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New gauge forces beyond the standard model*

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Abstract In this work we investigate the minimal and next to minimal new gauge forces beyond standard model by constructing the corresponding electroweak chiral Lagrangians. Some phenomenological constraints from the mass differences in the K^0–$\bar{K}^0$, B^0_d–$\bar{B}^0_d$, B^0_s–$\bar{B}^0_s$ systems and the corresponding CP violation parameter are discussed.

Key words new gauge forces, electroweak chiral Lagrangian

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1 Introduction

It is well-known that there are four different kinds of fundamental forces in our real world: strong, weak, electro-magnetic and gravitational forces. With the exception of the gravitational force, which is responsible for space-time structure, all of the other three ones are described within the framework of the standard model (SM). In this work, we will investigate a situation in which it is assumed that new forces beyond the known ones act between the constituents of matter. Ways to describe such forces and their properties will be discussed. With LHC a new generation of high energy colliders is starting to run and people eagerly expect to find new forces beyond those of the SM. Suppose that new forces beyond the SM do exist, then they should all be gauge forces since the existing forces are all gauge forces except for gravity. As gauge forces they are controlled by the corresponding gauge groups for which the minimal one is $U(1)$ and the next to the minimal is $SU(2)$. These are the two cases in which we are interested in this work. Gauge forces are transmitted by gauge particles and each generator of the gauge group is associated with a vector particle. We call the carriers of the new $U(1)$ gauge force Z' and of the new $SU(2)$ gauge force $W'^{\pm}, Z'$, respectively. Then the research on minimal and next to minimal new forces beyond the SM is equivalent to investigate the physics for Z' and $W'^{\pm}, Z'$. Since up to now Z' and $W'^{\pm}$ are not found experimentally, they should all be massive and the corresponding gauge symmetry must be broken. Due to the Higgs mechanism, the longitudinal components of these gauge particles are "would-be" Goldstone bosons. In the following we will discuss the two physical cases that Z' ($W'^{\pm}, Z'$) are lightest new particles beyond the SM which are expected to be observed at LHC. It is known that conventional longitudinal electroweak gauge boson scattering requires a neutral scalar, usually called Higgs, to keep the unitarity of the scattering amplitudes at the TeV energy region [1–5]. To keep the unitarity, we need also to add an extra neutral Higgs into our theory. So in this work, with exception of the particles which are already discovered by experiment, we investigate the following two cases:

1) A neutral higgs plus a Z' and the corresponding Goldstone boson $\phi^{0\prime}$;

2) A neutral higgs plus a $W^{\pm\prime}, Z'$ and the corresponding Goldstone bosons $\phi^{\pm\prime}, \phi^{0\prime}$.

Since we are mainly interested in the Z' ($W^{\pm\prime}, Z'$) physics, the Higgs particle (which may be or may be not be responsible for the symmetry breaking), up to now only plays a passive role. For simplicity, we

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ignore it at the first step of our research. There are various new physics models which include the Z′ or $W^{\pm\prime}$,Z′. We limit ourselves to the situation that except the neutral higgs, required to delay the unitarity violation, the Z′ or $W^{\pm\prime}$,Z′ are the lightest new particles. Further, we do not want to get involved into the details of spontaneous symmetry breaking which is responsible to generate all particle masses, and prefer instead a model independent description of the new forces among these particles. This leads us to build an electroweak chiral Lagrangian (EWCL) for Z′ and $W^{\pm\prime}$,Z′, that is, the conventional EWCL for the known particles [6, 7] with Z′ or $W^{\pm\prime}$,Z′ and their Goldstone bosons being added to it. The symmetry realization patterns are generalized from the original $SU(2)_L\otimes U(1)_Y\to U(1)_{em}$ for $W^\pm$,Z,γ to $SU(2)_L\otimes U(1)\otimes U(1)'\to U(1)_{em}$ for $W^\pm$,Z,γ,Z′ and to $SU(2)_1\otimes SU(2)_2\otimes U(1)\to U(1)_{em}$ for $W^\pm$,Z,γ,$W^{\pm,\prime}$,Z′.

To build such EWCLs for new forces, the key is to figure out the symmetry transformation rules for all fields including all gauge fields, the corresponding Goldstone boson fields and SM fermion fields. To achieve this, we first give a short review on the construction of the bosonic part of the conventional EWCL. Then this procedure is applied to build EWCLs including Z′ and $W^{\pm,\prime}$,Z′. Finally we discuss the fermionic part and give some phenomenological constraints.

