Alpha-decay branching ratios to high-lying excited-states of the ^{242}Cm → ^{238}Pu → ^{234}U → ^{230}Th → ^{226}Rn decay chain*

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Abstract We present a systematic calculation on the α-decay branching ratios to excited-states of an even-even α-decay chain ^{242}Cm→^{238}Pu→^{234}U→^{230}Th →^{226}Rn by the improved barrier penetration approach. The changes of the parities between the parent nuclei and the daughter nuclei are properly taken into account. The theoretical values are compared with the available experimental data and the deviation between them is within a factor of 5 in most cases.

Key words alpha-decay, branching ratios, high-lying excited states, parity

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1 Introduction

The investigation of α-decay has been a very active field of nuclear physics[1—14] since it was first observed by Becquerel in 1986 as an unknown radiation. The α-decay half-lives and decay energy are broadly researched in many references[6—14] by various approaches. Most of them are concentrated on the favored α-decays, e.g. the α-transitions from the ground-state of parent nuclei to the ground-state of daughter nuclei of even-even nuclei. But the study of the α-decay to excited states, especially to the high-lying excited states, is rare due to the complexity of the unfavored decays where the angular momentum of the α particle is not zero. From both the experimental and theoretical sides, the unfavored α-decay is an effective tool to probe the nuclear structures[11, 15, 16].

Recently we have proposed a simple barrier penetration approach to calculate the α-decay branching ratios to members of the ground-state rotational band and to excited 0^+ states of even-even nuclei[16]. The influence of the α-decay energy, the angular momentum of the α particle and the excitation probability of the daughter nucleus have been properly taken into account[16].

In this paper we will extend the barrier penetration approach to calculate the α-decay branching ratios to the ground-state rotational band as well as to the high-lying excited-states of even-even nuclei. It is known that the parities of both the ground-states of the parent nuclei and the ground-state rotational bands of the daughter nuclei are all positive in the α-decay of even-even nuclei. But the different high-lying excited-states of the daughter nuclei have different parities, i.e. positive or negative. It is expected that the changes of parities between the daughter and parent nuclei may affect the decay branching ratios, so we take the influence of the changes of the parities into account in this paper. We assume that the probability of parent nuclei to the positive-parity or negative-parity states of daughter nuclei are exponentially dependent on the changes of the parities. The calculations cover the ^{242}Cm→^{238}Pu→^{234}U→^{230}Th →^{226}Rn decay chain, showing good agreement with the experimental data. This is a generalization of our previous work and it is also a test of the barrier penetration approach.

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This paper is organized in the following way. In Sect. 2 we present the framework of the barrier penetration approach. Sect. 3 gives the numerical results and discussions. The summary is given in Sect. 4.

2 The framework of the barrier penetration approach

In this section we summarize the details of the barrier penetration approach to compute the α-decay branching ratios. Our calculations start with the radial Schrödinger equation[16]

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2\psi(r)}{\mathrm{d}r^2}+\left[U(r)+\frac{\hbar^2}{2\mu}\frac{\ell_i(\ell_i+1)}{r^2}\right]\psi(r)=E\psi(r), \tag{1}$$

where the centrifugal potential $\frac{\hbar^2}{2\mu}\frac{\ell_i(\ell_i+1)}{r^2}$ is included in the Schrödinger equation and $U(r)$ is the standard square well potential

$$U(r)=\begin{cases}-U_0 & (r<R_0)\\ Z_1Z_2e^2/r & (r\geqslant R_0).\end{cases} \tag{2}$$

Using the well known WKB technique, one can obtain the penetration probability of the α-particle[16]

$$P_\alpha(Q_\alpha,E_i^*,\ell_i)\propto\left|\frac{\psi_{\rm out}}{\psi_{\rm in}}\right|^2=\exp\left[-2\int_{R_0}^{R_{\rm out}}k(r)\mathrm{d}r\right], \tag{3}$$

with the wave number $k(r)$ defined by

$$k(r)=\sqrt{\frac{2\mu}{\hbar^2}}\left[\frac{Z_1Z_2e^2}{r}+\frac{\hbar^2}{2\mu}\frac{\ell_i(\ell_i+1)}{r^2}-(Q_\alpha-E_i^*)\right]^{\frac{1}{2}}, \tag{4}$$

