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An Abelian Ward identity and the vertex corrections to the color superconducting gap*

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Abstract We derive an Abelian-like Ward identity in the color superconducting phase and calculate vertex corrections to the color superconducting gap. Making use of the Ward identity, we show that subleading order contributions to the gap from vertices are absent for gapped excitations.

Key words dense quark matter, color superconductivity, quantum chromodynamics

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1 Introduction

Quark matter at large quark chemical potential μ is a weakly coupling system because momenta exchanged in the interaction between quarks near the Fermi surface is of the order μ, which makes the coupling constant g small due to the property of asymptotic freedom in quantum chromodynamics (QCD). In this case, the dominant interaction between two quarks is one-gluon exchange, which is attractive in the color-antitriplet channel. Consequently, at sufficiently low temperatures, the quark Fermi surface is unstable with respect to the formation of Cooper pairs [1, 2] which leads to the so-called color superconducting (CSC) state[3—13] (for reviews, see, e.g.[5, 14—21]).

In a superconductor, exciting particle-hole pairs costs at least an energy amount $2\phi_0$, where ϕ_0 is the value of the superconductor gap parameter at the Fermi surface for $T=0$ and can be computed from a gap equation derived under mean-field approximation which involves one-gluon exchange and bare quark-quark-gluon vertex. Schematically this gap equation can be written in the form[13, 22—26]

$$\phi_0 = g^2\left[\zeta\ln^2\left(\frac{\mu}{\phi_0}\right)+\beta\ln\left(\frac{\mu}{\phi_0}\right)+\alpha\right]. \qquad (1)$$

For a small value of the QCD coupling constant, $g\ll 1$, the solution is

$$\phi_0 = 2b\mu\exp\left(-\frac{c}{g}\right)[1+O(g)]. \qquad (2)$$

The first term in Eq. (1) contains two powers of the logarithm $\ln(\mu/\phi_0)$, one is the same as in BCS theory[1, 2] and the other is from the exchange of almost static magnetic gluons, which is a long-range interaction[9, 11—13]. The weak coupling solution Eq. (2) implies that this term contributes to the gap equation at the order $O(1)$. We call this term the leading order term. The value of the coefficient ζ determines the constant c in Eq. (2). The second term in Eq. (1) contains subleading contributions of the order $O(g)$ to the gap equation, characterized by a single power of the logarithm $\ln(\mu/\phi_0)\sim 1/g$. Part of it arises from the exchange of non-static magnetic and static electric gluons[13, 22—25]. Another source is the quark self-energy correction[23]. The coefficient β in Eq. (1) determines the constant b in Eq. (2). The term is called the subleading one. The third term in Eq. (1) summarizes the sub-subleading contributions of order $O(g^2)$ with neither a collinear nor a BCS logarithm. It was argued in Refs. [10, 13, 27] that at this order the gauge-dependent terms enter the QCD mean field gap equation. In the Coulomb gauge the authors of Ref. [28] showed that the gauge-dependent

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contribution arising from the gluon propagator appears at this order when the momentum arguments of the gap are put on the quasi-particle mass shell. In a covariant gauge one can see that the gauge dependence shows up at the subleading order and brings an additional factor $\exp(3/2\xi)$[11] to prefactor b. However, the gap parameter is in principle an observable quantity on the quasi-particle mass-shell, and thus gauge-independent. Therefore, the naive mean-field approach to the gap equation with bare qqg vertices is not enough to guarantee the gauge independence even at the subleading level.

A general way to study gauge independence is to make use of Ward identities. This approach has been frequently used to show the gauge independence of physical collective excitations in thermal gauge theories, like hot QCD[29—32]. However, we need to use a Nambu-Gorkov (NG) basis in the color superconducting phase, therefore it is desirable to derive a Ward identity in the NG basis with diquark condensates. Recently Gerhold and Rebhan[33] used the generalized Nielsen identities to give a formal proof that the fermionic quasiparticle dispersion relation in a color superconductor is gauge independent under the assumption that the 1PI part of variation induced by that of the gauge fixing function in the effective action has no singularities coinciding with those of the quark propagator. We have provided another proof of gauge independence of the 2SC gap by deriving a generalized Ward identity from QCD with the diquark condensate and by applying it to a gap equation[34].