2 Review the building of $SU(2)_L\otimes U(1)_Y$ chiral Lagrangian

For the conventional EWCL of $SU(2)_L\otimes U(1)_Y\to U(1)_{em}$ given by Ref. [6], we can label the $SU(2)_L\otimes U(1)_Y$ group elements as $(e^{i\theta^a t^a_L},e^{i\theta t})$ with hermitian t^a_L (θ^a) for $a=1,2,3$ and t (θ) being the generators (group parameters) of $SU(2)_L$ and $U(1)_Y$ respectively. The electromagnetic $U(1)_{em}$ group generator is $t_{em}\equiv t^3_L+t$ which results in the group element $(e^{i\theta_{em}t^3_L},e^{i\theta_{em}t})$ with the $U(1)_{em}$ group parameter θ_{em}. Group theory tells us that each breaking generator corresponds to a coset which can be represented by introducing a representative element in each coset. If we denote the representative element by n, then the rule of a mapping of n to n' under a group element g is $gn=n'h$ with h being an element of the unbroken subgroup. Fig. 1 should help to understand the meaning of the mathematical relation $gn=n'h$. Combining these group theoretic results with Goldstone's theorem which states that to each symmetry breaking generator there corresponds a massless

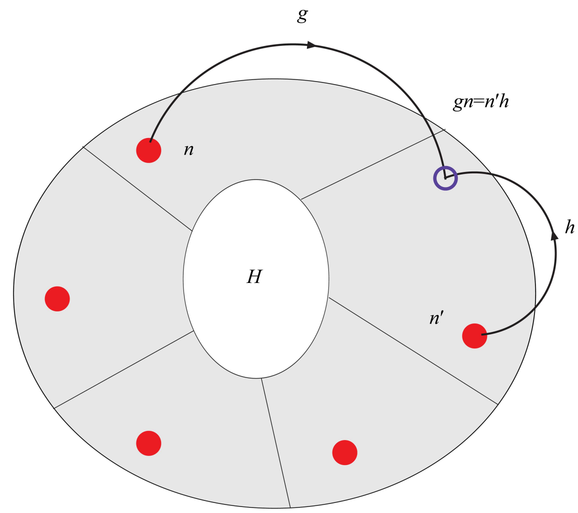

Fig. 1. Transformation law between the elements of a coset G/H of the group G, where $H\subset G$ is an invariant subgroup of G. Red bullets are the representative elements of the coset. Transformation rule: $gn=n'h$ with h belonging to H ($h\subset H$).

Goldstone particle, we find that the Goldstone boson fields related to the symmetry breaking generators can attain signifiance through the representative elements of the coset. We will define the representative element of the coset by $(U,1)$ with the unitary element U as a Goldstone field. Then we apply the transformation rule $gn=n'h$ to this $(U,1)$. With $SU(2)_L\otimes U(1)_Y$ and $g=(e^{i\theta^a t^a_L},e^{i\theta t})$ we obtain

$$(e^{i\theta^a t^a_L},\ e^{i\theta t})(U,1)\overset{gn=n'h}{=\!=\!=\!=\!=}\underbrace{(e^{i\theta^a t^a_L}Ue^{-i\theta t^3_L},\ 1)}_{U'}\underbrace{(e^{i\theta t^3_L},\ e^{i\theta t})}_{U(1)_{em}},\tag{1}$$

which leads to the transformation rule for the Goldstone field U under the $SU(2)_L\otimes U(1)_Y$ transformation,

$$U'=e^{i\theta^a t^a_L}\ U\ e^{-i\theta t^3_L}.\tag{2}$$

We can choose the Goldstone field U to be a 2×2 unimodular matrix which has three degrees of freedom matching the three breaking generators for $SU(2)_L\otimes U(1)_Y\to U(1)_{em}$. This choice of the Goldstone field in the two dimensional inner space corresponds to the choice of the generators $t^a_L=\tau^a/2$ and $t=1$. With (2) and the standard $SU(2)_L\otimes U(1)_Y$ transformation rule for electroweak gauge fields $W_\mu\equiv W^a_\mu\tau^a/2$ and B_μ, we can uniquely fix the covariant derivative for the Goldstone field U, $D_\mu U=\partial_\mu U+ig_2W_\mu U-iU\frac{\tau^3}{2}g_1B_\mu$. With this and Eq. (2) we can construct the $SU(2)_L\otimes U(1)_Y$ invariant EWCL composed from U and W_μ,B_μ as

