# Shape transitions in proton-rich Ho and Tm isotopes*

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**Abstract** Total Routhian Surface (TRS) calculations have been performed for even-even nuclei along proton drip line to study nuclear ground-state deformations, as well as the odd proton nuclei Ho and Tm isotopes. The drip line nuclei show the expected shape transition with the shell effects. Ground-state shape changes from prolate to oblate at $^{143}$Ho and $^{145}$Tm in these two isotopes, which is due to the $\gamma$ instability around $N=76$.

**Key words** $\gamma$ softness, proton drip line, proton emitter

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## 1 Introduction

The discovery of proton radioactivity is providing a unique opportunity to study the properties of nuclei around the proton drip-line which represents one of the fundamental limits of nuclear existence. Unlike that of the classical alpha decay, the proton decay rate is very sensitive to details of the nuclear structure, i.e., the onset of deformation and the occupation of certain Nilsson orbit.

In fact, the systematical study of proton decay half-lives[1] indicates that the transitions between spherical and deformed shapes among the proton emitters can lead to a sudden kink in the intrinsic proton-decay rates. Thus it is very important to determine the deformations and the configurations of the proton emitters.

In this work, the deformations of proton emitters and corresponding daughter nuclei are investigated systematically by the cranking shell model (CSM). The potential energy surfaces of the nuclei are plotted, which clearly show the occurrence of shape transitions (from spherical to deformed to spherical) along the proton drip line. Especially for the proton-rich Ho and Tm isotopes, shape transitions from prolate to oblate are noticeable. The competence between the two deformations leads to rather soft shapes for nuclei around $N$=76. The shape transitions could remarkably affect the nuclear structure of these nuclei, as well as the proton emission properties. The origin of the shape transition is also discussed.

## 2 The model

The ground-state total Routhian $E(Z,N,\hat{\beta})$ is composed by three components, the macroscopic liquid-drop energy[2], the microscopic shell correction[3—5] and pairing energy[6].

In our calculations, pairing correlations are self-consistently treated by solving the Hartree-Fock-Bogolyubov-like equations. In order to avoid the spurious pairing phase transition encountered in the BCS calculation the pairing is treated by the Lipkin-Nogami approach[6], in which the particle number is conserved approximately. The HFB-like equations have the following form:

$$\sum_{\beta>0}\left\{\left[(e_\alpha-\lambda)\delta_{\alpha\beta}-\omega(j_x)_{\alpha\beta}-G\rho^*_{\bar{\alpha}\bar{\beta}}+4\lambda_2\rho_{\alpha\beta}\right]U_{\beta k}-\Delta\delta_{\alpha\beta}V_{\bar{\beta}k}\right\} = E_k U_{\alpha k}\ , \tag{1}$$

$$\sum_{\beta>0}\left\{\left[(e_\alpha-\lambda)\delta_{\alpha\beta}+\omega(j_x)_{\alpha\beta}-G\rho_{\alpha\beta}+4\lambda_2\rho^*_{\bar{\alpha}\bar{\beta}}\right]V_{\bar{\beta}k}+\Delta^*\delta_{\alpha\beta}U_{\beta k}\right\} = E_k V_{\bar{\alpha}k}\ , \tag{2}$$

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where

$$\Delta \equiv G\sum_{\alpha>0}\kappa_{\alpha\bar{\alpha}}, \quad \lambda=\lambda_1+2\lambda_2(N+1), \quad E_\kappa=\varepsilon_\kappa-\lambda_2.$$

Here, $\varepsilon_\kappa$ is quasi-particle energy, $\rho = V^* * V^{\mathrm{T}}$, and $\kappa = V^*U^{\mathrm{T}}$ correspond to the density matrix and pairing tensor.

