

Effects of the density dependence of the symmetry energy on neutron stars^{*}

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Abstract In this paper, we include the density dependence behavior of the symmetry energy in the improved quark mass density dependent (IQMDD) model. Under the mean field approximation, this model is applied to investigate neutron star matter and neutron stars successfully. Effects of the density dependence of the symmetry energy on neutron stars are described.

Key words symmetry energy, neutron stars, relativistic mean field approximation, quark-meson coupling

PACS 24.10.Jv, 21.65.+f, 26.60.+c

1 Introduction

In recent years, the development of the radioactive ion beam (RIB) has provided scientists with opportunities for studying neutron-rich matter and the isospin degree of freedom [1–5]. Thus, researches on symmetry energy have been attracting more and more interest, which is important for the understanding of the radioactive nuclei properties in nuclear physics and many interesting issues in astrophysics [3, 4].

The density dependence of symmetry energy, especially its high density behavior, is poorly understood. Different results given by microscopic and/or phenomenological models can generally be classified into two kinds: one describes an increasing symmetry energy (E_{sym}) of baryon density (ρ_B) [1, 2], while the other predicts E_{sym} to decrease when $\rho_B > 2\rho_0$, where ρ_0 is the saturation density [4]. Using the following two forms, $E_{\text{sym}}^a = E_{\text{sym}}(\rho_0)u$ ($u \equiv \rho_B/\rho_0$) and $E_{\text{sym}}^b = E_{\text{sym}}(\rho_0)u(3-u)/2$, Li in Ref. [5] indicates that the neutron star becomes a pure neutron matter with the E_{sym}^b when $\rho_B > 3\rho_0$, while it becomes more proton-rich as the density increases with the E_{sym}^a . Our present work discusses neutron stars which are known to have extremely large inner densities. And it

is necessary to consider the high-density dependence of E_{sym} within the corresponding model, since it contributes to the equation of state (EOS).

To study nuclear matter, quantum hadrodynamics (QHD) is a pioneering framework describing the nuclear system as a relativistic many-body system of baryons and mesons. Since the quantum chromodynamics (QCD) of quarks and gluons is the fundamental theory of the strong interaction, it is natural to extend QHD to quark level. The first corresponding model, namely, the quark-meson coupling (QMC) model, is suggested by Guichon [6]. This describes the nuclear matter as a collection of nonoverlapping MIT bags, scalar σ mesons and vector ω mesons. Although QMC is successful in describing the physical properties of the nuclear system, two shortcomings arise from the MIT bag boundary. The first one is that QMC is a permanent quark confinement model, because the MIT boundary condition cannot be destroyed by temperature and density. The second one is its failure to do nuclear many-body calculations beyond the relativistic mean field approximation (MFA) by quantum field theory.

To overcome these two shortcomings, Wu et al. suggest an improved QMDD model [7–10]. Instead

Received 1 February 2010

^{*} Supported by National Natural Science Foundation of China (10535010, 10675090, 10775068, 10735010, 10975072), National Major State Basic Research and Development of China (2007CB815004), CAS Knowledge Innovation Project (KJCX2-SW-N02) and Research Fund of Doctoral Point (RFDP) (200700284016)

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of the MIT bag, they construct a Friedberg-Lee soliton bag after introducing the nonlinear interaction of σ mesons and $qq\sigma$ coupling. The interactions between quarks and mesons are then extended to the whole system. Since this model gives up the MIT bag constraint, it can well describe the quark deconfinement phase transition and do nuclear many-body calculations beyond the MFA.

In Ref. [8], the isospin vector ρ meson is included within IQMDD to discuss asymmetric nuclear matter, in particular neutron star matter, and reasonable results are given. However, the paper only employs $E_{\text{sym}}(\rho_0)$ to fit the coupling constant between quark and ρ meson, neglecting its behavior at other densities. In this work, we consider the density dependence of E_{sym} and discuss how it affects the neutron star properties.

This paper is organized as follows. In Section 2, we try to study the density dependence of nuclear symmetry energy. Then in the third section, the main formulae of the IQMDD model under MFA are given. Numerical results will be presented next. In the last section, a summary and some discussions are included.