$$\mathcal{L}^{\rm boson}_{SU(2)_{\rm L}\otimes U(1)_{\rm Y}} = -\frac{f^2}{4}{\rm tr}(V_\mu V^\mu)+\frac{f^2}{4}\beta_1[{\rm tr}(TV_\mu)]^2+\frac{1}{2}\alpha_1 g_2 g_1 B_{\mu\nu}{\rm tr}(TW^{\mu\nu})+ \frac{\rm i}{2}\alpha_2 g_1 B_{\mu\nu}{\rm tr}(T[V^\mu,V^\nu])+{\rm i}\alpha_3 g_2{\rm tr}(W_{\mu\nu}[V^\mu,V^\nu])+\alpha_4[{\rm tr}(V_\mu V_\nu)]^2+\alpha_5[{\rm tr}(V_\mu V^\mu)]^2+ \alpha_6{\rm tr}(V_\mu V_\nu){\rm tr}(TV^\mu){\rm tr}(TV^\nu)+\alpha_7{\rm tr}(V_\mu V^\mu){\rm tr}(TV_\nu){\rm tr}(TV^\nu)+\frac{1}{4}\alpha_8 g_2^2[{\rm tr}(TW_{\mu\nu})]^2+ \frac{\rm i}{2}\alpha_9 g_2{\rm tr}(TW_{\mu\nu}){\rm tr}(T[V^\mu,V^\nu])+\frac{1}{2}\alpha_{10}[{\rm tr}(TV_\mu){\rm tr}(TV_\nu)]^2+ \alpha_{11}g_2\epsilon^{\mu\nu\rho\lambda}{\rm tr}(TV_\mu){\rm tr}(V_\nu W_{\rho\lambda})+2\alpha_{12}{\rm tr}(TV_\mu){\rm tr}(V_\nu W^{\mu\nu})+\frac{1}{4}\alpha_{13}g_2 g_1\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}{\rm tr}(TW_{\rho\sigma})+ \frac{1}{8}\alpha_{14}g_2^2\epsilon^{\mu\nu\rho\sigma}{\rm tr}(TW_{\mu\nu}){\rm tr}(TW_{\rho\sigma})-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\frac{1}{2}{\rm tr}(W_{\mu\nu}W^{\mu\nu})+O(p^6), \quad (3)$$

where $T=U\tau^3U^\dagger$ and $V_\mu=(D_\mu U)U^\dagger$. This Lagrangian was first completely derived in Ref. [6].

3 $SU(2)_{\rm L}\otimes U(1)\otimes U(1)'$ chiral Lagrangian

With the experience in constructing a $SU(2)_{\rm L}\otimes U(1)_{\rm Y}$ invariant EWCL, we now generalize the process to the construction of a $SU(2)_{\rm L}\otimes U(1)\otimes U(1)'$ invariant EWCL. The group element now is $({\rm e}^{{\rm i}\theta^a t^a_{\rm L}+{\rm i}\theta' t'},{\rm e}^{{\rm i}\theta t})$ with an extra $U(1)'$ generator t' and a corresponding group paramerter θ'. The electromagnetic $U(1)_{\rm em}$ group generator now is $t_{\rm em}\equiv t^3_{\rm L}+t+ct'$ with an additional arbitrary parameter c, which results in the $U(1)_{\rm em}$ group element $({\rm e}^{{\rm i}\theta_{\rm em}(t^3_{\rm L}+ct')},{\rm e}^{{\rm i}\theta_{\rm em}t})$. We define the representative element by $(\hat{U},1)$ with $\hat{U}$ being the Goldstone boson field. Then the transformation rule $gn=n'h$ gives

$$({\rm e}^{{\rm i}\theta^a t^a_{\rm L}+{\rm i}\theta' t'},\ {\rm e}^{{\rm i}\theta t})(\hat{U},1)\overset{gn=n'h}{=\!=\!=\!=\!=} (\underbrace{{\rm e}^{{\rm i}\theta^a t^a_{\rm L}+{\rm i}\theta' t'}\hat{U}{\rm e}^{-{\rm i}\theta(t^3_{\rm L}+ct')}}_{\hat{U}'},\ 1)\underbrace{({\rm e}^{{\rm i}\theta(t^3_{\rm L}+ct')},\ {\rm e}^{{\rm i}\theta t})}_{U(1)_{\rm em}}, \quad (4)$$

which leads to the following transformation rule for the Goldstone field $\hat{U}$ under the $SU(2)_{\rm L}\otimes U(1)\otimes U(1)'$ transformation

$$\hat{U}'={\rm e}^{{\rm i}\theta^a t^a_{\rm L}+{\rm i}\theta' t'}\ \hat{U}\ {\rm e}^{-{\rm i}\theta(t^3_{\rm L}+ct')}\ . \quad (5)$$

