Study of shape isomer yields of ^{240}Am in the framework of a dynamical-statistical model

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Abstract A dynamical statistical model is used to analyze the experimental shape isomer yields data in the reaction d+^{240}Pu at E=20–29 MeV. The possibility of determining the nuclear dissipation is discussed. Comparison of the experimental data with the calculations leads to a value of the reduced dissipation coefficient β=0.45×10^{21} s^{-1} for the Am isotopes.

Key words fission, fission barrier, reduced dissipation coefficient

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1 Introduction

At present the double-humped fission barrier concept has firmly established itself in nuclear physics. The presence of the second deep minimum on the potential surface of a heavy nucleus upon deformations permits a unified explanation of a great deal of experimental data, in particular, the nature of spontaneously fissionable isomers, subbarrier fission resonances, etc.

Spontaneous fission of isomers in heavy nuclei and spontaneous fission half-lives of heavy nuclei in the ground state and in the isomeric state [1, 2] are very important subjects in nuclear physics.

The aim of this research is to analyze the shape isomer yields in the framework of a dynamical statistical model of nuclear fission of heavy elements [3] that allows us to obtain information about the magnitude of dissipation coefficient for Am isotopes. It should be stressed that in low excitation energies the shell effect is very important so we want to consider the effect of it on the fission barrier.

2 Details of the model and analysis of the experimental data

We combine a dynamical (Langevin) and statistical description of heavy ion induced fission so that in the first potential well we use a statistical model and at each time step $\hbar/\Gamma_{\rm tot}$ calculate the decay widths for emission of n, p, α, γ and the width of the decay channel related to passing the inner fission barrier. The probabilities of decay via different channels can be calculated by using a standard Monte Carlo cascade procedure where the kind of decay selected with the weights $\Gamma_{\rm i}/\Gamma_{\rm tot}$ (i = n, p, α, γ, fission) and $\Gamma_{\rm tot}=\sum_{\rm i}\Gamma_{\rm i}$. If a random choice of decay channel leads to the transition of the nucleus from the first potential well to the second one, further evolution of the nucleus is simulated in terms of the coupled Langevin equations. It should be stressed that simulation of the fission process of the nucleus in terms of Langevin equations also allows for the emission of n, p, α and γ quanta. The result of simulation of the nucleus evolution in the second potential well can be classified as follows: 1) overcoming the second barrier and reaching the scission point; 2) population of the second potential well and cooling there via particle or γ emission that this event is interpreted as the formation of shape isomers; 3) returning of the system into the first potential well.

In order to specify the shape collective coordinates for a dynamical description of nuclear fission, we use the shape parameters r, h and α as suggested by Brack et al. [4]. However, we will simplify the calculation by considering only the symmetric fission (α=0) and will further neglect the neck degree of freedom (h=0). Consequently, the one-dimensional potential

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in the Langevin equation will be defined as $V(r)$ and the coupled Langevin equations in one dimension will be given [5] as

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{1}{2}\left(\frac{p}{m(r)}\right)^2 \frac{\mathrm{d}m(r)}{\mathrm{d}r} - \frac{\mathrm{d}F}{\mathrm{d}r} - \beta(r)p + R(t),$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{p}{m(r)}. \quad (1)$$

The free energy of the system is denoted by F while $R(t)$ is a random delta–correlated force with the properties $\langle R(t)\rangle = 0$ and $\langle R(t)\,R(t')\rangle = 2\beta T\delta(t-t')$ and β is the damping coefficient. The collective inertia, m, was calculated in the frame of the Werner–Wheeler approach. The conservative forces are calculated by free energy of the excited nuclear system as

$$F(r,T,J) = V(r,T,J) - a(r)T^2, \quad (2)$$

where T is the nuclear temperature that is defined as

$$T = (E_{\mathrm{int}}/a(r))^2, \quad (3)$$

with

$$E_{\mathrm{int}} = E^* - p^2/(2m) - V(r,T,J) - E_{\mathrm{rot}}(J), \quad (4)$$

where E^* is the total excitation energy and $E_{\mathrm{rot}}(J)$ is the rotational energy.

The potential energy of a fissionable nucleus is calculated as the sum of the liquid drop potential energy $V_{\mathrm{ld}}(r,J)$ of a rotating nucleus with an angular momentum J and a shell correction δw,

$$V(r,J,T) = V_{\mathrm{ld}}(r,J) + \delta w(r)\left[1 + \exp\left(\frac{T-T_0}{d}\right)\right]^{-1}, \quad (5)$$

where r is the distance between the centers of mass of the forming fission fragments and the bracketed expression describes the damping of the shell effects with the growth of temperature T. The values of the parameters $T_0 = 1.75$ MeV and $d = 0.2$ MeV are taken from Ref. [6].

