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Entanglement in a two-qubit Heisenberg XXZ chain with different Dzyaloshinskii-Moriya couplings and an inhomogeneous magnetic field *

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Abstract Ground state entanglement and thermal entanglement of a two-qubit Heisenberg XXZ chain in the presence of the different Dzyaloshinski-Moriya interaction and inhomogeneous magnetic field are investigated. By the concept of concurrence, we find that the inhomogeneity of the magnetic field may make entanglement last for a long time and the critical temperature is dependent on J_z and b. The entanglement can be increased by increasing the temperature in some cases. We also find that the x-component parameter D_x has a higher critical temperature and more entanglement for a certain condition than the z-component parameter D_z.

Key words entanglement, inhomogeneous magnetic field, Dzyaloshinskii-Moriya couplings, concurrence, Heisenberg chain, thermal state

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1 Introduction

Quantum entanglement [1, 2] plays a central role in much quantum information processing. It has attracted much attention in recent years. The quantum entanglement in solid systems, such as spin chains [3], is an important emerging field. Spin chains are natural candidates for the realization of entanglement compared with other physics systems, and spin has been researched in many other systems, such as superconductors [4], quantum dots [5, 6], and trapped ions [7].

The Heisenberg spin system, which may be a suitable candidate to simulate the relation between qubits in a quantum computer, is an extensively studied solid-state system [8–19] which is simple but realistic. But only spin-spin interaction is considered in most of those studies; the effect of spin-orbit coupling on the entanglement is rarely included. In some recent papers, Ma and Wang considered spin-orbit entanglement in a (1/2, 1) mixed-spin chain. They studied the isotropic [12] and anisotropy [13] cases, respectively. But they have not considered the different Dzyaloshinskii-Moriya interaction. In this paper, we investigate the effects of the different Dzyaloshinskii-Moriya (DM) [20] couplings on the entanglement properties of quantum spin chains. This interaction has a number of important consequences and may cause a number of unconventional phenomena.

In this paper, the entanglement in a two-qubit Heisenberg XXZ chain with different Dzyaloshinskii-Moriya couplings and inhomogeneous magnetic field is investigated. In Section 2, we briefly give the model Hamiltonian and the definition of the concurrence. In Section 3, we investigate the properties of the ground states and the thermal states. Finally, Section 4 contains the concluding remarks.

2 General formalism

To be specific, we consider the case of the N spins of one-half anisotropic Heisenberg XXZ chain with DM interaction parameter D_x and D_z. The Hamiltonian is

$$H_x = \sum_n (J s_n^x s_{n+1}^x + J s_n^y s_{n+1}^y + J_z s_n^z s_{n+1}^z) + D_x (s_n^y s_{n+1}^z - s_n^z s_{n+1}^y) + (B+b) s_n^z + (B-b) s_n^z , \quad (1)$$

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$$H_z = \sum_n (J s_n^x s_{n+1}^x + J s_n^y s_{n+1}^y + J_z s_n^z s_{n+1}^z) + D_z (s_n^x s_{n+1}^y - s_n^y s_{n+1}^x) + (B+b) s_n^z + (B-b) s_n^z , \quad (2)$$

where J is the real coupling constant for the spin interaction. The chain is said to be anti-ferromagnetic for $J>0$ and ferromagnetic for $J<0$. The parameter J_z quantifies the anisotropy in the interaction. $D_{x,\ z}$ is the x/z component of the DM interaction. $B \geqslant 0$ is restricted, and the magnetic fields acting on the two spins have been parameterized in such a way that b controls the degree of inhomogeneity. We assume periodic boundary conditions, i.e., $N+1 \equiv 1$.

The state of a typical condensed-matter system at thermal equilibrium (temperature T) is $\rho = \exp(-\beta H)/Z$, where H is the Hamiltonian, $Z = \mathrm{tr}(\exp(-\beta H))$ is the partition function, and $\beta = 1/(kT)$, where k is the Boltzmann's constant. The entanglement associated with the thermal state ρ is referred to as thermal entanglement [21].

To quantify the amount of entanglement associated with ρ, we consider the concurrence, which is defined as [22, 23]

$$C = \max\left[0, 2\max(\lambda_i) - \sum \lambda_i\right], \quad (3)$$

where λ_i are the square roots of the eigenvalues of the matrix $R = \rho S \rho^* S$, in which ρ is the density matrix, $S = \sigma^y \otimes \sigma^y$, and the asterisk stands for the complex conjugate. As usual for entanglement measures, C ranges from 0 (no entanglement) to 1, when the two qubits are maximally entangled.

3 Results and discussion

We consider here the Hamiltonian for the $N=2$ case. In the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the Hamiltonian (1, 2) can be rewritten as

$$H_x = \begin{pmatrix} J_z + 2B & \mathrm{i}D_x & -\mathrm{i}D_x & 0 \\ -\mathrm{i}D_x & -J_z + 2b & 2J & \mathrm{i}D_x \\ \mathrm{i}D_x & 2J & -J_z - 2b & -\mathrm{i}D_x \\ 0 & -\mathrm{i}D_x & \mathrm{i}D_x & J_z - 2B \end{pmatrix}, \quad (4)$$

$$H_z = \begin{pmatrix} J_z + 2B & 0 & 0 & 0 \\ 0 & -J_z + 2b & 2J + 2\mathrm{i}D_z & 0 \\ 0 & 2J - 2\mathrm{i}D_z & -J_z - 2b & 0 \\ 0 & 0 & 0 & J_z - 2B \end{pmatrix}. \quad (5)$$

It is very tedious to write out the analytical results of concurrence, so we will first discuss the numerical results for the D_x case and then we will give a comparison of the two different interactions.

