Study on the Lorentz detuning and tuning of $\beta = 0.3$, 352 MHz spoke cavity

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Abstract The superconducting spoke cavity has been proposed to accelerate the proton in the low energy section of the high power proton linac for the Accelerator Driven Sub-critical System in China. In this paper, the basic geometric and RF parameters of a β=0.3, 352 MHz spoke cavity are given, and the Lorenz detuning and tuning are presented.

Key words Lorentz force detuning, spoke cavity, proton linac

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1 Introduction

The Accelerator Driven Sub-critical System (ADS) needs an intense beam proton accelerator. Because of the high beam power, the normal temperature accelerating structure DTL and EM quadrupole is hard to use to accelerate the intense beam for its heavy water-cooling burden in a narrow space. To tackle these problems, superconducting cavity and superconducting quadrupole are proposed. But when the beta value is less than 0.4, the conventional superconducting elliptical cavity which is often used in the electron accelerator can hardly be used because of its poor mechanical properties. So it is necessary to develop a new type of superconducting accelerating structure, and spoke cavity is a good candidate.

Within the framework of the Accelerator Driven Sub-critical System program in China, we choose three kinds of spoke cavities to accelerate the proton beam from 3.5 MeV to 150 MeV. In this paper, the optimization of the cavity shape and the Lorentz force detuning and tuning analysis of the β=0.3, 352 MHz 2-cell spoke cavity have been reached.

2 Design of a spoke cavity

According to the intense beam proton linac for ADS requirements, we have designed a β=0.3, f = 352 MHz 2-cell spoke cavity (Fig. 1 and Fig. 2). The length over both gaps is $2/3\beta\lambda$. the distance from gap-center to gap-center is $\beta\lambda/2$. The aperture radius is 2.5 cm. As the maximum magnetic field is at the spoke base, the maximum electric field is at the centre of the end wall or the race crook, which faces the end wall. So we can lower the ratio of the surface peak E&B fields to the accelerating field separately. In the process of optimization, we change the spoke base radius to optimize the maximum surface magnetic field and change the racetrack thickness and

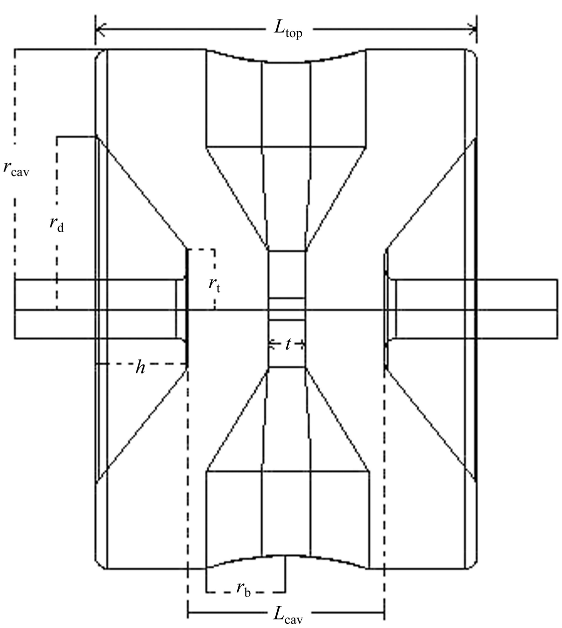

Fig. 1. Section of the spoke cavity.

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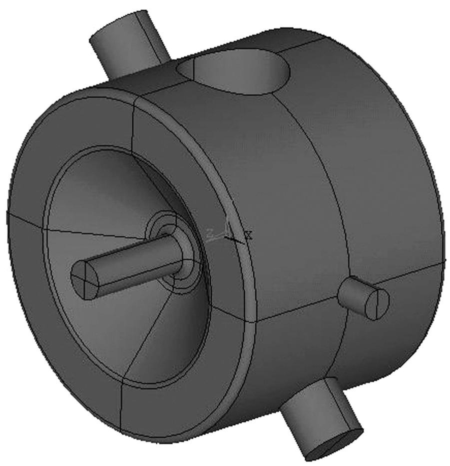

Fig. 2. Spoke cavity.

width to optimize the maximum surface electric field. Finally, the $E_{\mathrm{pk}}/E_{\mathrm{acc}}$ and $H_{\mathrm{pk}}/E_{\mathrm{acc}}$ reach 3.12 and 62.4 G/MV/m. The geometrical parameters of spoke cavity and basic RF parameters are listed in Table 1 and Table 2.