In the zero temperature limit, $\delta w(r)$ can be taken to be equal to the difference between the above approximation and $V_{\mathrm{ld}}(r, J=0)$. Therefore, if we approximate $V(r,\ J=0,\ T=0)$ in terms of another method, then we can calculate $\delta w(r)$ as

$$\delta w = V(r) - V_{\mathrm{ld}}(r). \quad (6)$$

The double-humped fission barrier, $V(r)$, can be approximated in terms of four smoothly joined parabolic segments [7] and determined by

$$V(r) = E_i \pm \frac{1}{2} m\omega_i^2 (r-r_i)^2, \quad (7)$$

where i=0, 1, 2, 3, and the negative sign applies to $i = 1$, 3 and the positive sign to $i = 0$, 2. E_i represents the minima and maxima of the potential, $\hbar\omega_i$ are their respective curvature energies, r_i represents the locations of the minima and maxima on the deformation axis (fission coordinate) and m is the inertial mass parameter of the fissioning nucleus.

It is clear that the double-humped fission barrier can be determined by twelve parameters; three of them describe each of the four parabolas. The values of the parameters E_0, E_1, E_2, E_3, $\hbar\omega_0$, $\hbar\omega_1$, $\hbar\omega_2$ and $\hbar\omega_3$ are taken from Refs. [8, 9]. The exact locations of the various minima and maxima as well as those of the points of intersection of the successive parabolic segments can then be expressed in terms of these eight parameters. These parameters can be determined by matching the potentials $V(r)$ as well as their first derivatives at the intersection points.

Figure 1 shows the results of calculation of the double-humped fission barrier and the shell correction for ^{240}Am, ^{241}Am, and ^{242}Am.

Typical Langevin trajectories calculated by Langevin equations are also presented in Fig. 2.

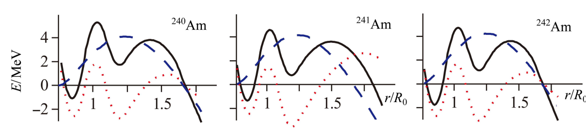

Fig. 1. Calculated double–humped fission barrier (solid curve), liquid-drop fission barrier (dashed curve) and shell correction (dotted curve) for ^{240}Am, ^{241}Am, ^{242}Am.

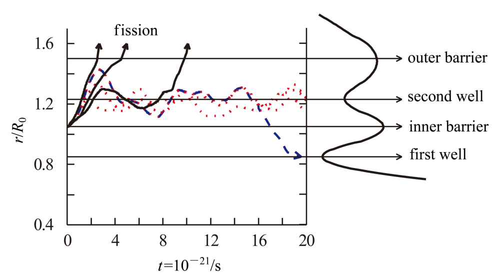

Fig. 2. Typical Langevin trajectories reach the scission point (solid curve), terminate in shape isomers (dotted curves) and return to the first potential well (dashed curve). R_0 is the radius of the spherical nucleus.

Decay widths for excited compound nuclei are calculated in the Hauser-Feschbach formalism [10], which gives the width of the disintegration of a compound nucleus C with spin J and at excitation energy E^*_{C} toward a nucleus B by the emission of a particle b of spin I_{b},

$$\Gamma_{\{\mathrm{C},J,E^*_{\mathrm{C}}\}\to\{b,I_{\mathrm{b}}\}} = \frac{1}{2\pi\rho^{\mathrm{g}}_{\mathrm{c}}(E^*_{\mathrm{C}},J)}\sum_{I_{\mathrm{B}}}(2I_{\mathrm{b}}+1)(2I_{\mathrm{B}}+1)$$

$$\times \int_{V_{\text{coul}}}^{E_{\mathrm{C}}^*} \mathrm{d}\varepsilon_{\mathrm{b}} \rho_{\mathrm{B}}^{\mathrm{g}}(E_{\mathrm{C}}^* - B_{\mathrm{b}} - \varepsilon_{\mathrm{b}}, I_{\mathrm{B}})$$
$$\times \sum_{S_{\mathrm{b}}=|I_{\mathrm{b}}-I_{\mathrm{B}}|}^{I_{\mathrm{b}}+I_{\mathrm{B}}} \sum_{l_{\mathrm{B}}=|J-S_{\mathrm{b}}|}^{J+S_{\mathrm{b}}} \tau_{l_{\mathrm{b}} S_{\mathrm{b}}}^{J}(b, \varepsilon_{\mathrm{b}}), \quad (8)$$

where $\tau_{l_{\mathrm{b}},S_{\mathrm{b}}}^{J}$ is the transmission coefficient and ρ^{g} the level density at the ground state. The variable ε_{b} is the kinetic energy of the evaporated particle b and B_{b} the binding energy of the particle b in the nucleus B.