In this paper, we present an investigation of the vertex contributions in the gap equation. The calculation is done in Nambu-Gorkov (NG) formalism in a super-phase with diquark condensates. We will show that this method is equivalent to and a good alternative to that used in Ref. [12] based on four-fermion scattering amplitudes. We found that there is a similar cancellation as in the normal phase between the Abelian and triple-gluon vertices, which leads to an Abelian-like Ward identity in the NG form except that an additional term appears in the super-phase. With this Ward identity, we finally show that the contributions from vertices to the gap equation are free of subleading terms for gapped modes.

In this paper four-momenta are denoted by capital letters, $K_\mu=(k_0,\boldsymbol{k})$, with $\boldsymbol{k}$ being a three-momentum of modulus $|\boldsymbol{k}|\equiv k$ and direction $\hat{\boldsymbol{k}}\equiv\boldsymbol{k}/k$. For the summation over Lorentz indices, we use a notation familiar from Minkowski space, with metric $g_{\mu\nu}=\mathrm{diag}(+,-,-,-)$. For simplicity and without ambiguity we always write Lorentz indices as subscripts.

2 Settings

In this section, we will give some preparation knowledge and conventions necessary to the calculation. Since we are concerned with the super-phase, it is quite natural to work in the NG basis. We will see that it is very convenient for describing the mass-shell condition for quasi-particles in the NG basis. In this paper we choose a special case for convenience, the color superconducting phase with two flavors (2SC). The calculation can be extended to other phases. We work in zero temperature.

In the NG basis the quark fields read

$$\Psi = \begin{pmatrix}\psi\\ \psi_\mathrm{c}\end{pmatrix}, \overline{\Psi}=(\overline{\psi}\ \overline{\psi}_\mathrm{c}), \tag{3}$$

where conjugate fields are given by $\psi_\mathrm{c}=C\overline{\psi}^\mathrm{T}$ and $\overline{\psi}_\mathrm{c}=\psi^\mathrm{T}C$ with charge conjugate matrix $C=\mathrm{i}\gamma^2\gamma_0$. The quark propagator inverse is

$$\boldsymbol{S}^{-1}(K) = \begin{pmatrix}S_{11}^{-1} & S_{12}^{-1}\\ S_{21}^{-1} & S_{22}^{-1}\end{pmatrix} = \boldsymbol{S}_0^{-1}+\boldsymbol{\Sigma}= \begin{pmatrix}S_{11}^{0\ -1} & 0\\ 0 & S_{22}^{0\ -1}\end{pmatrix}+\begin{pmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{pmatrix}, \tag{4}$$

where we use boldface letters to denote the NG matrices. The free part is given by $S_{11}^{0\ -1}(K)=\not{K}+\mu\gamma_0$ and $S_{22}^{0\ -1}(K)=\not{K}-\mu\gamma_0$. In the super-phase, the off-diagonal elements of self-energy $\boldsymbol{\Sigma}$ are proportional to the diquark condensate or the gap parameter,

$$\Sigma_{21} = J_3\tau_2\gamma_5\phi^\mathrm{e}\Lambda^\mathrm{e}$$

$$\Sigma_{12} = -J_3\tau_2\gamma_5\phi^{\mathrm{e}*}\Lambda^{-\mathrm{e}},$$

where the color part is chosen as $(J_3)_\mathrm{ij}=\mathrm{i}\epsilon_\mathrm{i3j}$, the third generator of $SO(3)$, to incorporate the pairing in the anti-symmetric channel between the red and green fundamental colors. The flavor part τ_2 is the second Pauli matrix representing the anti-symmetry in flavor space. The appearance of γ_5 implies that we only consider even-parity pairings. Then the quark propagators are given by finding the inverse of $\boldsymbol{S}^{-1}(K)$ in Eq. (4),

