of the distribution.

As discussed above the event-by-event fluctuation $\sigma^2 = \overline{p_t^2} - \overline{p_t}^2$ has been suggested by many authors as a possible signal for the critical point. It has the disadvantage of being contaminated by statistical fluctuations. So we turn to the first order and the higher than second order moments. The present available experimental data on $\langle p_t \rangle$ of the ϕ particle show a clear rise and subsequent saturation with the increasing of colliding energy, as depicted in Fig. 1. This monotonic behavior, if it also exists for other identified particles, e.g. pions, is encouraging. It indicates that possibly an abrupt jump has occurred at a certain energy.

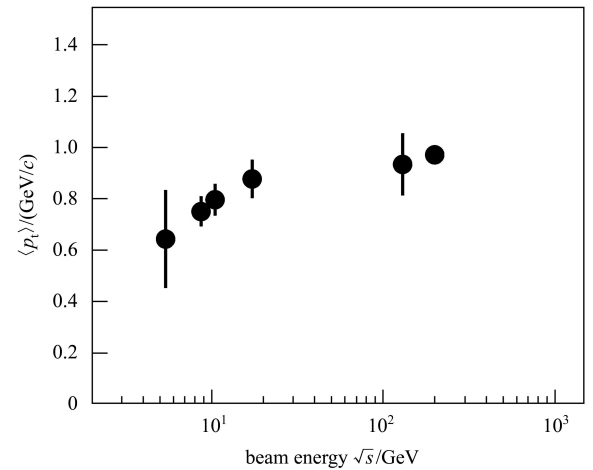


Fig. 1. The average transverse-momentum of ϕ in A+A collisions at different colliding energies. Data taken from Ref. [9].

Let us define the scaled n -th order moment and its reciprocal as

$$C_n = \frac{\langle p_t^n \rangle}{\langle p_t \rangle^n}, \quad D_n = (C_n)^{-1} = \frac{\langle p_t \rangle^n}{\langle p_t^n \rangle}. \quad (1)$$

Both of them are dimensionless and are, therefore, more suitable to serve as a characterizing variable.

The higher than second order scaled moments portray the shape of transverse momentum distribution. If there were a sudden change in the shape of the transverse momentum distribution when the energy increases passing through the critical point, then the higher order scaled moments of p_t might have a visible jump. If this phenomenon exists, it can serve as a good candidate of the critical-point signal.

Here comes another advantage of this approach in comparison with the usual one based on the large fluctuations at the critical point. As we know, the first round of energy scan will be performed with large steps between different colliding energies. If it is not by occasion that some energy used in the first round just locates at the vicinity of the fluctuation peak, we will see nothing in the first round and the subsequent scan has to be carried out in finer steps over the whole energy range. On the contrary, if the higher order scaled p_t moments have a sudden rise when passing through the critical point, then already in the first round of energy scan we will observe a rise of these moments, and most probably the critical point is located in the region of the moment-rising. Then the further scan could be concentrated in this region and the critical point, signaled by the abrupt jump of the scaled moments, can be caught easily in this way.

3 2-dimensional site-percolation model with temperature gradient

Let us use a simple well-defined model that has known critical behavior to exhibit the above argument. The model is a 2-dimensional site-percolation model^[10] with temperature gradient. We take a big circle of radius R and randomly locate in it n little circles of radius r ($r \ll R$). These little circles will be referred to as cells. If the number of cells is large enough, two or more cells may overlap. In this case, a connection is built among these cells and a cluster is formed. Clusters can be of various sizes. If there is a cluster extending from one side of the big circle to the other side, then it is referred to as an infinite cluster. The appearance of an infinite cluster is considered as the appearance of a new phase.

The size of the system (in the percolation-model sense) is defined as the radius of the big circle with that of the small circle as unit:

$$L = \frac{R}{r}. \quad (2)$$

The model parameter is chosen to be the sum of the

area of all the small circles and the area of the big circle:

$$\eta = \frac{n\pi r^2}{\pi R^2} = \frac{n}{L^2}. \quad (3)$$

In the real physical system the parameter η corresponds to the energy density of the system.