Table 1. The geometrical parameters of the cavity.

cavity radius (r_{cav})/cm	22.45
top cavity length (L_{top})/cm	33.03
cavity length (L_{cav})/cm	17.03
spoke radius at base (r_{b})/cm	3.5
end wall top plate radius (r_{t})/cm	5
end wall bottom radius (r_{d})/cm	15
end wall thickness (h)/cm	8
racetrack thickness (t)/cm	3.2

Table 2. RF parameters of spoke cavity.

f/MHz	352
$E_{\mathrm{pk}}/E_{\mathrm{a}}$	3.12
$(B_{\mathrm{pk}}/E_{\mathrm{a}})$/(G/MV/m)	62.4
$(r/Q)/\Omega$	154.2
T (@β=0.3)	0.8177

3 Lorentz force detuning

ADS operates in CW mode, so the cavity's frequency shift caused by Lorentz force doesn't change with the time. Lorentz force is the result of electromagnetic field in a cavity interacting with the RF wall current. The resulting pressure acting on the cavity wall is

$$p=\frac{1}{4}(\mu_0 H^2-\varepsilon_0 E^2)^{[1]}. \tag{1}$$

A small deformation of the cavity shape results in a shift of the cavity resonant frequency according to Slater's rule.

$$\frac{\Delta f}{f_0}=\frac{1}{4W}\int_{\Delta V}(\mu_0 H^2-\varepsilon_0 E^2)\mathrm{d}V\,. \tag{2}$$

where

$$W=\frac{1}{4}\int_{\Delta V}(\mu_0 H^2+\varepsilon_0 E^2)\mathrm{d}V\,. \tag{3}$$

W is the stored energy and f_0 is the resonant frequency of the unperturbed cavity. From Eq. (2) and hook's law we can get

$$\Delta f=K\cdot E_{\mathrm{acc}}^{2}{}^{[2]}\,. \tag{4}$$

K is the Lorentz force coefficient representing the mechanical property of the cavity. Given the accelerating gradient, the fewer the frequency shift, the better the mechanical property. Δf is proportional to the K, so we expect the Lorentz force coefficient to be as little as possible.

The niobium material property of the β=0.3, 352 MHz superconducting spoke cavity is showed in Table 3. Firstly, we choose the wall thickness as 2 mm. From Eq. (1) and the chosen accelerating gradient as 10 MV/m, we get the pressure on the inner wall surface of cavity. Inputting the pressure into the code which is written using the ANSYS, we get the deformation of the cavity's shape. The deformation of the cavity wall is shown in Fig. 3. Lorentz force gives the cavity's wall, crossing with the spoke, an inward perturbation, and the end face of the cavity's wall an outward perturbation. At the spoke base region the magnetic field is strong while the electric field is weak. At the centre of the end wall, the electric field is strong while the magnetic field is weak. From the perturbation theory of resonant cavity, we know that if the inward perturbation occurs in the strong electric field but weak magnetic field region, or the outward perturbation happens in the strong magnetic field but weak electric field region, the cavity's resonant frequency decreases. So the spoke cavity's resonant frequency will decline owing to the Lorentz force.

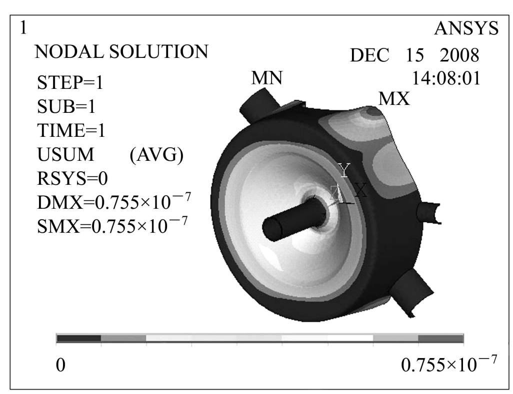

Fig. 3. Deformation of the cavity wall, MX and MN represent the maximum and minimum deformation.

