

Contributions of $qqqq\bar{q}$ components to nucleon spin structure^{*}

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Abstract The spin structure of nucleons is presented in the framework of an extended quark model which in addition to the conventional qqq structure also takes into account $qqqq\bar{q}$ admixtures in the nucleon wave functions, where the $qqqq\bar{q}$ components are in colored quark cluster configurations. The axial vector weak coupling constant and spin distributions for polarized nucleons as well as spin content are obtained for the lowest positive parity $qqqq\bar{q}$ configurations in flavor-spin dependent interaction. In particular, the contributions of the down and strange quarks to the proton spin and the sum rule for polarized neutron are negative, in agreement with recent experiments.

Key words five quark components, spin content, axial vector coupling constant, sum rule for polarized nucleon, nucleon

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1 Introduction

The extended quark model which takes into account the explicit coupling of the lowest positive parity $qqqq\bar{q}$ components to the conventional qqq structure was first proposed to study the baryon spectroscopy^[1] and electromagnetic transition properties^[2]; in particular, it help us to understand the long-standing puzzle of the low mass of the Roper resonance $P_{11}(1440)$ ^[3]. If this correct, one should also expect that the pentaquark components play an explicit role in the nucleon wave function. Indeed, the implication of these quark cluster configurations have been analyzed for the pentaquark components with hidden strangeness in the nucleons^[4, 5]. It seem to be feasible to account for the empirical indications for the positive strangeness magnetic moment of proton for the configurations that are expected to have the lowest energy in the flavor-spin hyperfine interaction. Further, it is shown^[6] that addition of the lowest positive $qqqq\bar{q}$ admixtures to the ground state baryon can improve the overall description of the magnetic moments of the octet and decuplet baryons.

Here the spin structure of nucleons is examined

in the quark model which in addition to the conventional qqq structure also takes into account $qqqq\bar{q}$ admixtures in the nucleon wave functions. It was shown^[7, 8] that spin structure of the nucleon is determined by the spin content of nucleon carried by all quarks, magnetic moments of the nucleon, and the sum rule of polarized protons and neutrons in the deep inelastic scattering. Previous works^[4–6, 9] have indicated that the $qqqq\bar{q}$ components in baryons seem to be mainly in colored quark cluster configurations rather than in baryon-meson configurations or in the form of a sea of quark-antiquark pairs. The formalism summarized in that references is elaborated and is now applied to the weak decay constant and sum rules for polarized nucleons in the deep inelastic scattering as well as spin contents.

2 Wave functions of nucleons

The nucleon wave function with the $qqqq\bar{q}$ components in addition to the conventional qqq structure may be expressed in the general form^[6]:

$$|N\rangle = \varepsilon |N, qqq\rangle + \sqrt{1-\varepsilon^2} |N, qqqq\bar{q}\rangle, \quad (1)$$

where ε and $\sqrt{1-\varepsilon^2}$ represent the probability amplitudes of qqq and $qqqq\bar{q}$ components. The wave

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function of the qqq components can be expressed in the form

$$|N, \text{qqq}\rangle = \frac{1}{\sqrt{2}} \left([21]_{f_1} [21]_{s_1} + [21]_{f_2} [21]_{s_2} \right) [3]_x [1^3]_c, \quad (2)$$

where the partitions $[3]_x$, $[1^3]_c$ and $[21]_{f,s}$ have been used to indicate the spatial, color (or flavor) $SU(3)$ and the spin $SU_s(2)$ symmetries. The explicit form of

flavor and spin wave functions in the qqq component are listed in Tables 1 and 2. One can construct the spatial wave function in terms of the internal coordinates ξ_i , which are in general defined by the constituent coordinates for the n -body system

$$\xi_k = \frac{1}{\sqrt{k+k^2}} \left[\sum_{i=1}^k \mathbf{r}_i - k \mathbf{r}_{k+1} \right], \quad k = 1, 2, \dots, n-1. \quad (3)$$

Table 1. The flavor wave functions of the qqq components with the flavor symmetry $[21]_f$, and of the qqqq̄ components with the flavor symmetry $[321]_f$ in which the qqqq has the symmetry $[22]_f$. The abbreviation notations $[q_1, q_2] = q_1 q_2 - q_2 q_1$ and $\{q_1, q_2\} = q_1 q_2 + q_2 q_1$ are used in table.