$$S_{11} = \frac{Z^2(q_0)L_\mathrm{i}\Lambda_\mathrm{q}^\mathrm{e}S_{22}^{0\ -1}}{q_0^2-[Z(q_0)\epsilon_\mathrm{q,ie}]^2},$$

$$S_{22} = \frac{Z^2(q_0)L_\mathrm{i}\Lambda_\mathrm{q}^{-\mathrm{e}}S_{11}^{0\ -1}}{q_0^2-[Z(q_0)\epsilon_\mathrm{q,ie}]^2},$$

$$S_{12} = -\frac{Z^2(q_0)J_3\tau_2\gamma_5\phi^{e*}\Lambda_q^e}{q_0^2-[Z(q_0)\epsilon_{q,e}]^2},$$

$$S_{21} = \frac{Z^2(q_0)J_3\tau_2\gamma_5\phi^{e}\Lambda_q^{-e}}{q_0^2-[Z(q_0)\epsilon_{q,e}]^2}, \quad (5)$$

where the repetition of indices implies summation if not indicated explicitly. Note that there are two branches of excitations, one is gapped denoted by $\mathrm{i}=1$, the other one is gapless denoted by $\mathrm{i}=0$. Then we have $\phi_1=\phi$ and $\phi_0=0$. The quasi-particle energies are $\epsilon_{q,ie}=\sqrt{(q-e\mu)^2+|\phi_i^e|^2}$. The color projectors are $L_1=J_3^2$ and $L_0=1-J_3^2$, corresponding to the gapped and gapless modes, respectively. The quark wave-function renormalization[23, 35—40] constant $Z(q_0)$ results from the diagonal part of the quark self-energy, Σ_{11} and Σ_{22}, and is given by $Z(q_0)=1-(g^2/18\pi^2)\ln(M^2/q_0^2)$ with $M^2=N_f g^2\mu^2/(2\pi^2)$, a scale characterizing Debye or Meissner screening. For two loop corrections to the gap equation in the NG basis, $Z(q_0)$ can be neglected because its contribution is beyond the sub-subleading order. The bare quark-gluon vertex is

$$\Gamma_\mu^{(0)a} = \begin{pmatrix} T^a & 0 \\ 0 & -T^{aT} \end{pmatrix}\gamma_\mu \equiv T^a\gamma_\mu\,. \quad (6)$$

Here we write it in a special way with the color and Dirac part separated. In this paper, we choose a covariant gauge, the hard dense loop (HDL) propagator for gluons.

In the NG basis, the mass shell condition for quasi-particles can be expressed as

$$S^{-1}(K_{on})\Psi(K_{on}) = 0\,,$$

$$\overline{\Psi}(K_{on})S^{-1}(K_{on}) = 0\,, \quad (7)$$

where $K_{on}=(\epsilon^e_{\boldsymbol{k},i},\boldsymbol{k})$ denotes on-shell momenta and $\Psi(K_{on})$ are on-shell wave functions. Note that these equations are in the matrix form. Hereafter we suppress the subscript of K_{on} for simplicity of notations, without ambiguity.

3 Vertex corrections

In this section we will investigate the vertex contributions in the gap equation. An Abelian-like Ward identity will also be derived explicitly from a cancellation between the Abelian and triple-gluon diagrams.

There are two 1-loop diagrams which provide corrections to the quark-gluon vertex, one is the Abelian diagram and the other one is from the triple gluon diagram, see Fig. 1. The full vertex is then the sum of the two 1-loop diagrams and the tree-level vertex.

Now we focus on the Abelian diagram in Fig. 1 denoted by $\mathrm{i}\Lambda^b_{1\sigma}$. Note that the gluon line in $\mathrm{i}\Lambda^b_{1\sigma}$ is a HDL-dressed propagator. Contracting $\mathrm{i}\Lambda^b_{1\sigma}$ with momentum P we have