2 Density dependence of the symmetry energy

The binding energy of asymmetric nuclear matter can be formally written as

$$E(\rho_B, \alpha) = E(\rho_B, \alpha = 0) + E_{\text{sym}}(\rho_B)\alpha^2, \quad (1)$$

where $\rho_B = \rho_n + \rho_p$ is the baryon density and $\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ denotes the isospin asymmetric parameter. Evidently the symmetry energy can be given by

$$E_{\text{sym}}(\rho_B) = \frac{1}{2} \frac{\partial^2 E(\rho_B, \alpha)}{\partial \alpha^2} \Big|_{\alpha=0}. \quad (2)$$

In the relativistic mean field model, the symmetry energy can be derived from Eq. (2) [11–14] as

$$E_{\text{sym}}(\rho_B) = \frac{k_F^2}{6E_F^*} + \frac{g_\rho^2}{12\pi^2} \frac{k_F^3}{m_\rho^2}, \quad (3)$$

where $E_F^* = \sqrt{k_F^2 + M_N^{*2}}$, $\rho_B = 2k_F^3/(3\pi^2)$. M_N^* refers to the nucleon effective mass and k_F is the fermi momentum.

At high density, the symmetry energy is dominated by the second term [11, 15], namely, $k_F \rightarrow \infty$, $E_{\text{sym}}(\rho_B) \rightarrow g_\rho^2 k_F^3/(12\pi^2 m_\rho^2)$. So when it comes to high density, such as the inner density of neutron stars, we will get too much symmetry energy [15].

As pointed out by Chen et al. in Ref. [1], the density dependence of E_{sym} mainly depends on that of the couplings between the isovector mesons and nucleon in the relativistic mean field (RMF) model. Using the same approach as Ref. [11], we try to modify the symmetry energy by introducing the density dependent coupling between quark and ρ meson in our calculations. As usual, we employ the density dependence form of nucleon- ρ coupling in Ref. [16] here, which reads

$$\Gamma_\rho(\rho_B) = \Gamma_\rho(\rho_0) \exp \left[-a_\rho \left(\frac{\rho_B}{\rho_0} - 1 \right) \right], \quad (4)$$

where a_ρ and $\Gamma_\rho(\rho_0)$ are to be determined by experimental data. As shown in the Numerical results section, the suprasaturation density dependence of E_{sym} becomes much softer.

3 Density dependent IQMDD

In this section, we include the density dependence of E_{sym} in the frame of the IQMDD model and then studies on neutron stars are carried out.

3.1 Main formulas of IQMDD

For details of IQMDD, one may turn to references [7–10]. Here we just outline its main formulae, including the density dependent ρ -nucleon (quark) coupling. The Lagrangian density within IQMDD is

$$\begin{aligned} \mathcal{L} = & \bar{\psi} [i\gamma^\mu \partial_\mu - m_q + g_\sigma^q \sigma - g_\omega^q \gamma^\mu \omega_\mu - g_\rho^q(\rho_B) \gamma^\mu \vec{\tau} \cdot \vec{\rho}_\mu] \psi \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ & + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu}, \end{aligned} \quad (5)$$

where

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} b \sigma^3 + \frac{1}{4} c \sigma^4 + B,$$

m_q ($q=u, d$) is given by the quark mass density dependent model, i.e. $m_q = B/n_q$, B denotes the bag constant, n_q is the quark number density. m_σ , m_ω and m_ρ are the corresponding masses of σ , ω and ρ mesons. g_σ^q and g_ω^q are the coupling constants between quark and σ , and quark and ω , respectively. $g_\rho^q(\rho_B)$ is the density dependent quark- ρ coupling. In QMC or IQMDD, it satisfies $g_\rho^q(\rho_B) = \Gamma_\rho(\rho_B)$ [15], therefore $g_\rho^q(\rho_B)$ has the same density dependence form as $\Gamma_\rho(\rho_B)$.

The meson and quark field equations of motion can be derived from Eq. (5) under MFA. The meson equations remain unchanged, just as that of Refs. [17, 18]. However, when we do the partial derivatives of L to the field $\bar{\psi}$, extra terms appear due

to the additional degree, namely, $\frac{\delta \mathcal{L}}{\delta \psi} = \frac{\partial \mathcal{L}}{\partial \psi} + \frac{\partial \mathcal{L}}{\partial \rho_B} \frac{\delta \rho_B}{\delta \psi}$ [19]. Here, only the vector density $\rho_B = \sqrt{j_\mu j^\mu}$ dependence is considered. Hence, the equation of motion for the quark reads as

$$[\gamma^\mu (i\partial^\mu - \Sigma_\mu - \Sigma_\mu^R) - (m_q - \Sigma_s)]\psi = 0. \quad (6)$$

The so-called rearrangement self-energy term Σ_μ^R is given by Ref. [20],

$$\Sigma_\mu^R = \frac{j_\mu}{\rho_B} \frac{\partial \Gamma_\rho}{\partial \rho_B} \bar{\psi} \gamma^\nu \frac{\tau}{2} \psi \cdot \vec{\rho}_\nu. \quad (7)$$