Let us consider the concurrence changing with temperature for x-component Dzyaloshinskii-Moriya couplings. In Fig. 1, we give the results at different nonuniform magnetic fields (Fig. 1(a), J_z=0.2, D_x=2, B=0.5) and anisotropy parameters (Fig. 1(b) D_x=2, B=1, b=3) entanglement changing with temperature. One can see that there exist critical temperatures T_c above which the entanglement vanishes. From Fig. 1(a) we find that with the increase in inhomogeneous magnetic field b, the ground state entanglement (T=0) gradually decreases. Though the entanglement is decreasing with the increase in b, the entanglement constant region is broadening. So with the increase in temperature we may get a long time entanglement by the increase in inhomogeneous magnetic field. We also find in the temperature range $T>6$ that the larger the value of b, the stronger the entanglement. In addition, T_c is improved with the increase in b. From Fig. 1(b) we find that in the temperature range ($0<T<3$), the larger the value in J_z, the weaker the entanglement. This condition is different from Ref. [11]. After this critical point, with the increase in J_z, the entanglement increases. And we also find that the critical temperature increases.

In Fig. 2, we analyse the differences between the x-component parameter and the z-component parameter for the temperature (Fig. 2(a), J_z=0.6, B=4, b=1.5) and magnetic field (Fig. 2(b) J_z=0.6, b=1, T=0.5). In Fig. 2(a) we find that the x-component parameter can enhance entanglement than the z-component parameter. The x-component parameter can raise the critical temperature to slow down the decrease in entanglement. We now consider the interesting case D_z=2. One notes that the concurrence is zero at zero temperature and the entanglement increases with the increase in temperature. There is a maximum value of concurrence at a finite temperature and the maximum value is due to the optimal mixing of all eigenstates in the system. In Fig. 2(b) we plot the concurrence as a function of the magnetic field B. From Fig. 2(b) we find that in the magnetic field range ($0<B<3$), the z-component entanglement is stronger than the x-component, after it the condition is reverse. We also notice that there exist critical magnetic fields B_c above which the entanglement vanishes. It is easy to see that for the same D_x and D_z, the x-component parameter D_x has a higher critical temperature and more entanglement for a certain condition than the z-component parameter D_z.

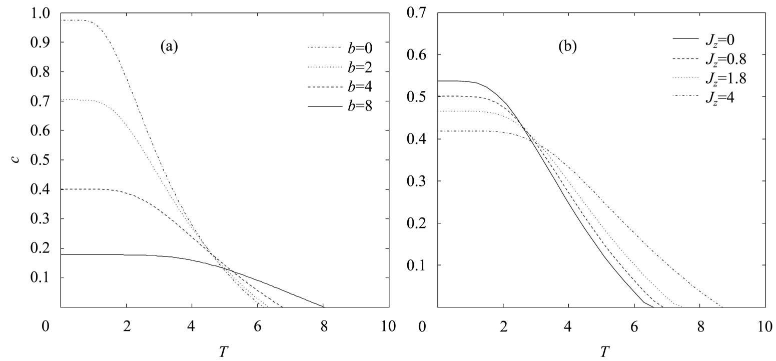

Fig. 1. The concurrence versus temperature for different b (left) and J_z (right) (J=1).

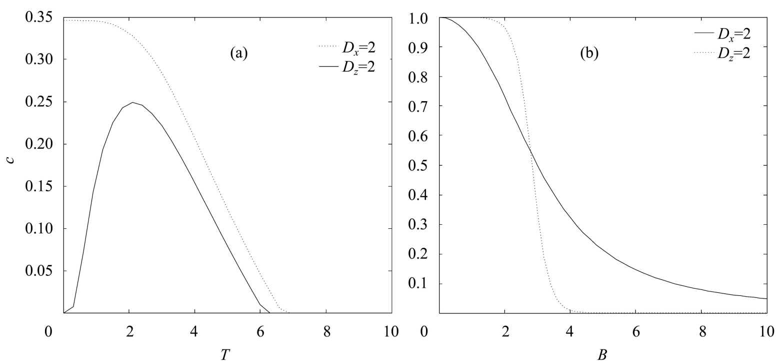

Fig. 2. The concurrence versus T (left) and B (right) for different DM couplings (J=1).

4 Conclusions

In this paper, we have studied the properties of the entanglement in a two-qubit Heisenberg XXZ chain with different Dzyaloshinskii-Moriya couplings and an inhomogeneous magnetic field. We give the concurrence forms and have obtained some numerical results of this model. Our results show that we may get a long time entanglement from the increase in inhomogeneous magnetic field. We find that the x-component parameter D_x has a higher critical temperature and more entanglement for a certain condition than the z-component parameter D_z. Thus, by changing the direction of the DM interaction, we can get a more efficient control parameter to increase the entanglement and the critical temperature.

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