$$\begin{aligned}\mathrm{i}P_\sigma\Lambda^b_{1\sigma} = g^3\int\frac{d^4P'}{(2\pi)^4}\,\mathrm{i}D^{HDL}_{\mu\nu}(P')\times \\ \mathrm{i}T^a\gamma_\mu\mathrm{i}S(K-P')\mathrm{i}T^b\,\not{P}\times \\ \mathrm{i}S(K-P-P')\mathrm{i}T^a\gamma_\nu\,, \end{aligned}\quad (8)$$

where $\mathrm{i}D^{HDL}_{\mu\nu}(P')$ is a HDL-dressed propagator. We note that $\not{P}$ can be written as

$$\not{P} = S^{-1}(K-P')-S^{-1}(K-P-P')\,. \quad (9)$$

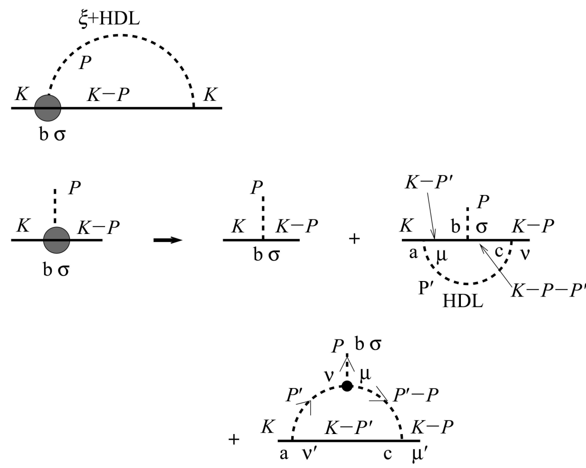

Fig. 1. Right-hand-side of the gap equation and the full vertex.

Here we have neglected the diagonal parts of self-energy Σ_{11} and Σ_{22} which is of the order $g\phi$ on the Fermi surface. The reason is that once $\Sigma_{11/22}$ are inserted into the vertex (the left full vertex in the first diagram of Fig. 1) in the gap equation whose right-hand-side is given by the first diagram of Fig. 1, one will see that the contribution is at most of the order $g^4\phi\ln^2\phi$ where the two logs are from the loop integral. We also assume that the gap is independent of the momentum within the range $|q-\mu|\lesssim g\mu$ around the Fermi surface, which means the gap is assumed to be constant in the gap equation with the momentum of the exchanged gluon being of the order $g\mu$. Inserting Eq. (9) into Eq. (8), we see that $S^{-1}(K-P-P')$ in Eq. (9) cancels $S(K-P-P')$ in Eq. (8). But for $S^{-1}(K-P')$, the procedure is a little more complicated. Because T^b is not commutable with $S(K-P')$, with a T^b in the middle, $S^{-1}(K-P')$ in Eq. (9) cannot directly touch $S(K-P')$ in Eq. (8). Making use of the commutator

$$[T^a,S^{-1}] = -\begin{pmatrix} 0 & A_{12} \\ A_{21} & 0 \end{pmatrix}\,, \quad (10)$$

where $A_{12}=(T^{\rm a}J_3+J_3T^{\rm aT})\tau_2\gamma_5\phi^{\rm e*}\Lambda^{-\rm e}$ and $A_{21}=(T^{\rm aT}J_3+J_3T^{\rm a})\tau_2\gamma_5\phi^{\rm e}\Lambda^{\rm e}$, we can move $S^{-1}(K-P')$ over $T^{\rm b}$ and cancel $S(K-P')$. Note that we have neglected the momentum dependence of the gap for the soft gluon exchange, therefore the above commutator is constant in the gap equation. Obviously the commutator only has off-diagonal elements which are of the order ϕ. After a short algebra, we obtain $\mathrm{i}P_\sigma\Lambda^{\rm b}_{1\sigma}(P,K,K-P)$ as

$$\begin{aligned}\mathrm{i}P_\sigma\Lambda^{\rm b}_{1\sigma} = & g[\mathrm{i}\Sigma(K)T^{\rm b}-T^{\rm b}\mathrm{i}\Sigma(K-P)]+\\ & gf^{\rm abc}T^{\rm a}[\Sigma_{\rm nc}(K)-\Sigma_{\rm nc}(K-P)]T^{\rm c}+\\ & I^{\rm b}_{\rm X}(K,K-P)\,,\end{aligned}\tag{11}$$

where $\Sigma_{\rm nc}$ is the quark self-energy with the color part factorized out as $\Sigma=T^{\rm a}\Sigma_{\rm nc}T^{\rm a}$.