where Z_1 and Z_2 are the charge numbers of the α-particle and the daughter nucleus, respectively. μ is the reduced mass of the α-core system and Q_α is the decay energy of the ground-state transition. E_i^* is the excitation energy of the i-th excited-state of the daughter nucleus and ℓ_i is the angular momentum carried by the α-particle. R_0 is the radius of the daughter nucleus ($R_0=1.2A_2^{1/3}$) and $R_{\rm out}$ is the outer classic turning point[16]. Usually the height of the centrifugal barrier at $r=R_0$ is very small compared with the Coulomb barrier[14]

$$\varepsilon=\frac{\hbar^2}{2\mu}\frac{\ell_i(\ell_i+1)}{R_0^2}:\frac{Z_1Z_2e^2}{R_0}\,. \tag{5}$$

By expanding the wave number $k(r)$ in powers of the small quantity ε, the penetration probability can be written in a simple form[14]

$$P_\alpha(Q_\alpha,E_i^*,\ell_i)=\exp\left[-\sqrt{\frac{2\mu}{\hbar^2}}\frac{Z_1Z_2e^2\pi}{(Q_\alpha-E_i^*)^{\frac{1}{2}}}\right]\times \exp\left[-\sqrt{\frac{\hbar^2}{2\mu}}\frac{2\ell_i(\ell_i+1)}{(Z_1Z_2e^2R_0)^{\frac{1}{2}}}\right], \tag{6}$$

where the first term represents the influence of the excitation energy E_i^* on the penetration factor and the second term denotes the influence of the non-zero angular momentum ℓ_i. We assume that the probability of the residual daughter nucleus to stay in its excited states obeys the Boltzmann distribution

$$w_i(E_i^*)=\exp[-c_1E_i^*], \tag{7}$$

where E_i^* is the excitation energy of the i-th excited-state and c_1 is a free parameter. The value of parameter c_1 was fixed to 1.5 in our previous work[16]. Here we still choose 1.5 for c_1. The spins of all the parent nuclei are zero and the parities are positive as they are the ground-states of even-even nuclei. But the parities of the different excited-states of the same daughter nucleus may be different, positive or negative. It is expected that the changes of parities between the parent and daughter nuclei may affect the decay branching ratios. Here we assume that the probability of the parent nuclei to positive-parity or negative-parity states of the daughter nuclei is exponentially dependent on the changes of the parities. So we define

$$w_i(P_i)=\exp[-c_2(1-P_i)], \tag{8}$$

where P_i denotes the parity of the daughter nucleus. If the parity of the i-th excited state of the daughter nucleus is positive we define P_i to equal 1. Or P_i is equal to -1 if the parity is negative. And $1-P_i$ denotes the changes of the parities between the parent and daughter nucleus where the value 1 represents the positive parity of the parent nucleus. c_2 is also a free parameter here and it is fixed to 2.0 in this paper. It is reasonable to take the influence of the parity into account in physics and this will lead to good agreement between theory and experiment. Now we define I_i as the product of the penetration factor, the excitation probability[16] and the parity probability.

$$I_i=w_i(P_i)w_i(E_i^*)P_\alpha(Q_\alpha,E_i^*,\ell), \tag{9}$$

which denotes the total probability of α-transition from the ground-state of the parent nucleus to the i-th excited state of the daughter nucleus. It is very convenient to estimate the influence of these factors on the hindered α-transitions from I_i. With the help of I_i, the branching ratios of α-decay to the excited-states of the daughter nucleus can be written as[16]

$$\begin{aligned}
b^{0}_{\text{g.s.}}\% &= I_0/(I_0+I_1+I_2+\cdots+I_i+\cdots)\times 100\% \\
b^{1}_{\text{e.s.}}\% &= I_1/(I_0+I_1+I_2+\cdots+I_i+\cdots)\times 100\% \\
b^{2}_{\text{e.s.}}\% &= I_2/(I_0+I_1+I_2+\cdots+I_i+\cdots)\times 100\% \\
&\cdots \\
b^{i}_{\text{e.s.}}\% &= I_i/(I_0+I_1+I_2+\cdots+I_i+\cdots)\times 100\% \\
&\cdots . \qquad (10)
\end{aligned}$$

