Influence of radial magnetic field on emittance in the median plane at CYCIAE-100

ZHONG Jun-Qing(钟俊晴)[1;1)] AN Shi-Zhong(安世忠)[1] YAO Hong-Juan(姚红娟)[1]
ZHANG Tian-Jue(张天爵)[1] BI Yuan-Jie(毕远杰)[1,2]

1 (China Institute of Atomic Energy, Beijing 102413, China)
2 (Department of Engineering Physics Tsinghua University, Beijing 100084, China)

Abstract A 75—100 MeV H- compact cyclotron CYCIAE-100 is being constructed at China Institute of Atomic Energy (CIAE). About 200 μA proton beam will be provided by CYCIAE-100. The imperfection of magnetic fields will remarkably affect the acceleration orbit and beam envelope in CYCIAE-100. The effects to the accelerating beam by the imperfection fields, especially the field components B_r on the mid-plane will be analyzed in detail with tracking code COMA. Poles misalignment that causes magnetic imperfection will be described in the paper. According to the simulation results, the tolerance of the poles machining and assembly will be illustrated in this paper.

Key words cyclotron, circulating emittance, radial field

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1 Introduction

Four straight sectors and the variable hill gap are chosen in magnet design of CYCIAE-100. The angle of sector is 47° and the gap is 6 cm in the central region and reduced gradually with radius. The main parameters of magnet in CYCIAE-100 are referred in Zhang's paper[1] of this conference.

Poles misalignment will cause the magnetic imperfection. The emittance growth caused by the imperfection magnetic fields, especially the field components B_r on the mid-plane and the 1st harmonic of vertical field, will be analyzed in detail with tracking code COMA[2]. According to the simulation results, the tolerance of the poles machining and assembly will be illustrated in this paper.

2 The equation of motion with B_r component

If the mirror symmetry of poles in cyclotron is violated, the radial component of the field will be produced in the median plane. The radial field can induce the shift of orbit centre in vertical orientation, and this shift will induce a beam blowup at relevant vertical resonances. The equation[3] of motion for z in case of a B_r is given by

$$\frac{\mathrm{d}^2 z}{\mathrm{d}\theta^2}+\nu_z^2 z=\frac{r_0}{\bar{B}_z}B_r\,, \tag{1}$$

where r_0 is the nominal radius, ν_z is the vertical tune and $\bar{B}_z$ the average field. The radial field can be written as a Fourier series in calculation,

$$B_r=\sum_{k=0}^{\infty}B_{rk}\cos k\theta\,.$$

So the relation between one of the Fourier components and the vertical motion will be expressed by

$$\frac{\mathrm{d}^2 z}{\mathrm{d}\theta^2}+\nu_z^2 z=\frac{r_0}{\bar{B}}B_{rk}\cos k\theta\,. \tag{2}$$

The general solution of this equation is

$$z(\theta) = z_0\cos\nu_z\theta+\frac{z_0'}{\nu_z}\sin\nu_z\theta+ \frac{r_0 B_{rk}}{\bar{B}_z(\nu_z^2-k^2)}[\cos k\theta-\cos\nu_z\theta], \tag{3}$$

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1) E-mail: zhongjunqing123@hotmail.com

where z_0 and z_0' are the initial position and the velocity of orbit centre $\left(\frac{\partial z}{\partial \theta}\Big|_{\theta=0}\right)$ respectively.