The term $I^{\rm b}_X(K,K-P)$ with $X=[S^{-1},T^{\rm b}]$ is

$$\begin{aligned}I^{\rm b}_{\rm X} = & -g^3\int\frac{\mathrm{d}^4P'}{(2\pi)^4}D^{\rm HDL}_{\mu\nu}(P')T^{\rm a}\gamma_\mu S(K-P')\times\\ & [T^{\rm b},S^{-1}]S(K-P-P')T^{\rm a}\gamma_\nu\,.\end{aligned}\tag{12}$$

Note that this term results from the superphase because of the non-vanishing commutator $[T^{\rm b},S^{-1}]$ which is proportional to the diquark condensate. The similar term is there even in QED[41].

Now we look at another one-loop diagram with triple-gluon vertex in Fig. 1, which reads

$$\begin{aligned}P_\sigma\mathrm{i}\Lambda^{\rm b}_{2\sigma} = & g^3P_\sigma\int\frac{\mathrm{d}^4P'}{(2\pi)^4}\mathrm{i}D^{\rm HDL}_{\nu'\nu}(P')\mathrm{i}D^{\rm HDL}_{\mu'\mu}(P'-P)\times\\ & \mathrm{i}V^{\rm abc}_{\nu\sigma\mu}(P',-P,-(P'-P))\times\\ & T^{\rm a}\mathrm{i}\gamma_{\nu'}\mathrm{i}S(K-P')T^{\rm c}\mathrm{i}\gamma_{\mu'}\,.\end{aligned}\tag{13}$$

The triple-gluon vertex $\mathrm{i}V^{\rm abc}_{\nu\sigma\mu}$ is composed of two parts, the bare and the HDL one $\mathrm{i}V=\mathrm{i}V^{(0)}+\mathrm{i}V^{\rm HDL}$.

First let us focus on the bare vertex, we decompose the bare vertx into a transverse $V^{(0)\rm F}$ and a longitudinal part $V^{(0)\rm P}$,which is $\mathrm{i}V^0=\mathrm{i}V^{(0)\rm F}+\mathrm{i}V^{(0)\rm P}$, and each part reads

$$\begin{aligned}\mathrm{i}V^{(0)\rm F} = & -gf^{\rm abc}[(-2P'+P)_\sigma g_{\mu\nu}+2P_\mu g_{\sigma\nu}]\,,\\ \mathrm{i}V^{(0)\rm P} = & -gf^{\rm abc}[P'_\nu g_{\sigma\mu}+(P'-P)_\mu g_{\sigma\nu}]\,.\end{aligned}\tag{14}$$

Contracted with two HDL-propagators in Eq. (13) $V^{(0)\rm P}$ gives zero due to the fact that the HDL-propagator has the transverse property, while $V^{(0)\rm F}$ satisfies a Ward identity as follows

$$P_\sigma\mathrm{i}V^{(0)\rm F}_{\nu\sigma\mu}=(P^2-2P\cdot P')g_{\mu\nu}=\left[(P'-P)^2-P'^2\right]g_{\mu\nu}\,,\tag{15}$$

where we have factorized out a constant $-gf^{\rm abc}$.

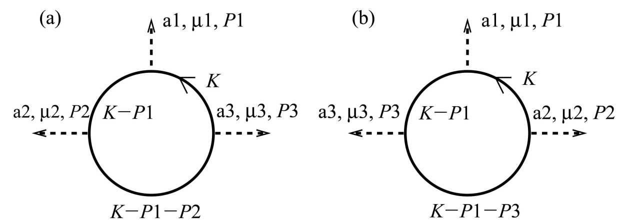

Fig. 2. HDL-resummed triple-gluon vertices.