Table 3. Niobium mechanical property[1].

Young' modulus	10^3 GPa
Poisson's ratio	0.359
density	8560 kg/m^3

The relevant resonate frequency shift can be calculated by changing the accelerating gradient. The relationship between the resonate frequency shift Δf ($\Delta f = f - f_0$) and accelerating gradient E_a is shown in Fig. 4.

We can see that the frequency shift is proportional to the accelerating gradient. With the increase of accelerating gradient, the resonant frequency decreases. When the accelerating gradient reaches 20 MV/m, the frequency shift reaches −189 Hz.

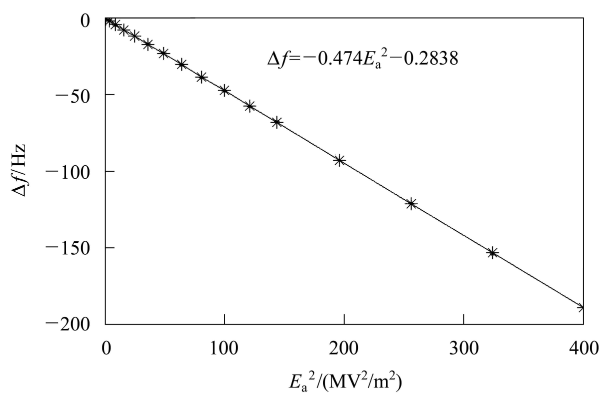

Fig. 4. Frequency shift vs. accelerating gradient.

The fitting curve of the Δf and E_a is

$$\Delta f = -0.4747 E_{\rm acc}^2 - 0.2838\,. \tag{5}$$

The obtained Lorentz force coefficient is −0.4747. Take the β=0.45, f = 704 MHz single cell elliptical cavity as an example, its Lorentz force coefficient is 85.396. Because its beta value and frequency are different from the β=0.35, 352 MHz spoke cavity, when the frequency of elliptical cavity decreases, the equator radius must be increased, and when the beta value is reduced the accelerating gap must be decreased. All these will cause the Lorentz force coefficient of the elliptical cavity to become higher. So the mechanical stability of the spoke cavity is better than that of the elliptical cavity which has the same frequency and beta value.

4 Cavity stiffening

As for the static Lorentz detuning, the mechanical stability of the un-stiffening 2 mm thick spoke cavity is very good. But when the cavity operates in a pulse mode, the frequency detuning changes with the time, this is called dynamic Lorentz detuning. And the response speed of the static Lorentz tuning system cannot match the time-varying detuning. So a fast Lorentz tuning system such as the frequency regulation system by phase-locked loop must be used. However the frequency adjustment range of the fast Lorentz tuning system is limited, so it is needed to reduce the frequency detuning by stiffening the cavity. Meanwhile, stiffening the cavity can eliminate the mechanical vibration. One of the effective methods of stiffening the cavity is to increase the thickness of the cavity wall.

Figure 5 shows how the Lorentz detuning coefficient varies with the cavity wall thickness. We can see that the absolute Lorentz coefficient value decreases with the wall thickness increase. There is a linear relation between the Lorentz force coefficient and the square of the thickness reciprocal value, which is described as

$$K = -1.808(1/h^2) + 0.0201\,. \tag{6}$$

When the cavity wall thickness increases to 4 mm, the absolute Lorentz force coefficient value is reduced to 0.1342. Increasing the cavity wall thickness effectively enhances the mechanical stability of the spoke cavity

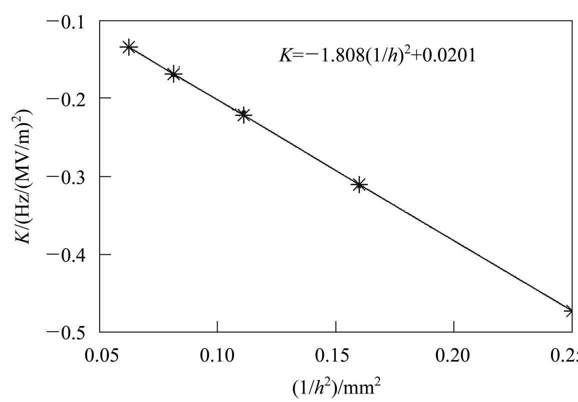

Fig. 5. Lorentz factor vs. wall thickness.