3 Influence of uniform distribution of radial field on emittance

We assume that, initially, the orbit centre is in the median plane of CYCIAE-100, i.e. $z_0 = z_0' = 0$, and the radial field B_r is uniformly distributed in median plane at 2 Gauss, i.e., the $k=0$ in formula (3). We can obtain the relation between the shift of orbit centre in vertical and the rotation angle from the expressions and COMA code. We calculate and simulate the shift of the particle whose energy is 60.4 MeV, then, the vertical resonance and the average radius are 0.645 and 147.2 centimeters respectively. The results of shift orbit centre in vertical from formula and COMA code are shown in Fig. 1. We will find that the result from formula is identical with that from COMA. In order to calculate the influence of radial field on emittance, we assume the initial emittance of beam is 2 πmm·mrad, the radial field is 8 Gauss, and the position of centre particle in phase space is shown in Fig. 2. The initial and circulating emittance are shown in Fig. 3. The value of circulating emittance is 8.13 πmm·mrad and the ratio of the emittance growth is 306%. According to the relation between the emittance growth and the shift of orbit centre,

$$\Delta z = \left(f^{\frac{1}{2}} - 1\right) \times z_i \,, \tag{4}$$

where z_i is beam size related to the initial normalized emittance. We will get the shift $\Delta z = 3.71$ mm from (4). However, after rotating for 31 turns, the average shift of the orbit centre from COMA is 3.74 mm at initial position i.e., θ=0° in cyclotron. So when there is uniform radial field in median plane, the circulating emittance can be expressed by:

$$\Delta Z = \frac{1}{n}\sum_{m=1}^{n}\frac{r_0 B_r}{\bar{B}\nu_z^2}(1-\cos 360\nu_z m), \tag{5}$$

$$\varepsilon_c = \varepsilon_0\left(1+\frac{\Delta z}{z_0}\right)^2 . \tag{6}$$

where n, ε_0 and the ε_c are the rotation turns, the initial emittance and the circulating emittance respectively.

We will find that the energy is higher and the ratio of emittance growth is bigger, when the initial emittance and B_r are given. Initial emittance and Br are 2 πmm·mrad and 2 gauss, when the beam energy is chosen to be 23 MeV, 60.4 MeV and 97.8 MeV, the corresponding ratio of emittance growth is 40%, 58% and 67.7% respectively.

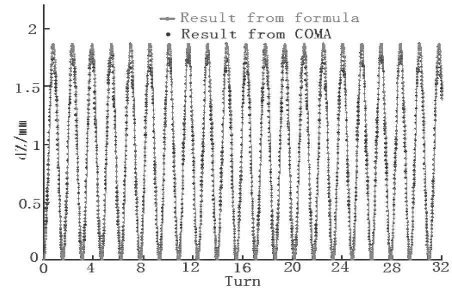

Fig. 1. The vertical shift of centre particle.

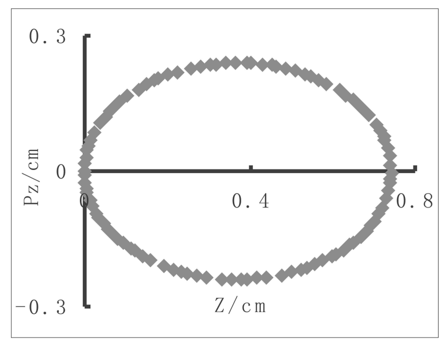

Fig. 2. The position of the centre particle with rotated turns.

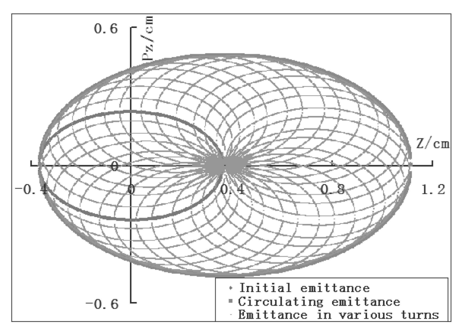

Fig. 3. The circulating emittance and initial emittance.

4 Influence of the first harmonic radial field on emittance

In this part, we will consider the influence of the first harmonic radial field on the emittanc, and the radial field is assumed B_r=11 cosθ and the energy of the particle is 60.4 MeV. Then after rotating for 31 turns, the average vertical shift of the orbit centre will be derived from the formulas of (3) and (5),

$$\Delta\bar{Z} = \frac{1}{n}\sum_{m=1}^{n}\frac{r_0 B_{r1}}{\bar{B}(\nu_z^2-1)}(\cos 360m - \cos 360\nu_z m) = -3.68 \text{ mm}.$$

When we assume the initial emittance is 2 πmm·mrad, the circulating emittance is 8.1 πmm·mrad, and the ratio of emittance growth is about 300%, the vertical shift of orbit centre derived from formula (4) is −3.68 mm. The vertical shift of the orbit centre is shown in Fig. 4, and the circulating and initial emittance are shown in Fig. 5.