Because neutron star matter can be regarded as infinite nuclear matter, in the RMF approximation, the rearrangement term is reduced to

$$\Sigma_0^R = \frac{\partial \Gamma_\rho}{\partial \rho_B} \rho_3 \frac{\bar{\rho}}{2}, \quad \Sigma_{1,2,3}^R = 0, \quad (8)$$

where $\rho_3 = \rho_p - \rho_n$; $\bar{\rho} = g_\rho \rho_3 / (2m_\rho^2)$. The rearrangement term does not affect the energy density of infinite nuclear matter but contributes to both the pressure density and the chemical potentials [12, 18]. This conclusion is also valid for the neutron star matter.

3.2 Study of neutron stars

Now we turn to the study of neutron stars within the modified IQMDD model. Considering the simplest neutron star, two basic assumptions are the charge neutrality and its β -equilibrium condition. Here, we mainly emphasize the important equations and formulae concerned, and the main formulae of neutron stars can be obtained in Ref. [8].

As is well known, the rearrangement self-energy term contributes to the chemical potentials of the nucleon, which are written in the following form,

$$\mu_n = \sqrt{K_F^{n^2} + M_N^{*2}} + g_\omega \bar{\omega} - \frac{1}{2} \Gamma_\rho(\rho_B) \bar{\rho} + \Sigma_0^R, \quad (9)$$

$$\mu_p = \sqrt{K_F^{p^2} + M_N^{*2}} + g_\omega \bar{\omega} + \frac{1}{2} \Gamma_\rho(\rho_B) \bar{\rho} + \Sigma_0^R, \quad (10)$$

where K_F^p and K_F^n refer to the Fermi momentum of protons and neutrons, respectively.

The EOS of neutron stars can be calculated from

$$\begin{aligned} \varepsilon = & \frac{\gamma_N}{(2\pi)^3} \left(\int_0^{K_F^p} + \int_0^{K_F^n} \right) \sqrt{M_N^{*2} + p^2} dp^3 \\ & + \frac{g_\omega^2}{2m_\omega^2} \rho^2 + \frac{1}{2} m_\sigma^2 \bar{\sigma}^2 + \frac{1}{3} b \bar{\sigma}^3 + \frac{1}{4} c \bar{\sigma}^4 + \frac{\Gamma_\rho(\rho_B)^2}{8m_\rho^2} \rho_3^2 \\ & + \frac{1}{\pi^2} \sum_l \int_0^{k_l} \sqrt{k^2 + m_l^2} k^2 dk, \end{aligned} \quad (11)$$

$$\begin{aligned} p = & \frac{1}{3} \frac{\gamma_N}{(2\pi)^3} \left(\int_0^{K_F^p} + \int_0^{K_F^n} \right) \frac{p^2}{\sqrt{M_N^{*2} + p^2}} dp^3 \\ & + \frac{g_\omega^2}{2m_\omega^2} \rho^2 - \frac{1}{2} m_\sigma^2 \bar{\sigma}^2 - \frac{1}{3} b \bar{\sigma}^3 - \frac{1}{4} c \bar{\sigma}^4 + \frac{\Gamma_\rho(\rho_B)^2}{8m_\rho^2} \rho_3^2 \\ & + \frac{1}{3\pi^2} \sum_l \int_0^{k_l} \frac{k^4}{\sqrt{k^2 + m_l^2}} dk + \rho_B \Sigma_0^R. \end{aligned} \quad (12)$$

Using the Oppenheimer and Volkoff equations [8],

$$\begin{aligned} \frac{dp(r)}{dr} = & -\frac{Gm(r)\varepsilon}{r^2} \left(1 + \frac{p}{\varepsilon C^2} \right) \left(1 + \frac{4\pi r^3 p}{m(r)C^2} \right) \\ & \times \left(1 - \frac{2Gm(r)}{rC^2} \right)^{-1}, \end{aligned} \quad (13)$$

$$dM(r) = 4\pi r^2 \varepsilon(r) dr, \quad (14)$$

where G is the gravitational constant and C is the velocity of light. We can study the physical behavior of neutron stars within IQMDD.

4 Numerical results

Before carrying out concrete numerical calculations, we need to explain the parameters used in this model. First, the masses of mesons are fixed as $m_\omega = 783$ MeV, $m_\rho = 770$ MeV and $m_\sigma = 509$ MeV, respectively. Fitting the nucleon mass $M_N = 939$ MeV, we obtain $B = 174$ MeV fm⁻³. $g_\omega^a = 2.44$, $g_\sigma^a = 4.67$ and $b = -1460$ MeV are chosen to reproduce the saturation properties of neutron matter: the binding energy per particle $E/A = -15$ MeV and the compressibility $K = 210$ MeV at the density $\rho_0 = 0.15$ fm⁻³.