The fission width is calculated by the Bohr formula [11],

$$\Gamma_{\{\mathrm{C},J,E_{\mathrm{C}}^*\}\to\{f,J,E_{\mathrm{f}}\}} = \frac{1}{2\pi\rho^{\mathrm{g}}(E_{\mathrm{C}}^*, J)} \int_{-B_{\mathrm{f}}}^{E_{\mathrm{C}}^* - B_{\mathrm{f}}} \mathrm{d}\varepsilon_{\mathrm{f}} \rho^{\mathrm{s}}(E_{\mathrm{C}}^* - B_{\mathrm{f}} - \varepsilon_{\mathrm{f}}, J) \tau_{\mathrm{f}}(\varepsilon_{\mathrm{f}}), \quad (9)$$

where ρ^{s} and ρ^{g} are the level densities of the nucleus at the ground and saddle points, respectively. The transmission coefficient in terms of the Hill and Wheeler formula [12] can be defined as

$$\tau_{\mathrm{f}}(\varepsilon_{\mathrm{f}}) = \frac{1}{1 + \mathrm{e}^{-2\pi\varepsilon_{\mathrm{f}}/(\hbar\omega_{\mathrm{sd}})}}, \quad (10)$$

where parameter ω_{sd} is the curvature of the fission barrier at the saddle point.

The width of the gamma emission is calculated by

$$\Gamma_{\{\mathrm{C},J,E_{\mathrm{C}}^*\}\to\gamma} = \frac{C}{\rho_{\mathrm{c}}^{\mathrm{g}}(E_{\mathrm{C}}^*, J)} \times \sum_{|J-1|}^{J+1} \int_{0}^{E^*} \mathrm{d}\varepsilon \rho_{\mathrm{B}}(E_{\mathrm{C}}^* - \varepsilon, I) \varepsilon^3. \quad (11)$$

The constant C for the above equation is obtained by requiring the expression to give the observed total radiation widths for the slow-neutron resonances.

In terms of the above described model, we analyzed the experimental data on isomer/prompt ratios for ^{240}Am produced in the reaction d+^{238}PU at bombarding energies of 20–29 MeV. It should be stressed that in calculations we investigated the probability of the production isomers form after the emission of two neutrons. In the dynamical branch and in Langevin equations the reduced friction parameter is considered as a free parameter. In Fig. 3, the results of the calculations in terms of various values of β are presented.

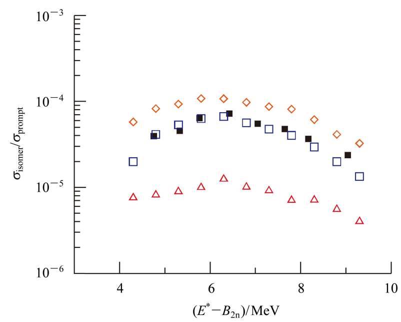

Fig. 3. Yields of the shape isomer ^{240}Am in the reaction d+^{240}Pu, ■ experimental data [9], ◇, □, △ calculation results with $\beta = 0.60\times10^{22}$ s^{-1}, $\beta = 0.45\times10^{21}$ s^{-1} and $\beta = 0.20\times10^{20}$ s^{-1}, respectively.

It can be seen that the calculations are very sensitive to the magnitude of the dissipation coefficient and in terms of $\beta = 0.45\times10^{21}$ s^{-1} the calculation is in agreement with the experimental data very well.

3 Conclusion

A dynamical statistical model was used to analyze the experimental shape isomer yield data in the reaction d+^{240}Pu at E = 20–29 MeV. The possibility of determining the nuclear dissipation was discussed and it shows that the suitable value of the reduced dissipation coefficient for the Am isotopes is $\beta = 0.45\times10^{21}$ s^{-1}. The reduced dissipation coefficient obtained is consistent with the under damped collective motion and is in agreement with the ones obtained on the basis of the analysis of the experimental information of the fission probability at low excitation energies ($\beta = 0.5\times10^{21}$ s^{-1}) [13].

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