3 Numerical calculations and results

We systematically calculate the α decay branching ratios to the members of the ground-state rotational bands as well as to the high-lying excited states of the daughter nuclei of the ^{242}Cm→^{238}Pu→^{234}U→^{230}Th →^{226}Rn decay chain. The results are given in Tables 1—4. All the experimental data are taken from Ref. [15]. The first column is the serial number and i represents the i-th excited state of the corresponding daughter nucleus, e.g. $i=2$ denotes the second excited-state of the relevant daughter nucleus. The second column marks the spins and parities of the corresponding excited states of the daughter nucleus. Note that the spins of all the parent nuclei are zero and the parities are positive. The third column marks the parity of the daughter nucleus. As presented in Sect. 2, if the parity of the daughter nucleus is positive P_i will be equal to 1 or it will be −1. In Column 4, we list the excited energy of the daughter nucleus. Experimental and theoretical branching ratios are given in the fifth and sixth columns, respectively. The last column is the proportion of the experimental branching ratios to the corresponding theoretical ones. It is easy to understand that if the proportion is close to value 1.0, it means the theoretical values agree well with the experimental data.

Before we present the detailed theoretical results, we would like to discuss the influence of different parities on the α-decay penetration probability. It is well known that the parity is conserved in the process of α-decay, if the ground-state of even-even nucleus (0^+ state) decays to the positive-parity-state of the daughter nucleus, the value of the angular momentum of the α-particle will be even, otherwise it will be odd. In our calculations, the influence of the different angular momentum of the α-particle has been taken into account (see in Eq. (6)). When the nucleus decays from the ground-state (0^+ state) to the negative-parity-state, the different nuclear structure configurations between the initial and final states may greatly reduce the decay probability. Before we introduce the factor $\exp[-c_2(1-P_i)]$ the theoretical results for the decays to the negative-parity-states of the daughter nuclei agree less with the data than the positive-parity-states and they are generally larger than the data. This implies that the penetration probabilities of α-decay become smaller in the parity-changing decays. Based on this experimental fact and qualitative analysis, we introduce the factor $\exp[-c_2(1-P_i)]$. Besides ^{242}Cm decay chains, we have performed primary calculations on other decay chains, e.g. ^{244}Cm decay chains, ^{236}Pu decay chains and ^{230}U decay chains which give similar results. We will make more detailed theoretical investigations in our future study.

Table 1. Experimental and calculated branching ratios of α-decay to the members of the ground-state rotational band and to the high-lying excited states of the daughter nucleus for ^{242}Cm. The ground-state to ground-state α-decay energy is $Q_\alpha = 6.216$ MeV.

i	I_i	P_i	E^*_i	b^i(%)(Expt.)	b^i(%)(Calc.)	b^i(Expt.)/b^i(Calc.)
0	0^+	1	0.000	74.0	76.0	0.97
1	2^+	1	0.044	25.0	22.7	1.10
2	4^+	1	0.146	0.035	1.332	0.03
3	6^+	1	0.303	0.0031	0.0152	0.20
4	8^+	1	0.513	2.0×10^{-5}	3.2×10^{-5}	0.63
5	1^-	−1	0.605	2.4×10^{-4}	2.0×10^{-4}	1.20
6	3^-	−1	0.661	1.2×10^{-5}	2.9×10^{-5}	0.41
7	5^-	−1	0.763	2.0×10^{-7}	9.2×10^{-7}	0.22
8	0^+	1	0.942	5.2×10^{-5}	5.9×10^{-5}	0.88
9	1^-	−1	0.963	1.1×10^{-6}	6.2×10^{-7}	1.77
10	2^+	1	0.983	1.6×10^{-6}	1.6×10^{-5}	0.10
11	2^+	1	1.029	3.4×10^{-6}	7.4×10^{-6}	0.46
12	4^+	1	1.126	3.4×10^{-7}	3.3×10^{-7}	1.03
13	0^+	1	1.229	5.1×10^{-7}	4.1×10^{-7}	1.24
14	2^+	1	1.264	4.8×10^{-7}	1.2×10^{-7}	4.00

In Table 1, we list the experimental and theoretical branching ratios for ^{242}Cm. Experimentally the branching ratios for ^{242}Cm have been measured up to the 14th excited-state[15]. It can be seen from Table 1 that the calculated results for the ground-state rotational band (i = 0, 1, 2, 3, 4) are the same as our previous calculation[16] and the abnormity of the 4^+ state[16] also exists. For higher excited-states, the proportions of the last column are all larger than 0.2 and less than 5.0 with the exception of the tenth excited-state (2^+ state). This means that most of the experimental branching ratios for ^{242}Cm are well reproduced within a factor of 5.