The ground state of an odd particle system is described by the one quasi-particle state, $\hat{\alpha}_j^\dagger|\mathrm{BCS}\rangle$, where $|\mathrm{BCS}\rangle$ is the ground state of the neighboring even-even nuclei. The formalism can be extended to the case of one quasi-particle states in odd-$A$ systems by a modification of density, $\rho$, and pair density, $\kappa$, due to the blocked quasi-particle, in the following way[6]:

$$\tilde{\rho}_{\alpha\beta} = \rho_{\alpha\beta}-(V^*_{\alpha j}V_{\beta j}-U_{\alpha j}U^*_{\beta j}), \tag{3}$$

$$\tilde{\kappa}_{\alpha\beta} = \kappa_{\alpha\beta}-(V^*_{\alpha j}U_{\beta j}-V^*_{\beta j}U_{\alpha j}). \tag{4}$$

In our calculations the monopole pairing strength $G$ is determined by the average gap method[7, 8] and quadruple strengths are obtained by restoring the Galilean invariance broken by the seniority pairing force[9, 10]. The deformation of a state is determined by minimizing the calculated TRS.

## 3 Results and discussion

The Total-Routhian-Surface (TRS) calculations[11] have been performed to study ground-state deformations of proton-rich nuclei, from $Z = 24$ to $Z = 82$. The ground-state quadrupole deformations of nuclei from magic number $Z = 28$ to $Z = 82$ are shown in Fig. 1, where only the axial symmetry deformation is shown in the graph for clarity. In this case, the near prolate triaxial deformations, $-30° < \gamma < 30°$ and for $-90° < \gamma < -30°$ are displayed as positive $\beta_2$ and negative $\beta_2$, respectively.

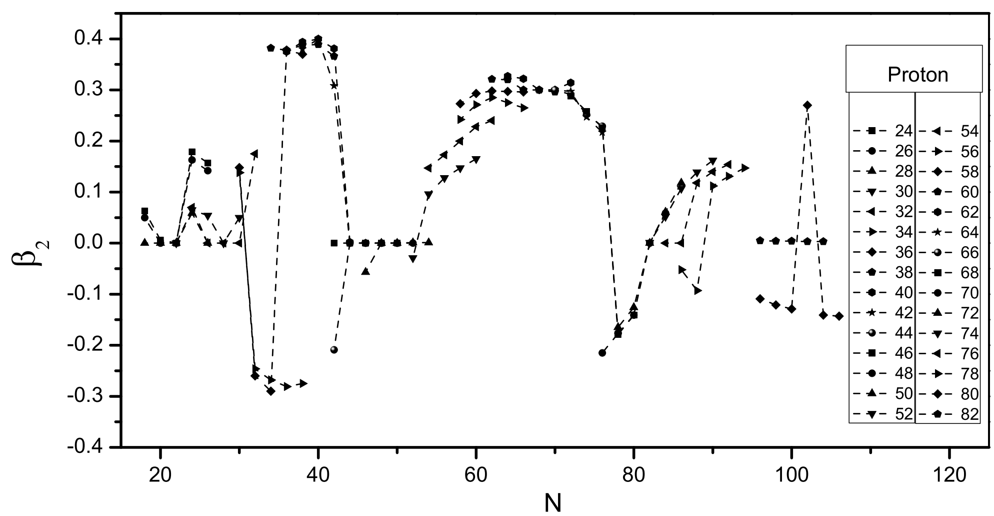


Fig. 1. The ground-state deformation of proton-drip line isotopes with each dot on one dashed line representing one isotope.

The ground-state deformation of nuclei changes in the nuclei near the proton drop line. The systematic evolvement could largely be understood by the shell effect. For nuclei with magic proton number, the deformations are relatively small. For other isotopes the nuclei become near spherical at neutron magic numbers $N = 28, 50, 82$. The deformations become notable for the nuclei with neutron number between magic numbers. $\beta_2$ of every isotopes have their maxima in the middle of the shell.