We can choose the Goldstone field $\hat{U}$ to be a 2×2 unitary matrix which has four degrees of freedom matching the four breaking generators for $SU(2)_{\rm L}\otimes U(1)\otimes U(1)'\to U(1)_{\rm em}$. This choice of the Goldstone field in the two dimensional inner space corresponds to the generators $t^a_{\rm L}=\tau^a/2$, $t=t'=1$ (Note, according to our arrangement of the group elements, t and t' are placed in different spaces, so $t=t'=1$ will not cause any confusion). With Eq. (5) and the standard transformation $SU(2)_{\rm L}\otimes U(1)\otimes U(1)'$ rule for the electroweak gauge fields W_μ, B_μ and the extra $U(1)'$ gauge field X_μ, we can derive the covariant derivative for the Goldstone field $\hat{U}$, $D_\mu\hat{U}=\partial_\mu\hat{U}+{\rm i}(g_2W_\mu+g'_1X_\mu)\hat{U}-{\rm i}\hat{U}\left(\frac{\tau^3}{2}g_1+cg_1\right)B_\mu$. With this and Eq. (5), the $SU(2)_{\rm L}\otimes U(1)\times U(1)'$ invariant EWCL composed from U and W_μ,B_μ,X_μ is obtained as

$$\mathcal{L}^{\rm boson}_{SU(2)_{\rm L}\otimes U(1)\otimes U(1)'} = -\frac{f^2}{4}{\rm tr}(\hat{V}_\mu\hat{V}^\mu)+\frac{f^2}{4}\beta_1[{\rm tr}(T\hat{V}_\mu)]^2+\frac{f^2}{4}\beta_2{\rm tr}(\hat{V}_\mu){\rm tr}(T\hat{V}_\mu)+\frac{f^2}{4}\beta_3[{\rm tr}(\hat{V}_\mu)]^2-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}- \frac{1}{2}{\rm tr}[W_{\mu\nu}W^{\mu\nu}]-\frac{1}{4}X_{\mu\nu}X^{\mu\nu}+\frac{1}{2}\alpha_1 gg'B_{\mu\nu}{\rm tr}[TW^{\mu\nu}]+\frac{\rm i}{2}\alpha_2 g'B_{\mu\nu}{\rm tr}[T[\hat{V}^\mu,\hat{V}^\nu]]+ {\rm i}\alpha_3 g{\rm tr}[W^{\mu\nu}[\hat{V}^\mu,\hat{V}^\nu]]+\alpha_4{\rm tr}[\hat{V}_\mu\hat{V}_\nu]{\rm tr}[\hat{V}^\mu\hat{V}^\nu]+\alpha_5{\rm tr}[\hat{V}_\mu\hat{V}^\mu]{\rm tr}[\hat{V}^\nu\hat{V}_\nu]+ \alpha_6{\rm tr}[\hat{V}_\mu\hat{V}_\nu]{\rm tr}[T\hat{V}^\mu]{\rm tr}[T\hat{V}^\nu]+\alpha_7{\rm tr}[\hat{V}_\mu\hat{V}^\mu]{\rm tr}[T\hat{V}_\nu]{\rm tr}[T\hat{V}^\nu]+\frac{1}{4}\alpha_8 g^2{\rm tr}[TW_{\mu\nu}]{\rm tr}[TW^{\mu\nu}]+ \frac{\rm i}{2}\alpha_9 g{\rm tr}[TW^{\mu\nu}]{\rm tr}[T[\hat{V}_\mu,\hat{V}_\nu]]+\frac{1}{2}\alpha_{10}{\rm tr}[T\hat{V}^\mu]{\rm tr}[T\hat{V}^\nu]{\rm tr}[T\hat{V}_\mu]{\rm tr}[T\hat{V}_\nu]+ \alpha_{11}g\epsilon^{\mu\nu\rho\lambda}{\rm tr}[T\hat{V}_\mu]{\rm tr}[\hat{V}_\nu W_{\rho\lambda}]+\alpha_{12}g{\rm tr}[T\hat{V}^\mu]{\rm tr}[\hat{V}^\nu W_{\mu\nu}]+\alpha_{13}gg'\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}{\rm tr}[TW_{\rho\lambda}]+ \alpha_{14}g^2\epsilon^{\mu\nu\rho\lambda}{\rm tr}[TW_{\mu\nu}]{\rm tr}[TW_{\rho\lambda}]+\alpha_{15}{\rm tr}[\hat{V}_\mu]{\rm tr}[T\hat{V}^\mu]{\rm tr}[T\hat{V}_\nu]{\rm tr}[T\hat{V}^\nu]+ \alpha_{16}{\rm tr}[\hat{V}_\mu]{\rm tr}[T\hat{V}^\mu]{\rm tr}[\hat{V}_\nu\hat{V}^\nu]+\alpha_{17}{\rm tr}[\hat{V}_\mu]{\rm tr}[T\hat{V}_\nu]{\rm tr}[\hat{V}^\mu\hat{V}^\nu]+ \alpha_{18}{\rm tr}[\hat{V}_\mu]{\rm tr}[\hat{V}_\nu]{\rm tr}[T\hat{V}^\mu]{\rm tr}[T\hat{V}^\nu]+\alpha_{19}{\rm tr}[\hat{V}_\mu]{\rm tr}[\hat{V}_\nu]{\rm tr}[\hat{V}^\mu\hat{V}^\nu]+$$