The experimental data and the theoretical results for ^{238}Pu are given in Table 2. From Table 2, we can find that the theoretical α-decay branching ratios to the ground-state rotational band (from i=0 to i=4) are also consistent with our previous results[16]. The largest factor between the theoretical values and experimental branching ratios is equal to 10.0 for the fifth excited-state (1^- state). But the experimental branching ratios to the rest excited-states are well reproduced within a factor of 5. As predictions we calculate the branching ratio to the eleventh excited-state which hasn't been measured experimentally. The experiment hasn't provided an accurate branching ratio but an approximate one for the last excited-state of ^{238}Pu[15] and we denote this by symbol a. Our barrier penetration approach also gives the predicted value here. It is interesting to compare these theoretical predictions with future experimental observations.

In Table 3, we compare the experiment branching ratios with the calculated ones for ^{234}U. It is easy to see from Table 3 that the calculated decay branching ratios to the ground-state and to the first three excited-states are close to the experimental data within a factor of 2 except for the fourth excited-state (I_i=0^+ state). Overall the agreement between the theoretical results and the experimental values is satisfactory. The predicted branching ratio for the fifth excited-state is also given by our barrier penetration approach.

Table 2. The same as Table 1, but for ^{238}Pu. The ground-state to ground-state α-decay energy is Q_α = 5.593 MeV. Here a represents the case where the experiment hasn't provided accurate branching ratios but approximate ones for the corresponding states. b denotes that the experimental branching ratio for the corresponding state is still unknown.

i	I_i	P_i	E_i^*	b^i(%)(Expt.)	b^i(%)(Calc.)	b^i(Expt.)/b^i(Calc.)
0	0^+	1	0.000	70.91	77.40	0.92
1	2^+	1	0.043	28.98	21.52	1.35
2	4^+	1	0.143	0.105	1.063	0.10
3	6^+	1	0.296	0.0030	0.0093	0.32
4	8^+	1	0.497	6.8×10^{-6}	1.4×10^{-5}	0.49
5	1^-	−1	0.786	2.2×10^{-5}	2.2×10^{-6}	10.00
6	0^+	1	0.810	5.0×10^{-5}	9.5×10^{-5}	0.53
7	3^-	−1	0.849	9.0×10^{-8}	2.4×10^{-7}	0.38
8	2^+	1	0.852	4.2×10^{-6}	2.3×10^{-5}	0.18
9	2^+	1	0.927	1.2×10^{-5}	5.5×10^{-6}	2.18
10	4^+	1	0.948	2.5×10^{-7}	8.4×10^{-7}	0.30
11	4^+	1	1.024	b	1.9×10^{-7}	*
12	0^+	1	1.045	1.2×10^{-6}	1.0×10^{-6}	1.20
13	2^+	1	1.085	$\sim1.1\times10^{-6a}$	2.4×10^{-7}	*

Table 3. The same as Table 1, but for ^{234}U. The ground-state to ground-state α-decay energy is Q_α = 4.859 MeV. Here a represents the case where the accurate branching ratio hasn't been measured experimentally.

i	I_i	P_i	E_i^*	b^i(%)(Expt.)	b^i(%)(Calc.)	b^i(Expt.)/b^i(Calc.)
0	0^+	1	0.000	71.38	82.78	0.86
1	2^+	1	0.053	28.42	16.82	1.69
2	4^+	1	0.174	0.20	0.40	0.50
3	1^-	−1	0.508	4×10^{-5}	8.0×10^{-5}	0.50
4	0^+	1	0.635	2.6×10^{-5}	2.9×10^{-5}	0.09
5	2^+	1	0.678	$\sim7\times10^{-6a}$	6.0×10^{-5}	*

Table 4. The same as Table 1, but for ^{230}Th. The ground-state to ground-state α-decay energy is Q_α = 4.770 MeV. Here a denotes the cases where the experiment hasn't provided accurate branching ratios but approximate ones for the corresponding states.