Figure 2 is the dependence of the probability P_∞ of the appearance of an infinite cluster on parameter η . It can be seen from the figure that, when η is small, an infinite cluster almost does not appear, but as η increases when η goes near to a certain value η_c , the probability P_∞ of an infinite cluster increases gradually to unity. The bigger the system, the sharper the rising of P_∞ . When $L \rightarrow \infty$, the value of P_∞ jumps suddenly from 0 to 1. The value of η where this jump appears marks the critical point and is denoted by η_c . In our system, η_c is about 1.12. It is noticeable that all the values of P_∞ for various system sizes intersect at $\eta = \eta_c$. This is a basic property of the percolation model and can be utilized to locate the critical value η_c .

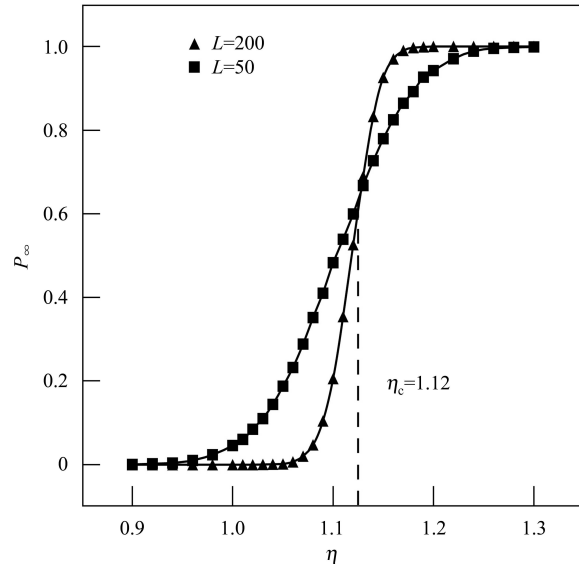


Fig. 2. The dependence of P_∞ on η .

The step-function singularity of P_∞ ¹⁾ is the critical phenomenon in the model. This phenomenon in coordinate space can be observed in the momentum space provided the transverse momentum distribution has a sudden change when energy increases passing through the critical point. In order to show this explicitly through a simple example we construct a toy model by adding a temperature gradient into the above-mentioned percolation model, assuming that the center of the big circle is at a higher temperature $T_{\max} = 400$ MeV, and the fringe gets a lower

1) A real singularity appears only for an infinite system. For a finite-size system it is smoothed to a rapid rise, cf. Fig. 2.