$$\alpha_{20}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[\hat V^\mu]\mathrm{tr}[T\hat V_\nu]\mathrm{tr}[T\hat V^\nu]+\alpha_{21}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[\hat V^\mu]\mathrm{tr}[\hat V_\nu\hat V^\nu]+\alpha_{22}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[\hat V^\mu]\mathrm{tr}[\hat V_\nu]\mathrm{tr}[T\hat V^\nu]+$$
$$\alpha_{23}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[\hat V_\nu]\mathrm{tr}[\hat V^\mu]\mathrm{tr}[\hat V^\nu]+gg''\alpha_{24}X_{\mu\nu}\mathrm{tr}[TW^{\mu\nu}]+g'g''\alpha_{25}B_{\mu\nu}X^{\mu\nu}+$$
$$\alpha_{26}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]\mathrm{tr}[T[\hat V_\rho,\hat V_\lambda]]+\mathrm{i}g'\alpha_{27}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]B_{\rho\lambda}+$$
$$\mathrm{i}g\alpha_{28}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]\mathrm{tr}[TW_{\rho\lambda}]+g\alpha_{29}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[\hat V_\nu W_{\rho\lambda}]+$$
$$\mathrm{i}g''\alpha_{30}\epsilon^{\mu\nu\rho\lambda}X_{\mu\nu}\mathrm{tr}[T[\hat V_\rho,\hat V_\lambda]]+\mathrm{i}g''\alpha_{31}X_{\mu\nu}\mathrm{tr}[T[\hat V^\mu,\hat V^\nu]]+g''\alpha_{32}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]X_{\rho\lambda}+$$
$$\alpha_{33}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]\mathrm{tr}[T[\hat V^\mu,\hat V^\nu]]+g'g''\alpha_{34}\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}X_{\rho\lambda}+gg''\alpha_{35}\epsilon^{\mu\nu\rho\lambda}X_{\mu\nu}\mathrm{tr}[TW_{\rho\lambda}]+$$
$$\mathrm{i}g'\alpha_{36}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]B^{\mu\nu}+\mathrm{i}g\alpha_{37}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]\mathrm{tr}[TW^{\mu\nu}]+g\alpha_{38}\mathrm{tr}[\hat V^\mu]\mathrm{tr}[\hat V^\nu W_{\mu\nu}]+$$
$$g''\alpha_{39}\mathrm{tr}[\hat V_\mu]\mathrm{tr}[T\hat V_\nu]X^{\mu\nu}+\mathrm{i}g\alpha_{40}\mathrm{tr}[\hat V^\mu]\mathrm{tr}[T\hat V^\nu W_{\mu\nu}]+O(p^6)\,, \tag{6}$$

where we have identifying g_2 with g, g_1' with $-g''$, g_1 with g', cg_1 with $\tilde g'$, $T=\hat U\tau^3\hat U^\dagger$ and $\hat V_\mu=(D_\mu\hat U)\hat U^\dagger$. This Lagrangian combined with a neutral higgs field was given in Ref. [8].

4 $SU(2)_1\otimes SU(2)_2\otimes U(1)$ chiral Lagrangian – bosonic part

For the case of the $SU(2)_1\otimes SU(2)_2\otimes U(1)$ invariant EWCL the group elements become $(\mathrm{e}^{\mathrm{i}\theta_1^a t_1^a},\ \mathrm{e}^{\mathrm{i}\theta_2^a t_2^a},\ \mathrm{e}^{\mathrm{i}\theta t})$, where t_1^a and t_2^a (θ_1^a and θ_1^a) for $a=1,2,3$ are the $SU(2)_1$ and $SU(2)_2$ generators (group parameters) respectively. The electromagnetic $U(1)_{\rm em}$ group generator now becomes $t_{\rm em}\equiv t_1^3+t_2^3+t$ which results in the group element $(\mathrm{e}^{\mathrm{i}\theta_{\rm em}t_1^3},\mathrm{e}^{\mathrm{i}\theta_{\rm em}t_2^3},\mathrm{e}^{\mathrm{i}\theta_{\rm em}t})$. We define the representative element with $(U_1,U_2,1)$ with U_1,U_2 being the Goldstone boson fields. The transformation rule $gn=n'h$ gives then,