	Young partition	p	n
qqq	$[21]_1$	$\frac{1}{\sqrt{6}} (\{u, u\}d - \{u, d\}u)$	$\frac{1}{\sqrt{6}} (\{u, d\}d - \{d, d\}u)$
	$[21]_2$	$\frac{1}{\sqrt{2}} [u, d]u$	$\frac{1}{\sqrt{2}} [u, d]d$
qqqq̄	$[321]_1$	$\frac{1}{6\sqrt{2}} [(\{u, d\}, \{u, s\}) - \{\{u, u\}, \{d, s\}\}] \bar{s}$ $- (\{u, u\}, \{d, d\}) - \{\{u, d\}, \{u, d\}\}] \bar{d}$	$\frac{1}{6\sqrt{2}} [(\{u, u\}, \{d, d\}) - \{\{u, d\}, \{u, d\}\}] \bar{u}$ $- (\{u, d\}, \{d, s\}) - \{\{u, s\}, \{d, d\}\}] \bar{s}$
	$[321]_2$	$\frac{1}{2\sqrt{6}} [\{u, d\}, [s, u]] \bar{s} - \{u, d\}, [u, d]] \bar{d}$	$\frac{1}{2\sqrt{6}} [\{u, d\}, [u, d]] \bar{u} - \{u, d\}, [d, s]] \bar{s}$

Table 2. The spin-up wave functions of the qqq components with the symmetry $[21]_s$, and of the qqqq̄ components with the spin symmetry $[32]_s$ in which the qqqq has the symmetry $[22]_s$.

	Young partition	nucleon
qqq	$[21]_1$	$\frac{1}{\sqrt{6}} (\{\uparrow, \uparrow\} \downarrow - \{\uparrow, \downarrow\} \uparrow)$
	$[21]_2$	$\frac{1}{\sqrt{2}} [\uparrow, \downarrow] \uparrow$
qqqq̄	$[32]_1$	$\frac{1}{2\sqrt{3}} (\{\uparrow\uparrow, \downarrow\downarrow\} - \{\uparrow\downarrow, \uparrow\downarrow\} \{\uparrow\downarrow, \downarrow\uparrow\}) \uparrow$
	$[32]_2$	$\frac{1}{2} [\uparrow\downarrow, \uparrow\downarrow] [\uparrow\downarrow, \downarrow\uparrow] \uparrow$

The wave function for the qqqq̄ components with the total spin $\frac{1}{2}$ and the positive parity is formed in the similar way to the qqq component. Since the total qqqq̄ wave function is a $[222]_c$ color singlet and the anti-quark is in the conjugate color symmetry $[11]_c$, the color part of the qqqq system has to be in a $[211]_c$ triplet. The qqqq wave function is required to be completely antisymmetric, hence the corresponding orbital-flavor-spin part have mixed symmetry $[31]_{xfs}$. Since the quark and antiquark have opposite intrinsic parity, the positive parity demands that the qqqq̄ components have to be in the P -state. For the qqqq system, if the qqqq are in spatially S -wave ground state with spatial symmetry $[4]_x$, the allowed flavor-spin part is in the $SU_{fs}(6)$ symmetry $[31]_{fs}$. In contrast, if the qqqq are in a P -wave state $[31]_x$, there are several allowed $SU_{fs}(6)$ symmetries: $[4]_{fs}$, $[31]_{fs}$, $[22]_{fs}$ and $[211]_{fs}$. These flavor-spin multiplet can be split by the spin-dependent hyperfine interaction between the quarks, only these states with lowest energy

may be expected to become the component of nucleons. The flavor-spin interaction generated by the one-meson-exchange can give the correct ordering of the baryon spectrum in all flavor sectors, indicating that antisymmetric flavor and spin configurations have the lowest energy^[1]. Consequently for the baryon octet, the lowest energy qqqq system is expected to have the symmetry $[31]_x [211]_c [4]_{fs} [22]_f [22]_s$, which also be suggested by the experimental data on the strangeness magnetic moment of the proton if one of the quarks is strange^[4, 5]. Thus, the wave functions of the qqqq̄ components are given in the form^[4-6]

$$|N, \text{qqqq}\bar{q}\rangle = \sum_{m, s_z} \sum_{i, j, k, l} C_{1m, \frac{1}{2} s_z}^{\frac{1}{2} \frac{1}{2}} C_{[211]_i, [31]_j}^{[1^4]_{ex}} C_{[22]_k [22]_l}^{[4]_{fs}} \times \psi_{c_i} \psi_{x_j, 1m} \psi_{f_k} \psi_{s_l, \frac{1}{2} s_z}. \quad (4)$$

where the sum over i, j, k , and l run over the configurations of $[211]_c$, $[31]_x$, $[22]_f$ and $[22]_s$ for which the corresponding Clebsch-Gordan coefficients are non-vanishing. The spatial wave functions ψ_x transform as three component of the permutation group S_4 symmetry $[31]_x$. The qqqq̄ flavor and spin-up wave functions are listed in Tables 1 and 2.