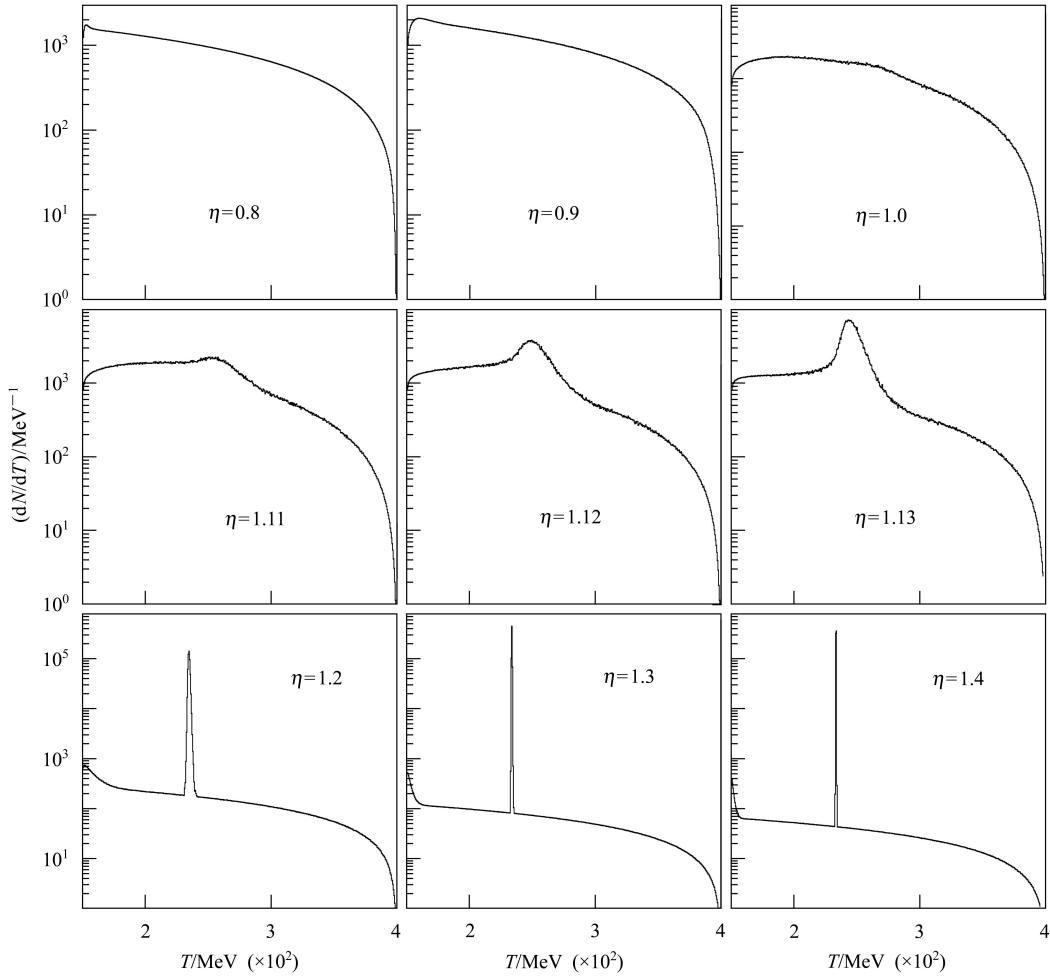


Fig. 3. The distribution of cell temperature for different η .

temperature $T_{\min} = 150$ MeV. A linear temperature gradient is assumed along the radius of the big circle.

We further assume that each cluster has arrived at thermal equilibrium and all the cells in it have the same temperature. The temperature of the i th cluster is determined by the distance to the center of the big circle of its center of mass defined as

$$r_i^{\text{cm}} = \frac{1}{n_i} \sum_j r_{ij}, \quad (4)$$

where r_{ij} is the distance of the j th cell in cluster i to the center of the big circle, and n_i is the number of cells in cluster i . Thus, the temperature of all the cells in cluster i is:

$$T_i = T_{\max} - \frac{r_i^{\text{cm}}}{R} (T_{\max} - T_{\min}). \quad (5)$$

Figure 3 is the distributions of the number of cells at different temperatures. In this and the following figures, we will fix the size of the system and assign L the value 500.

From the figures we can see that, with increasing η , when $\eta \approx 1.12$ a peak appears suddenly in the dis-

tribution around $T = 233$ MeV, and the height of the peak increases very fast.

The transverse momentum of the cells in a cluster can be calculated from the thermal equilibrium momentum distribution, i.e. the Boltzmann distribution,

$$\frac{dN}{2\pi p_t dp_t} = \frac{N}{2\pi m k T} \exp\left(\frac{-p_t^2}{2m k T}\right), \quad (6)$$

where k is the Boltzmann constant, and m is the mass of the cell. In our calculation, we assume the mass of cell to be the mass of pion, equal to 135 MeV. Calculating the transverse momentum of each cell from this distribution, the p_t distributions are obtained and shown in Fig. 4.

It can be seen from Fig. 4 that the p_t distributions for $\eta < 1.15$ and $\eta > 1.2$ fall in two groups with a gap in between. This indicates that there is an abrupt change in the shape of p_t distribution when η increases passing through the critical point.