$$(\mathrm{e}^{\mathrm{i}\theta_1^a t_1^a},\ \mathrm{e}^{\mathrm{i}\theta_2^a t_2^a},\ \mathrm{e}^{\mathrm{i}\theta t})(U_1,U_2,1)\overset{gn=n'h}{=\!=\!=\!=}$$
$$(\underbrace{\mathrm{e}^{\mathrm{i}\theta_1^a t_1^a}U_1\mathrm{e}^{-\mathrm{i}\theta t_1^3}}_{U_1'},\underbrace{\mathrm{e}^{\mathrm{i}\theta_2^a t_2^a}U_2\mathrm{e}^{-\mathrm{i}\theta t_2^3}}_{U_2'},1)\underbrace{(\mathrm{e}^{\mathrm{i}\theta t_1^3},\ \mathrm{e}^{\mathrm{i}\theta t_2^3},\ \mathrm{e}^{\mathrm{i}\theta t})}_{U(1)_{\rm em}}\,, \tag{7}$$

which leads to the following rules for the Goldstone fields U_1,U_2 under the $SU(2)_1\otimes SU(2)_2\otimes U(1)$ transformation

$$U_1'=\mathrm{e}^{\mathrm{i}\theta_1^a t_1^a}\,U_1\,\mathrm{e}^{-\mathrm{i}\theta t_1^3}\,,\quad U_2'=\mathrm{e}^{\mathrm{i}\theta_2^a t_2^a}\,U_2\,\mathrm{e}^{-\mathrm{i}\theta t_2^3}\,. \tag{8}$$

We can choose the Goldstone fields U_1 and U_2 to be the 2×2 unimodular matrices with totally six degrees of freedom matching the six breaking generators of $SU(2)_1\otimes SU(2)_1\otimes U(1)\to U(1)_{\rm em}$. With Eq. (8) and the standard $SU(2)_1\otimes SU(2)_2\otimes U(1)$ transformation rule for the electroweak gauge fields $W_{1,\mu},W_{2,\mu},B_\mu$, we can fix the covariant derivatives for the Goldstone fields U_i as $D_\mu U_i=\partial_\mu U_i+\mathrm{i}g_iW_{i,\mu}U_i-\mathrm{i}U_i\frac{\tau^3}{2}gB_\mu$ for $i=1,2$. With this and Eq. (8), one can construct the $SU(2)_1\otimes SU(2)_2\times U(1)$ invariant EWCL from the fields U_1,U_2 and $W_{1,\mu},W_{2,\mu},B_\mu$ as

$$\mathcal{L}^{\rm boson}_{SU(2)_1\otimes SU(2)_2\otimes U(1)}=$$
$$-\frac14 f_1^2\mathrm{tr}(X_1^\mu)^2-\frac14 f_2^2\mathrm{tr}(X_2^\mu)^2+\frac12\kappa f_1f_2\mathrm{tr}(X_1^\mu X_2^\mu)+\frac14\beta_{1,1}f_1^2[\mathrm{tr}(\tau^3X_{1,\mu})]^2+\frac14\beta_{2,1}f_2^2[\mathrm{tr}(\tau^3X_{2,\mu})]^2+$$
$$\frac12\tilde\beta_1f_1f_2[\mathrm{tr}(\tau^3X_{1,\mu})][\mathrm{tr}(\tau^3X_2^\mu)]-\frac14W^a_{1,\mu\nu}W_1^{\mu\nu,a}-\frac14W^a_{2,\mu\nu}W_2^{\mu\nu,a}-\frac14B_{\mu\nu}B^{\mu\nu}+$$
$$\sum_{i=1,2}\Big[\frac12\alpha_{i,1}gB_{\mu\nu}\mathrm{tr}(\tau^3\overline W_i^{\mu\nu})+\mathrm{i}\alpha_{i,2}gB_{\mu\nu}\mathrm{tr}(\tau^3X_i^\mu X_i^\nu)+2\mathrm{i}\alpha_{i,3}\mathrm{tr}(\overline W_{i,\mu\nu}X_i^\mu X_i^\nu)+$$
$$\alpha_{i,4}[\mathrm{tr}(X_{i,\mu}X_{i,\nu})]^2+\alpha_{i,5}[\mathrm{tr}(X_{i,\mu}^2)]^2+\alpha_{i,6}\mathrm{tr}(X_{i,\mu}X_{i,\nu})\mathrm{tr}(\tau^3X_i^\mu)\mathrm{tr}(\tau^3X_i^\nu)+$$
$$\alpha_{i,7}\mathrm{tr}(X_{i,\mu}^2)[\mathrm{tr}(\tau^3X_{i,\nu})]^2+\frac14\alpha_{i,8}[\mathrm{tr}(\tau^3\overline W_{i,\mu\nu})]^2+\mathrm{i}\alpha_{i,9}\mathrm{tr}(\tau^3\overline W_{i,\mu\nu})\mathrm{tr}(\tau^3X_i^\mu X_i^\nu)+$$
$$\frac12\alpha_{i,10}[\mathrm{tr}(\tau^3X_{i,\mu})\mathrm{tr}(\tau^3X_{i,\nu})]^2+\alpha_{i,11}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\tau^3X_{i,\mu})\mathrm{tr}(X_{i,\nu}\overline W_{i,\rho\lambda})+$$
$$2\alpha_{i,12}\mathrm{tr}(\tau^3X_{i,\mu})\mathrm{tr}(X_{i,\nu}\overline W_i^{\mu\nu})+\frac14\alpha_{i,13}g\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}\mathrm{tr}(\tau^3\overline W_{i,\rho\sigma})+$$