Remarkable shape transitions from prolate to oblate emerge in Kr, Er and Hg isotopes. The sudden changes in ground-state shapes have two origins. For Kr, and Hg isotopes, the changes are caused by the competitions of energetically favored state in two different shapes. The TRS diagrams for $^{74}$Kr is shown in Fig. 2 on the left, two minima exist in a prolate state $\beta_2 = 3.8$, $\gamma = 0°$ and an oblate state $\beta_2 = 3.2$, $\gamma = -60°$. The energy difference between the two states is less than 1 MeV. In $^{70}$Kr and $^{72}$Kr oblate state become the ground states. The difference in energy between prolate and oblate state becomes negative, when the neutron number become more then 34 in $^{74}$Kr. This effect brings about the shape coexistence phenomena in Kr isotopes[12]. However, the shape transition in Er isotopes is caused by a transitional nuclei, whose TRS rather flat in $\gamma$ direction at $\beta_2 = 2.25$. In Er isotopes, $^{142}$Er has a stable prolate shape, with $\beta_2 = 0.25$. For $^{142}$Er an oblate minimum is found in the TRS graph. The shape transition happened in $^{144}$Er, which is shown in Fig. 2 on the right. $^{144}$Er is $\gamma$ soft, the energy difference between prolate state and oblate state is less than 0.5 MeV.

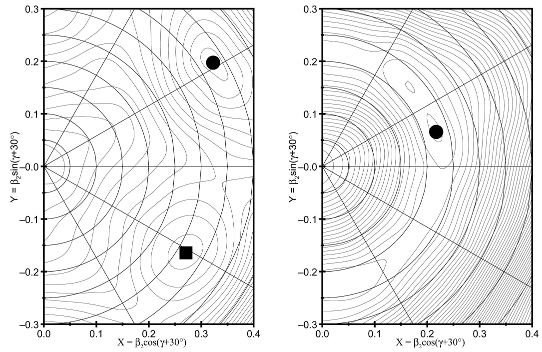

Fig. 2. TRS graph of $^{74}$Kr (left), and $^{144}$Er (right). The circle and square represent the overall minimum and the second minimum, respectively, and the contours are at 200 keV intervals.

We restrict ourselves to two isotopic series of odd proton nuclei, Tm and Ho, using blocking calculations. Shape transitions are also found in the two isotopes, the ground-state shapes change from prolate in $^{143}$Ho and $^{145}$Tm to oblate in heavier isotopes. Softness in the $\gamma$ direction appears in $^{143}$Ho and $^{145}$Tm, which is consistent with their isotones $^{144}$Er. This could be explained by the low density of neutron single-particle state in the vicinity of the deformation $\beta \approx 0.2$ and $\gamma = 30^\circ$ for $N = 76$[13]. The softness in $N = 76$ nuclei would result in the triaxial deformation[14, 15].

Composition of the blocked quasi-particle level of the valence proton in Ho and Tm isotopes are shown in Fig. 3. The composition of the quasi-particle state for the unpaired proton is influenced significantly by the nuclear shapes which change with neutron numbers.

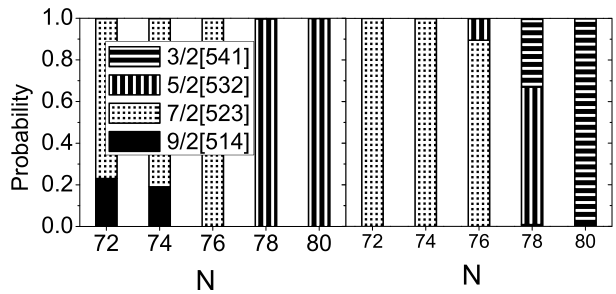


Fig. 3. Composition of the blocked quasi-particle level for the unpaired proton in $^{143}$Ho (left) and $^{145}$Tm (right).

## 4 Summary

In conclusion, TRS calculations have been done for proton-rich nuclei. We find that some isotopes have sudden change in the ground-state shape. The changes could be understood in the following two cases. First, like Kr isotopes, it could be explained by the competition of different nuclear structures. This would lead to the shape coexistences in the transitional nuclei. Another reason for the sudden changes are $\gamma$ deformations. Our calculations indicate $^{143}$Ho and $^{145}$Tm are $\gamma$ soft. The results indicate that shape transitions and $\gamma$ softness in nuclei near proton drip line could significantly influence the composition of the quasi-particle state for the unpaired proton. Thus the study of deformations are important in proton emitters.

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