5 Static Lorentz tuning

When the detuning appends, the efficiency of the power feeding will decrease and the beam quality will worsen. In order to avoid this phenomenon, it is necessary to adjust the frequency to its un-detuning frequency. For the dynamic Lorentz detuning, the frequency regulation by the phase-locked loop can be used. And for the static Lorentz detuning, the mechanical method such as squeezing or stretching the end face of the cavity is usually used. We will discuss the static Lorentz tuning of the spoke cavity. According to the perturbation theory[3] of resonant cavity, if we give the cavity a linear perturbation, which could offset the deformation of the cavity wall caused by the Lorentz force, then the frequency will be adjusted to the operating frequency. Here we impose stretch force on the end wall face to produce the frequency shift of the cavity.

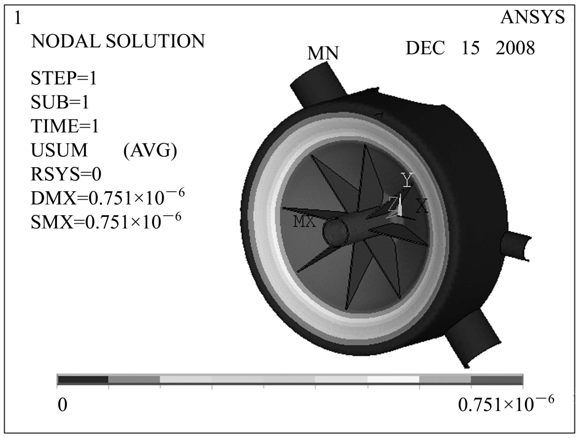

Fig. 6. Deformation under stretch force. MX and MN represent the maximum and minimum deformation.

As the symmetry of the cavity, half of the cavity is taken to analyze. The stretch force is applied to the cross section of the beam tube, with the cross section of the cavity fixed in z direction. The corresponding frequency shift can be gotten by changing the stretch force. The deformation of the cavity wall and the frequency shift under deferent stretch forces are shown in Fig. 6 and Fig. 7. In Fig. 6, the maximum deformation on the end wall is about the order of magnitude of 10^{-7}. Under the micro-displacement of the cavity wall, the frequency shift has a linear relation with the stretch force (P), which is described as

$$\Delta f = 0.0179P - 0.3747\,. \tag{7}$$

Eq. (7) tells us that we can employ a mechanical tuner to adjust the static Lorentz force detuning.

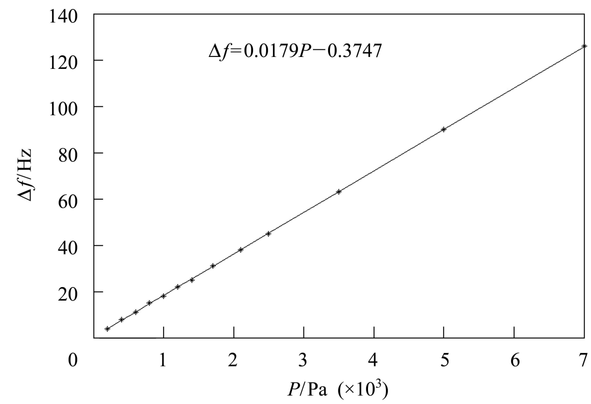

Fig. 7. Frequency shift vs. stretch force.

6 Conclusion

Lorentz force detuning is an important problem in the superconducting cavity. Through the calculation of spoke cavity, the RF parameters and Lorentz force factor are achieved. And finally, the reinforcement scheme and static Lorenz tuning measure are determined. It can be seen from analysis that the spoke cavity has a high accelerating gradient and good mechanical property in the low energy SC structure. So, the spoke cavity is a good choice in the low energy section of the high power proton linac.

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