Then we look at the HDL-resummed triple-gluon vertices as illustrated in Fig. 2. The first contribution is from Fig. 2(a)

$$\begin{aligned}\mathrm{i}V_1 = & \frac{1}{2}g^3R^{\rm a_1a_2a_3}\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{Tr}[\gamma_{\mu_1}S_0(K_1)\times\\ & \gamma_{\mu_2}S_0(K_2)\gamma_{\mu_3}S_0(K)]\,,\end{aligned}\tag{16}$$

where $K_1=K-P_1$, $K_2=K-P_1-P_2$ and $R^{\rm a_1a_2a_3}\equiv\mathrm{Tr}[T^{\rm a_1}T^{\rm a_2}T^{\rm a_2}]$. Note that we used the free quark propagator without condensate and self-energy correction, because the condensate and self-energy would contribute at a higher order. There is a factor of 1/2 in the front due to the usage of the NG basis. It is easier to work in the normal basis and get rid of the factor 1/2. Contracting $\mathrm{i}V^{\rm HDL}$ with $P_{1\mu_1}$ we obtain

$$\begin{aligned}\mathrm{i}P_{1\mu_1}V_1 = & g^3R^{\rm a_1a_2a_3}\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{Tr}[\not{P}_1S_0(K_1)\times\\ & \gamma_{\mu_2}S_0(K_2)\gamma_{\mu_3}S_0(K)]\,.\end{aligned}\tag{17}$$

We can rewrite $\not{P}_1$ as $S_0^{-1}(K)-S_0^{-1}(K_1)$ in the above formula and get

$$\mathrm{i}P_{1\mu_1}V_1=\mathrm{i}gR^{\rm a_1a_2a_3}\left[\Pi^{\rm nc}_{\mu_2\mu_3}(P_3)-\Pi^{\rm nc}_{\mu_2\mu_3}(P_2)\right]\,,\tag{18}$$

where $\Pi^{\rm nc}_{\mu\nu}$ is the polarization tensor without the color part and

$$\begin{aligned}\Pi^{\rm nc}_{\mu_2\mu_3}(P_2) = & \mathrm{i}g^2\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{Tr}[S_0(K-P_1)\gamma_{\mu_2}\times\\ & S_0(K-P_1-P_2)\gamma_{\mu_3}]\,,\\ \Pi^{\rm nc}_{\mu_2\mu_3}(P_3) = & \mathrm{i}g^2\int\frac{\mathrm{d}^4K}{(2\pi)^4}\mathrm{Tr}[\gamma_{\mu_2}S_0(K-P_3)\times\\ & \gamma_{\mu_3}S_0(K)]\,,\end{aligned}\tag{19}$$

where we have used $P_3=P_1+P_2$. Another diagram Fig. 2(b) is the same as Fig. 2(a) except that labels 2 and 3 are interchanged, which gives

$$\mathrm{i}P_{1\mu_1}V_2=\mathrm{i}gR^{\rm a_1a_3a_2}[\Pi^{\rm nc}_{\mu_2\mu_3}(P_2)-\Pi^{\rm nc}_{\mu_2\mu_3}(P_3)]\,.\tag{20}$$

The sum of the two diagrams gives

$$\mathrm{i}P_{1\mu_1}V^{\rm HDL;a_1a_2a_3}_{\mu_1\mu_2\mu_3}=-gf^{\rm a_1a_2a_3}\left[\Pi^{\mu_2\mu_3}(P_3)-\Pi^{\mu_2\mu_3}(P_2)\right]\,,\tag{21}$$

where $\Pi_{\mu_2\mu_3}$ is the polarization tensor with the color factor and $f^{a_1a_2a_3} \equiv -\mathrm{iTr}[T^{a_1}(T^{a_2}T^{a_3} - T^{a_3}T^{a_2})]$. We add Eqs. (21) and (15) together. By doing so, we make replacement $a_1a_2a_3 \rightarrow \mathrm{bac}$, $\mu_1\mu_2\mu_3 \rightarrow \sigma\nu\mu$, $P_1, P_2, P_3 \rightarrow P, -P', P'-P$, and obtain