3 Spin structure of nucleons

In the constituent quark model, the explicit gluon degree of freedom is usually neglected and its effect is included in the quark interaction term. In fact, it was found that the available data do not require the gluon to be polarized. The nucleon spin is the sum of

quark spin and orbital angular momentum:

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G (=0) + L_q. \quad (5)$$

The contributions of all the quarks to the nucleon spin for the wave function (1) is

$$\Delta \Sigma = \langle \sum_i \sigma_i \rangle_N = \varepsilon^2 - \frac{1}{3}(1 - \varepsilon^2). \quad (6)$$

Thus, the quark spin content depends on the probability amplitude of the qqq component only. The nucleon spin carried by internal orbital angular momenta only come from the qqqq \bar{q} components in the nucleon wave function. The contribution of internal orbital angular momenta to the nucleon spin can be given by the matrix element of the sum of the z -component of all the quark orbital angular momenta,

$$L_q = (1 - \varepsilon^2) \langle \sum_i L_i^z \rangle_N = \frac{2}{3}(1 - \varepsilon^2). \quad (7)$$

The nucleon spins carried by all the quarks are connected with the number of spin-up [$n(q_\uparrow)$] and spin-down [$n(q_\downarrow)$] quarks in spin-up nucleon. If $\Delta q = n(q_\uparrow) - n(q_\downarrow) + n(\bar{q}_\uparrow) - n(\bar{q}_\downarrow)$, $q = u, d, s$, then

$$\Delta \Sigma = \Delta u + \Delta d + \Delta s \quad (8)$$

with

$$\Delta u = \frac{4}{3}\varepsilon^2, \quad \Delta d = -\frac{1}{3}\varepsilon^2 - \frac{2}{9}(1 - \varepsilon^2), \quad \Delta s = -\frac{1}{9}(1 - \varepsilon^2) \quad (9)$$

for the proton.

The weak decay constant is given by matrix element of the operator $\sum_i \tau_z^i \sigma_z^i$ for proton wave function.

For the wave function (1), we obtain

$$(g_A/g_V)_{n \rightarrow p} = \Delta u - \Delta d = \frac{5}{3}\varepsilon^2 + \frac{2}{9}(1 - \varepsilon^2). \quad (10)$$

It is easy to verify that when there is no the qqqq \bar{q} contribution (i.e., $\varepsilon = 1$), the qqq model results, $(g_A/g_V)_{n \rightarrow p} = 5/3$ follows. For spin distributions in the proton and neutron, which determine the sum rule in the deep inelastic scattering^[8], we have

$$I_1^p = \int g_1^p dx = \frac{1}{2} \langle \sum_i e_i^2 \sigma_z^i \rangle_p = \frac{5}{18}\varepsilon^2 - \frac{1}{54}(1 - \varepsilon^2), \quad (11)$$

$$I_1^n = \int g_1^n(x) dx = \frac{1}{2} \langle \sum_i e_i^2 \sigma_z^i \rangle_n = -\frac{1}{18}(1 - \varepsilon^2). \quad (12)$$

One can see that the qqq quark model gives $\int g_1^p(x) dx = 5/18$ and $\int g_1^n(x) dx = 0$. Including the qqqq \bar{q} contributions, however, $\int g_1^{p,n}(x) dx$ could be different from their qqq values, and in particular, $\int g_1^n(x) dx$ could be negative. However, one can verify that the Bjorken sum rule is still satisfied:

$$\int [g_1^p(x) - g_1^n(x)] dx = \frac{1}{6}(g_A/g_V)_{n \rightarrow p}. \quad (13)$$

To obtain a numerical estimation, we use the observed weak decay constant to determine the parameter. To reproduce data $g_A/g_V = 1.26$ ^[10], we have $\varepsilon^2 = 0.718$. The light flavor sea quark asymmetry in the proton can be given for the wave function (1) as $\bar{d} - \bar{u} = \frac{2}{3}(1 - \varepsilon^2) = 0.188$, which is consistent with the observed value $\bar{d} - \bar{u} = 0.118 \pm 0.012$ ^[11]. From (9), the total quark spin contributions in the proton are $\Delta u = 0.957$, $\Delta d = -0.302$, and $\Delta s = -0.031$ which is within the ranges of the observed values $\Delta u = 0.842 \pm 0.004$, $\Delta d = -0.427 \pm 0.004$, and $\Delta s = -0.085 \pm 0.013$ ^[12], respectively. The spin contributions from both \bar{d} and \bar{s} antiquarks were found to be negative, while that from \bar{u} is very uncertain even as to its sign and could be far less polarized than \bar{d} and \bar{s} ^[13]. The qqqq \bar{q} component gives $\Delta \bar{u} = 0$, $\Delta \bar{d} = -\frac{2}{9}(1 - \varepsilon^2) = -0.063$, and $\Delta \bar{s} = -\frac{1}{9}(1 - \varepsilon^2) = -0.031$. The contribution of internal orbital angular momenta to the proton spin is $L_q = 0.188$. The proton and neutron spin distributions are $I_1^p = 0.194$ and $I_1^n = -0.016$, which are consistent with the observed values $I_1^p = 0.1211 \pm 0.0025$ and $I_1^n = -0.0268 \pm 0.0035$ ^[12] for the proton and neutron, respectively. Note here that the flavor symmetry breaking due to heavier s quark mass than u, d quarks has not been taken into account. It has been shown^[4, 5] that such a flavor-breaking effect can firmly improve the fit of strangeness magnetic moment and strangeness spin content to the experimental data.

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