4.1 Symmetry energy

For nuclear symmetry energy, $\Gamma_\rho(\rho_0) = 8.92$ and $a_\rho = -0.243$ can be derived from $\rho_0 = 0.15$ fm⁻³, $E_{\text{sym}} = 32.5$ MeV; $\rho_B = 0.10$ fm⁻³ and $E_{\text{sym}} = 25.8$ MeV [11, 14, 21]. Fig. 1 describes the density dependence of the symmetry energy. By and large, our model shows a relatively softer symmetry energy compared with that in the constant coupling IQMDD model. We may see that large discrepancies exist, especially at suprasaturation densities. This means that the density dependent coupling succeeds in softening the high density behavior of E_{sym} .

Around saturation density ρ_0 , $E_{\text{sym}}(\rho_B)$ can be expanded up to the second order in terms of ρ_B as

$$\begin{aligned} E_{\text{sym}}(\rho_B) = & E_{\text{sym}}(\rho_0) + \frac{L}{3} \left(\frac{\rho_B - \rho_0}{\rho_0} \right) \\ & + \frac{K_{\text{sym}}}{18} \left(\frac{\rho_B - \rho_0}{\rho_0} \right)^2, \end{aligned} \quad (15)$$

where L and K_{sym} read as [12]

$$L = 3\rho_0 \left. \frac{\partial E_{\text{sym}}(\rho_B)}{\partial \rho_B} \right|_{\rho_B=\rho_0}, \quad (16)$$

$$K_{\text{sym}} = 9\rho_0^2 \left. \frac{\partial^2 E_{\text{sym}}(\rho_B)}{\partial \rho_B^2} \right|_{\rho_B=\rho_0}. \quad (17)$$

The present L is predicted to be 62.6 MeV, right in the range of $L = (88 \pm 25)$ MeV calculated from the recent isospin data [18]. The isospin dependent part of the isobaric incompressibility $K_{\text{asy}} = K_{\text{sym}} - 6L = -386$ MeV is consistent with the extracted value $K_{\text{asy}} = -500 \pm 50$ MeV from the isospin diffusion [17, 22] and $K_{\text{asy}} = -550 \pm 100$ MeV favored by the giant monopole resonance in Sn isotopes [1]. Comparisons of these physical quantities in different models [12, 16, 23–25] are made in Table 1. One can see that the calculated results are satisfactory.

Table 1. Values of some quantities in several typical theoretical model predictions.

	ρ_0/fm^{-3}	E_0/MeV	K/MeV	J/MeV	L/MeV	$K_{\text{asy}}/\text{MeV}$	Ref.
MDI($x=1$)	0.16	-16.0	211	30.6	16.4	-369	[12]
TW99	0.15	-16.2	241	32.8	55	-454	[16]
CDM3Y6	0.17	-15.9	252	29.8	62.5	-336	[23]
M3Y-P5	0.16	-16.1	235	30.9	27.9	-384	[24]
FSUGold	0.15	-16.3	230	37.3	119	-384	[25]
IQMDD	0.15	-15.0	210	32.5	62.6	-386	

4.2 Neutron stars

First, we display the particle populations in Fig. 2 with respect to the baryon density in the interior of neutron stars. One can find that although neutrons dominate the nucleonic component of neutron stars, some protons still exist. The proton fraction is important for the cooling mechanism of neutron stars through a Direct Urca process permitted only when the

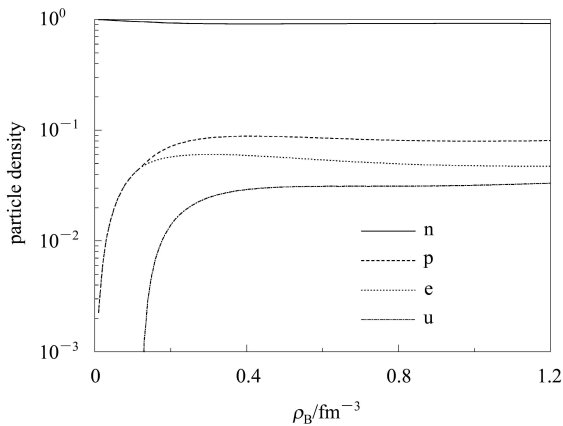


Fig. 2. The neutron (solid), proton (dashed), electron (dotted) and muon (dash-dot) densities in neutron stars as functions of the baryon density.