$$\begin{aligned} P_\sigma \mathrm{i}V^{\mathrm{abc}}_{\nu\sigma\mu} = {} & P_\sigma[\mathrm{i}V^{(0)\mathrm{F;abc}}_{\nu\sigma\mu} + \mathrm{i}V^{\mathrm{HDL;abc}}_{\nu\sigma\mu}] = \\ & -gf^{\mathrm{abc}}\{[(P'-P)^2 - P'^2]g_{\mu\nu} - \\ & [\Pi_{\mu\nu}(P'-P) - \Pi_{\mu\nu}(P')]\} = \\ & -gf^{\mathrm{abc}}[D^{-1}_{\mathrm{HDL}\,\mu\nu}(P') - \\ & D^{-1}_{\mathrm{HDL}\,\mu\nu}(P'-P)]\,. \end{aligned} \tag{22}$$

Substituting the above equation back into Eq. (13) we get

$$P_\sigma \mathrm{i}\Lambda^{\mathrm{b}}_{2\sigma}(K, K-P) = -gf^{\mathrm{abc}}T^{\mathrm{a}}[\Sigma_{\mathrm{nc}}(K) - \Sigma_{\mathrm{nc}}(K-P)]T^{\mathrm{c}}\,. \tag{23}$$

Combining Eq. (11) with (23),we obtain

$$\begin{aligned} P_\sigma \mathrm{i}\Lambda^{\mathrm{b}}_\sigma(K, K-P) = {} & g[\mathrm{i}\Sigma(K)T^{\mathrm{b}} - T^{\mathrm{b}}\mathrm{i}\Sigma(K-P)] + \\ & I^{\mathrm{b}}_{\mathrm{X}}(K, K-P)\,. \end{aligned} \tag{24}$$

We can add the bare vertex $\mathrm{i}g\gamma_\sigma T^{\mathrm{b}}$ to $\mathrm{i}\Lambda^{\mathrm{b}}_\sigma(K, K-P)$ and get the full vertex $\mathrm{i}\Gamma^{\mathrm{b}}_\sigma = \mathrm{i}gT^{\mathrm{b}}\gamma_\sigma + \mathrm{i}\Lambda^{\mathrm{b}}_\sigma$ which is the blob in the first diagram of Fig. 1:

$$\begin{aligned} P_\sigma \mathrm{i}\Gamma^{\mathrm{b}}_\sigma = {} & \mathrm{i}g[S^{-1}(K)T^{\mathrm{b}} - T^{\mathrm{b}}S^{-1}(K-P)] + \\ & I^{\mathrm{b}}_{\mathrm{X}}(K, K-P)\,. \end{aligned} \tag{25}$$

Here we have used Eq. (4) and the property that S_0^{-1} is commutable with T^{b}.

With the identity in Eq. (25) we can evaluate the gauge dependent part or the ξ part in the first diagram of Fig. 1,

$$\begin{aligned} I^\xi \sim {} & -\xi g^2 \int \frac{\mathrm{d}^4P}{(2\pi)^4}\frac{1}{P^4} \times \\ & [S^{-1}(K)T^{\mathrm{b}} - T^{\mathrm{b}}S^{-1}(K-P) - \\ & \mathrm{i}gI^{\mathrm{b}}_{\mathrm{X}}(K, K-P)]S(K-P)T^{\mathrm{b}}\gamma_\rho P_\rho\,. \end{aligned} \tag{26}$$

The first term inside the square brackets is vanishing when sandwiched between on-shell wave functions. The second term is also zero due to $\int \mathrm{d}^4 P P_\rho / P^4 = 0$. The contribution from $I^{\mathrm{b}}_{\mathrm{X}}$ is of the sub-subleading order for gapped modes in the gap equation.