According the results derived from COMA code, we find an interesting phenomenon. With the phase of the 1st harmonic θ_0, connecting the centre of initial emittance and that of circulating emittance with a straight line, the angle of the straight line and Z axis is θ_0, and the circulating emittance is changed with θ_0. This phenomenon is shown in Fig. 6.

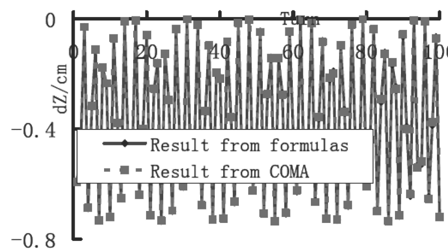

Fig. 4. The vertical shift of centre particle in 0° angle.

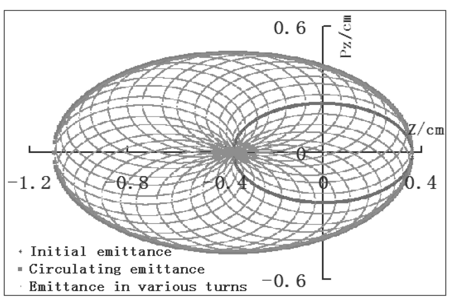

Fig. 5. The circulating emittance and initial emittance.

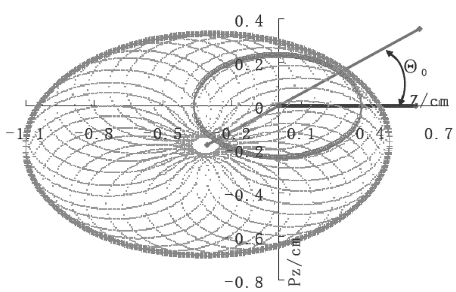

Fig. 6. The circulating and initial emittance of $B_r = 11\cos(\theta-\theta_0)$.

When the phase of the 1st harmonic is 270° or 90°, the circulating emittance is the maximum value of 13 πmm·mrad. Meanwhile the phase of the 1st harmonic is 0° or 180°, the circulating emittance is the minimum value 8.1 πmm·mrad.

5 Simulation on the relationship between error of the mirror symmetry and B_r

In order to obtain the permissible tolerance of the poles machining and assembly in construction, the relationship between the radial field and error of the poles misalignment in radial will be obtained by FEM code. The 2D model is chosen to reduce the error of meshing in simulation, and the plane of pole symmetry is modeled in FEM code. The errors of poles misalignment in radial are assumed 1 mm, 2 mm and 3 mm respectively. The results of B_r and corresponding positions are shown in Table 1.

Table 1. The relationship between error and B_r.

error/mm	0	1	2	3	4
B_r/Gauss	0	1.2	1.4	1.26	1.64
radius/cm	all	191	190	189	189

6 Conclusion

The energy is higher and the ratio of emittance growth is bigger, when the initial emittance and radial field components B_r are given. So, if the ratio of emittance growth is kept less than 50% with the initial emittance of 2 πmm·mrad, the uniform radial field components will be kept less than 1.5 Gauss at the highest energy, and the amplitude of the 1st harmonic is less than 1.9 Gauss. The $\nu_r = 1$ resonance driven by the 1st harmonic of vertical field will induce the emittance growth[4] in CYCIAE-100 too. So the error of poles misalignment should be less than 0.2 mm in radial at pole machined and assembled. Moreover we should pay more attention to the phase of the 1st harmonic during shimming, i.e. to avoid the phase of 270° or 90°.

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