$$\frac{1}{8}\alpha_{i,14}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}(\tau^3\overline{W}_{i,\mu\nu})\mathrm{tr}(\tau^3\overline{W}_{i,\rho\sigma})\Big]+\mathrm{i}\tilde{\alpha}_2 gB_{\mu\nu}\mathrm{tr}(\tau^3X_1^\mu X_2^\nu)+2\mathrm{i}\tilde{\alpha}_{3,1}\mathrm{tr}(\overline{W}_{1,\mu\nu}X_2^\mu X_2^\nu)+$$
$$2\mathrm{i}\tilde{\alpha}_{3,2}\mathrm{tr}(\overline{W}_{2,\mu\nu}X_1^\mu X_1^\nu)+2\mathrm{i}\tilde{\alpha}_{3,3}\mathrm{tr}(\overline{W}_{1,\mu\nu}X_1^\mu X_2^\nu)+2\mathrm{i}\tilde{\alpha}_{3,4}\mathrm{tr}(\overline{W}_{2,\mu\nu}X_2^\mu X_1^\nu)+$$
$$\tilde{\alpha}_{4,1}\mathrm{tr}(X_{1,\mu}X_{1,\nu})\mathrm{tr}(X_2^\mu X_2^\nu)+\tilde{\alpha}_{4,2}[\mathrm{tr}(X_{1,\mu}X_{2,\nu})]^2+\tilde{\alpha}_{4,3}\mathrm{tr}(X_{1,\mu}X_{2,\nu})\mathrm{tr}(X_2^\mu X_1^\nu)+$$
$$\tilde{\alpha}_{4,4}\mathrm{tr}(X_{1,\mu}X_{2,\nu})\mathrm{tr}(X_2^\mu X_2^\nu)+\tilde{\alpha}_{4,5}\mathrm{tr}(X_{2,\mu}X_{1,\nu})\mathrm{tr}(X_1^\mu X_1^\nu)+\tilde{\alpha}_{5,1}\mathrm{tr}(X_{1,\mu}^2)\mathrm{tr}(X_{2,\nu}^2)+$$
$$\tilde{\alpha}_{5,2}[\mathrm{tr}(X_{1,\mu}X_2^\mu)]^2+\tilde{\alpha}_{5,3}\mathrm{tr}(X_{1,\mu}X_2^\mu)\mathrm{tr}(X_{2,\nu}^2)+\tilde{\alpha}_{5,4}\mathrm{tr}(X_{2,\mu}X_1^\mu)\mathrm{tr}(X_{1,\nu}^2)+$$
$$\tilde{\alpha}_{6,1}\mathrm{tr}(X_{1,\mu}X_{1,\nu})\mathrm{tr}(\tau^3X_2^\mu)\mathrm{tr}(\tau^3X_2^\nu)+\tilde{\alpha}_{6,2}\mathrm{tr}(X_{2,\mu}X_{2,\nu})\mathrm{tr}(\tau^3X_1^\mu)\mathrm{tr}(\tau^3X_1^\nu)+$$
$$\tilde{\alpha}_{6,3}\mathrm{tr}(X_{1,\mu}X_{2,\nu})\mathrm{tr}(\tau^3X_1^\mu)\mathrm{tr}(\tau^3X_2^\nu)+\tilde{\alpha}_{6,4}\mathrm{tr}(X_{1,\mu}X_{2,\nu})\mathrm{tr}(\tau^3X_2^\mu)\mathrm{tr}(\tau^3X_1^\nu)+$$
$$\tilde{\alpha}_{6,5}\mathrm{tr}(X_{1,\mu}X_{2,\nu})\mathrm{tr}(\tau^3X_2^\mu)\mathrm{tr}(\tau^3X_2^\nu)+\tilde{\alpha}_{6,6}\mathrm{tr}(X_{2,\mu}X_{1,\nu})\mathrm{tr}(\tau^3X_1^\mu)\mathrm{tr}(\tau^3X_1^\nu)+$$
$$\tilde{\alpha}_{6,7}\mathrm{tr}(X_{1,\mu}X_{1,\nu})\mathrm{tr}(\tau^3X_1^\mu)\mathrm{tr}(\tau^3X_2^\nu)+\tilde{\alpha}_{6,8}\mathrm{tr}(X_{2,\mu}X_{2,\nu})\mathrm{tr}(\tau^3X_2^\mu)\mathrm{tr}(\tau^3X_1^\nu)+$$
$$\tilde{\alpha}_{7,1}\mathrm{tr}(X_{1,\mu}^2)[\mathrm{tr}(\tau^3X_{2,\nu})]^2+\tilde{\alpha}_{7,2}\mathrm{tr}(X_{2,\mu}^2)[\mathrm{tr}(\tau^3X_{1,\nu})]^2+$$
$$\tilde{\alpha}_{7,3}\mathrm{tr}(X_{1,\mu}X_2^\mu)\mathrm{tr}(\tau^3X_{1,\nu})\mathrm{tr}(\tau^3X_2^\nu)+\tilde{\alpha}_{7,4}\mathrm{tr}(X_{1,\mu}X_2^\mu)[\mathrm{tr}(\tau^3X_{2,\nu})]^2+$$