In order to find a variable that characterizes this abrupt change of p_t distribution, we calculate the moments of p_t distribution of first, second and third

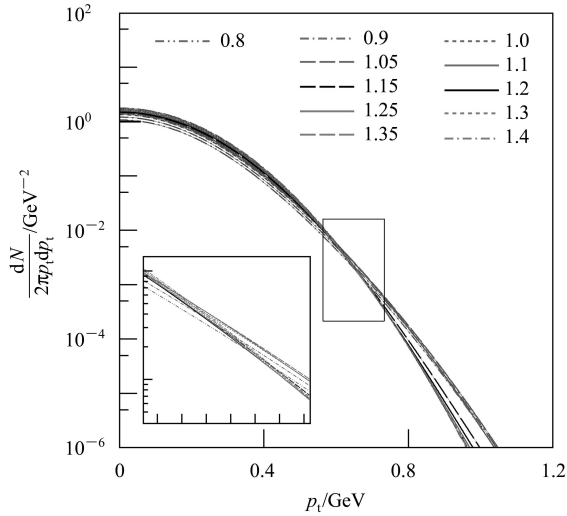


Fig. 4. (Color online) The distribution of cell transverse-momentum for different η (from $\eta = 0.8$ to $\eta = 1.4$).

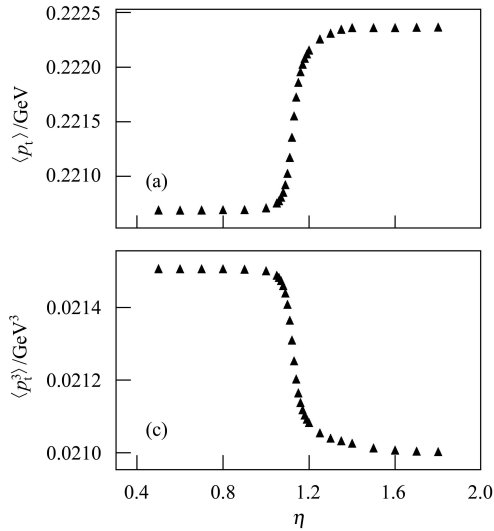


Fig. 5. The dependence on η of the 1st (a), 2nd (b) and 3rd (c) order moments of p_t distribution.

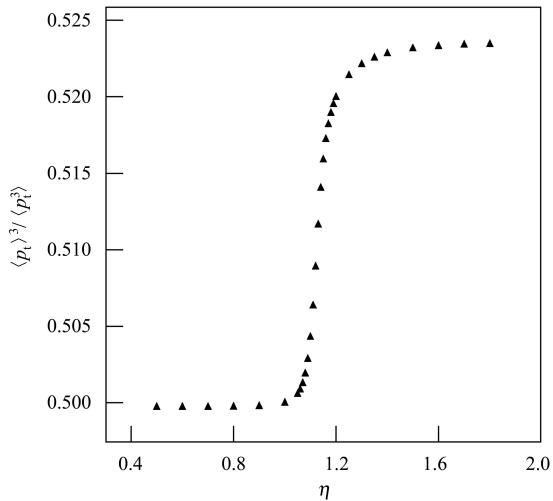
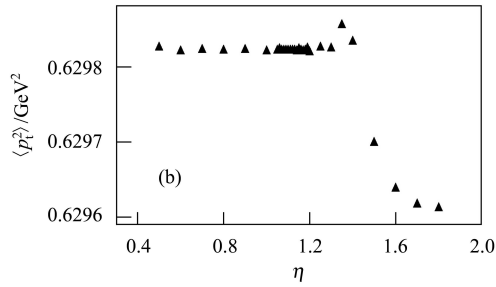


Fig. 6. The dependence of D_3 on η .

orders, as shown in Fig. 5. It can be seen from the figures that both the first and the third order moments $\langle p_t \rangle$ and $\langle p_t^3 \rangle$ change abruptly at the critical point, while the second order moment $\langle p_t^2 \rangle$ does not.