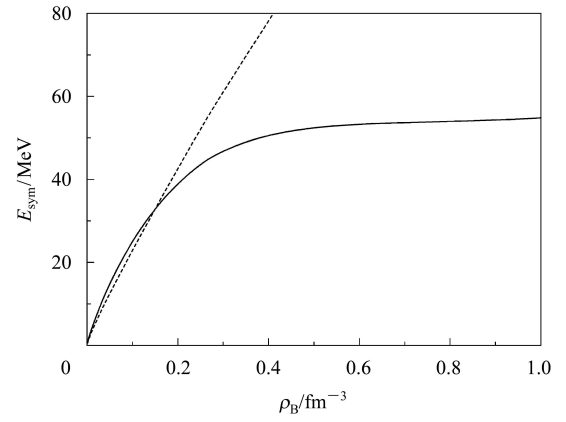


Fig. 1. Density dependence of the symmetry energy. The solid line is our present result. The broken line is the constant coupling case.

proton fraction exceeds 1/9 [22, 26]. We cannot claim that such a process is impossible since we have not taken hyperons and the isovector-scalar δ meson into account. They are assumed to increase the proton fraction [27] and the DU process may happen as well.

We plot the star mass ratio M/M_\odot versus central baryon density in Fig. 3. The maximum mass is

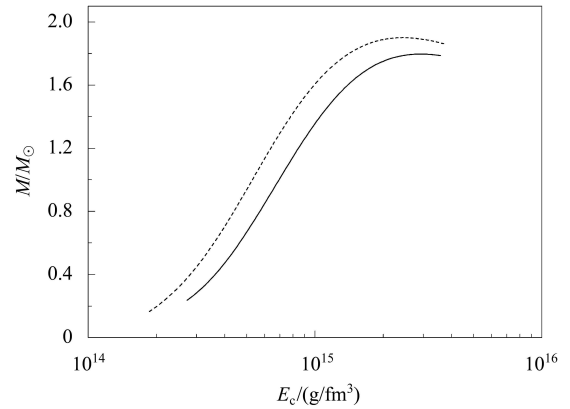


Fig. 3. Neutron star mass as a function of the central density. The solid line and the broken one are the density dependent coupling case and the constant coupling case, respectively.

about $1.79M_{\odot}$, rather smaller than the predicted value from the constant coupling case. Our result is reasonable when compared with $1.58 \pm 0.18M_{\odot}$ from PSR J0437-4715.

In Fig. 4, the masses of stars versus their radii are shown. The radius of a $1.4M_{\odot}$ neutron star is one of the most important astrophysical quantities. From Fig. 4, we can find that it is reduced from 13.38 km [7] to 12.74 km. This is reasonable since this radius

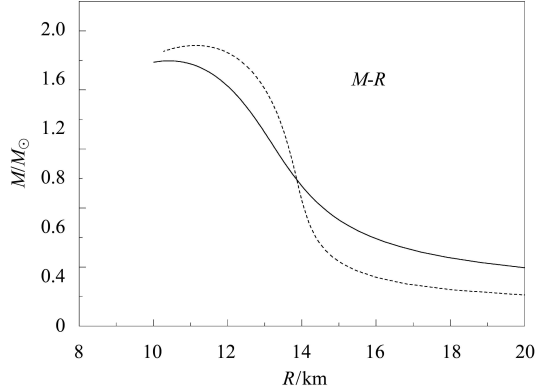


Fig. 4. The neutron star mass-radius relation. The solid line describes the density dependent coupling case and the broken one describes the constant coupling case.

tightly connects with the high-density EOS [11, 26] and a softer symmetry energy is expected to lead to a smaller neutron star radius. We may end this section with a conclusion that the density dependence of the symmetry energy plays an important role in the neutron EOS, thus neutron star predictions.

5 Summary and discussions

Recently, the knowledge of the symmetry energy has been the focus of attention in both nuclear physics and astrophysics, especially its suprasaturation density behavior. In this work, including the density dependent ρ -nucleon (quark) coupling, we are successful in giving a relatively softer symmetry energy. The neutron star maximum mass [28] and the radius of the $1.4M_{\odot}$ neutron star are to some extent reduced, and the effects of the density dependence of the symmetry energy on neutron stars are recognized. Because of the linear relationship between the slope of the symmetry energy and the neutron-skin thickness of heavy nuclei [12, 18, 28], we hope to pin down the accurate density dependence form of E_{sym} in the near future. This will enable us to constrain neutron star observations.

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