Having $P_\sigma \mathrm{i}\Lambda^{\mathrm{b}}_\sigma$ in Eq. (24), can one derive $\mathrm{i}\Lambda^{\mathrm{b}}_\sigma$? In principle, one cannot. But at the limit $P \rightarrow 0$, one can derive the leading contribution of $\mathrm{i}\Lambda^{\mathrm{b}}_\sigma$. Note that there is a subtlety in defining the limit $P \rightarrow 0$ because it is involved in two different types $p_0 \rightarrow 0$, $\boldsymbol{p} \rightarrow 0$ and $\boldsymbol{p} \rightarrow 0$, $p_0 \rightarrow 0$, which lead to different results for $\mathrm{i}\Lambda^{\mathrm{b}}_\sigma$. We take the first limit for both sides of Eq. (24) to extract $\mathrm{i}\Lambda^{\mathrm{b}}_{\mathrm{i}}$ as follows

$$\mathrm{i}\Lambda^{\mathrm{b}}_{\mathrm{i}} = \lim_{\boldsymbol{p}\rightarrow 0}\lim_{p_0 \rightarrow 0} \mathrm{i}\Lambda^{\mathrm{b}}_{\mathrm{i}}(K, K-P) \sim \frac{\partial}{\partial \boldsymbol{p}_{\mathrm{i}}}(\mathrm{r.h.s.})\,. \tag{27}$$

We consider the gapped modes. The contribution of $I^{\mathrm{b}}_{\mathrm{X}}(K, K-P)$ can be proved to be beyond the subleading order. Here we consider the first line in Eq. (24),

$$-\mathrm{i}gT^{\mathrm{b}}\frac{\partial}{\partial \boldsymbol{p}_{\mathrm{i}}}\Sigma(K-P)\bigg|_{P=0} \sim 0\,, \tag{28}$$

which means both the diagonal and off-diagonal parts are zero. We know that the diagonal parts $\Sigma_{11/22} \sim g^2(k_0 - p_0)\ln(|k_0 - p_0|/M)$ which has no dependence on spatial momentum. The off-diagonal parts $\Sigma_{12/21}$ are actually proportional to the diquark condensate which we assume in this paper do not have momentum dependence or the momentum dependence is of a higher order. For gapped modes, the derivative of the second term of Eq. (24) will also give zero from the leading contribution. We now take the second limit for both sides of Eq. (24) to extract $\mathrm{i}\Lambda^{\mathrm{b}}_0$:

$$\mathrm{i}\Lambda^{\mathrm{b}}_0 = \lim_{p_0 \rightarrow 0}\lim_{\boldsymbol{p}\rightarrow 0} \mathrm{i}\Lambda^{\mathrm{b}}_0(K, K-P) \sim \frac{\partial}{\partial p_0}(\mathrm{r.h.s.})\,. \tag{29}$$

The derivative of the first line in Eq. (24) gives

$$-\mathrm{i}gT^{\mathrm{b}}\frac{\partial}{\partial p_0}\Sigma(K-P)\bigg|_{P=0} \sim g\gamma_0 \ln\phi\,, \tag{30}$$

while the last term of Eq. (24) still gives zero. We can insert a non-zero value of Λ^{b}_0 into the gap equation where Λ^{b}_0 couples to the Debye screened electric gluon whose contribution turns out to be of the sub-subleading order.

4 Summary and conclusion

We explicitly derived an Abelian-like Ward identity in a color superconducting phase from Feynman diagrams, similar to the identity obtained by Nambu in normal superconductivity[41]. The identity arises from a cancellation between the Abelian diagram and the triple-gluon one. The same Ward identity was derived in Ref. [34] in a path integral approach. The identity has one additional term proportional to the Cooper condensate compared with that in the normal phase, which is related to the gauge dependent part or ξ part in the one-loop vertex correction in the color superconducting phase in the Nambu-Gorkov basis. We finally conclude that the vertex corrections are free of the subleading contribution to the color superconducting gap by showing that the contribution of the additional term is beyond the subleading order for the gapped modes in the gap equation. The method proposed in this paper is equivalent to Ref. [12]. The difference between these two approaches is that our approach works in both super and normal phases while the approach of Ref. [12] is based on the quark-quark scattering amplitude in the normal phase. One of the advantages of our approach is its compact form in the NG basis, where a set of component diagrams including those with and without normal phase correspondences can be assembled into a single diagram in the NG matrix form.

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