Although the changes of the first and third order moments $\langle p_t \rangle$ and $\langle p_t^3 \rangle$ are in the reverse direction, they occur at one and the same point, i.e. the critical point. The dependence of D_3 defined in Eq. (1) on η , shown in Fig. 6, represents a clear step-function shape. In this figure the abscissa η can be changed to the energy density ε and the latter in turn is related to the colliding energy $\sqrt{s_{NN}}$ in heavy ion collision experiments. We see that a sudden jump of D_3 as the increasing of $\sqrt{s_{NN}}$ can be used as an appropriate signal of the critical point in momentum space.

As stated above a strict step-function singularity exists only in an infinite system. In the real case of heavy ion collisions the system is of finite size. The



rise of the characteristic variable, D_3 , may not be very sharp and may be hard to be recognized. To solve this problem we can run the experiments with nuclei of various sizes. The intersection of the experimental results will mark the place of the critical point, cf. Fig. 2.

4 Summary and conclusion

In this letter we try to utilize the sudden change of some variable(s), instead of the large fluctuations, at the critical point to find an appropriate variable for locating this point. The advantage of such an approach is that it is free from the trouble on the elimination of statistical fluctuations and that the possible

region where the critical point locates can easily be fixed already in the first rounds of the energy scan. Using a site-percolation model with temperature gradient we found that the third order scaled moment of p_t distribution (or its reciprocal) can possibly serve for this purpose.

The model used in this letter is far from a realistic model of relativistic heavy ion collisions, but the final result is largely model-independent. Checking the whole derivation carefully we can see that the essential point for D_3 to serve as an appropriate signal for

the critical point is the fact that the shape of p_t distribution changes abruptly at the critical point. The higher order scaled moments (or their reciprocal) represent the shape of p_t distribution and, therefore, can serve as an appropriate signal. An abrupt change in the p_t -distribution shape when temperature increases passing through the critical point is probable for the system produced in relativistic heavy ion collisions. The higher order scaled moment or its reciprocal is, therefore, a possible candidate for the critical point signal in these collisions.

References

- 1 Adams J et al. (STAR Collab.). Nucl. Phys. A, 2005, **757**: 102—183; Adcox K et al. (PHENIX Collab.). Nucl. Phys. A, 2005, **757**: 184—283
- 2 Stephanov M. Prog. Theor. Phys. Suppl., 2004, **153**: 139—156; Stephanov M. Int. J. Mod. Phys. A, 2005, **20**: 4387—4392
- 3 Stephanov M, Rajagopal K, Shuryak E. Phys. Rev. Lett., 1998, **81**: 4816—4819
- 4 Mitchell J (PHOENIX Collab.). Proceeding of Science, 2006, **CPOD2006**: 019; Sorensen P (STAR Collab.). Proceeding of Science, 2006, **CPOD2006**: 020
- 5 Stephanov M, Rajagopal K, Shuryak E. Phys. Rev. D, 1999, **60**: 114028
- 6 Voloshin S A, Koch V, Ritter H G. Phys. Rev. C, 1999, **60**: 024901
- 7 Adcox K et al. (PHENIX Collab.). Phys. Rev. C, 2002, **66**: 024901; CERES Collab.. Nucl. Phys. A, 2003, **727**: 97—119
- 8 FU Jing-Hua, LIU Lian-Shou. Phys. Rev. C, 2003, **68**: 064904; LIU Lian-Shou, FU Jing-Hua. Chin. Phys. Lett., 2004, **21**: 1467; XU Ming-Mei, LIU Lian-Shou. Int. J. Modern Phys. A, 2006, **21**: 5241
- 9 Adams J et al. (STAR Collab.). Phys. Lett. B, 2005, **612**: 052302
- 10 Christensen K, Moloney N R. Complexity and Criticality. London: Imperial College Press, 2005. 30—38