i	I_i	P_i	E_i^*	b^i(%)(Expt.)	b^i(%)(Calc.)	b^i(Expt.)/b^i(Calc.)
0	0^+	1	0.000	76.3	86.6	0.88
1	2^+	1	0.068	23.4	13.2	1.78
2	4^+	1	0.212	$\sim 0.12^a$	0.20	*
3	1^-	−1	0.254	0.03	0.01	3.00
4	3^-	−1	0.322	9.7×10^{-4}	9.9×10^{-4}	0.98
5	6^+	1	0.417	8.0×10^{-6}	3.2×10^{-4}	0.03
6	5^-	−1	0.446	8.9×10^{-6}	1.2×10^{-5}	0.74
7	0^+	1	0.825	$\sim3.4\times10^{-6\,a}$	3.8×10^{-6}	*
8	2^+	1	0.874	$\sim1.4\times10^{-6\,a}$	0.6×10^{-6}	*

The comparison between the calculated branching ratios and the experimental values for ^{234}U is listed in Table 4. From Table 4, one can find that good agreement is successfully obtained by the barrier penetration approach for ^{234}U as most of the factors between the theoretical values and experiment data are less than 3 with the exception of the fifth excited-state (6^+ state). The experiment doesn't provide exact branching ratios to the second, the seventh and the eighth excited-states for ^{230}Th[15] and we list the corresponding theoretical predictions here. It is easy to find that the orders of magnitude of these calculated values are completely consistent with the experimental approximate data which may reflect the validity of our barrier penetration approach.

4 Summary

To conclude, we take the changes of the parities between parent and daughter nuclei into account based on our previous barrier penetration approach and calculate the α-decay branching ratios to high-lying excited-states of an even-even α-decay chain of ^{242}Cm→^{238}Pu→^{234}U→^{230}Th →^{226}Rn. The α-decay branching ratios to different excited-states of the daughter nuclei vary in a very wide range e.g. from 74.0% to 2.0×10^{-7}% (see Table 1). Unexpectedly most of the experiment data are well reproduced within a factor of 5 in our calculations and the largest factor is not much larger than 10. So it proves that our barrier penetration approach is very useful for the study of the α-decay branching ratios of even-even nuclei. We also give the predicted branching ratios where the experimental data haven't been provided or only approximate values are given by experiment. It is interesting to compare them with future experimental observations.

References

1 Gamov G. Z. Phys., 1928, **51**: 204
2 Condon E U, Gurney R W. Nature, 1928, **122**: 439
3 Buck B, Merchant A C, Perez S M. Phys. Rev. C, 1992, **45**: 2247
4 Buck B, Merchant A C, Perez S M. At. Data Nucl. Data Tables, 1993, **54**: 53
5 Buck B, Merchant A C, Perez S M. Phys. Rev. Lett., 1994, **72**: 1326
6 XU C, REN Z Z. Phys. Rev. C, 2006, **73**: 041301(R); XU C, REN Z Z. Phys. Rev. C, 2006, **74**: 014304
7 XU C, REN Z Z. Phys. Rev. C, 2007, **75**: 044301; XU C, REN Z Z. Phys. Rev. C, 2007, **76**: 027303
8 XU F R, PEI J C. Phys. Lett. B, 2006, **642**: 322
9 XU C, REN Z Z. Nucl. Phys. A, 2005, **753**: 174; XU C, REN Z Z. Nucl. Phys. A, 2005, **760**: 303
10 XU C, REN Z Z. Phys. Rev. C, 2006, **73**: 037302
11 Delion D S, Sandulescu A, Greiner W. Phys. Rev. C, 2004, **69**: 044318; Delion D S, Peltonen S, Suhonen J. Phys. Rev. C, 2006, **73**: 014315
12 Royer G, ZHANG H F. Phys. Rev. C, 2008, **77**: 037602
13 Mohr P. Phys. Rev. C, 2006, **73**: 031301(R)
14 Gamov G, Critchfueld C L. Theory of atomic nucleus and nuclear energy-sources. Oxford: The Clarendon Press, 1949. Vol. III
15 Firestone R B, Shirley V S, Baglin C M, Frank Chu S Y, Zipkin J. Table of Isotopes (Electronic version). 8th ed. New York: Wiley Interscience, 1996
16 XU C, REN Z Z. Nucl .Phys. A, 2006, **778**: 1