$$\tilde{\alpha}_{7,5}\mathrm{tr}(X_{2,\mu}X_1^\mu)[\mathrm{tr}(\tau^3X_{1,\nu})]^2+\tilde{\alpha}_{7,6}\mathrm{tr}(X_{1,\mu}^2)\mathrm{tr}(\tau^3X_{1,\nu})\mathrm{tr}(\tau^3X_2^\nu)+$$
$$\tilde{\alpha}_{7,7}\mathrm{tr}(X_{2,\mu}^2)\mathrm{tr}(\tau^3X_{2,\nu})\mathrm{tr}(\tau^3X_1^\nu)+\frac{1}{4}\tilde{\alpha}_8\mathrm{tr}(\tau^3\overline{W}_{1,\mu\nu})\mathrm{tr}(\tau^3\overline{W}_2^{\mu\nu})+$$
$$\mathrm{i}\tilde{\alpha}_{9,1}\mathrm{tr}(\tau^3\overline{W}_{1,\mu\nu})\mathrm{tr}(\tau^3X_2^\mu X_2^\nu)+\mathrm{i}\tilde{\alpha}_{9,2}\mathrm{tr}(\tau^3\overline{W}_{2,\mu\nu})\mathrm{tr}(\tau^3X_\mathrm{L}^\mu X_\mathrm{L}^\nu)+$$
$$\mathrm{i}\tilde{\alpha}_{9,3}\mathrm{tr}(\tau^3\overline{W}_{1,\mu\nu})\mathrm{tr}(\tau^3X_1^\mu X_2^\nu)+\mathrm{i}\tilde{\alpha}_{9,4}\mathrm{tr}(\tau^3\overline{W}_{2,\mu\nu})\mathrm{tr}(\tau^3X_2^\mu X_1^\nu)+$$
$$\frac{1}{2}\tilde{\alpha}_{10,1}[\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(\tau^3X_{2,\nu})]^2+\frac{1}{2}[\tilde{\alpha}_{10,2}[\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(\tau^3X_2^\mu)]^2+$$
$$\frac{1}{2}\tilde{\alpha}_{10,3}\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(\tau^3X_2^\mu)[\mathrm{tr}(\tau^3X_{2,\nu})]^2+\frac{1}{2}\tilde{\alpha}_{10,4}\mathrm{tr}(\tau^3X_{2,\mu})\mathrm{tr}(\tau^3X_1^\mu)[\mathrm{tr}(\tau^3X_{1,\nu})]^2+$$
$$\tilde{\alpha}_{11,1}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(X_{2,\nu}\overline{W}_{2,\rho\lambda})+\tilde{\alpha}_{11,2}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\tau^3X_{2,\mu})\mathrm{tr}(X_{1,\nu}\overline{W}_{1,\rho\lambda})+$$
$$\tilde{\alpha}_{11,3}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(X_{1,\nu}\overline{W}_{2,\rho\lambda})+\tilde{\alpha}_{11,4}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\tau^3X_{2,\mu})\mathrm{tr}(X_{2,\nu}\overline{W}_{1,\rho\lambda})+$$
$$\tilde{\alpha}_{11,5}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(X_{2,\nu}\overline{W}_{1,\rho\lambda})+\tilde{\alpha}_{11,6}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\tau^3X_{2,\mu})\mathrm{tr}(X_{1,\nu}\overline{W}_{2,\rho\lambda})+$$
$$2\tilde{\alpha}_{12,1}\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(X_{2,\nu}\overline{W}_2^{\mu\nu})+2\tilde{\alpha}_{12,2}\mathrm{tr}(\tau^3X_{2,\mu})\mathrm{tr}(X_{1,\nu}\overline{W}_1^{\mu\nu})+$$
$$2\tilde{\alpha}_{12,3}\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(X_{1,\nu}\overline{W}_2^{\mu\nu})+2\tilde{\alpha}_{12,4}\mathrm{tr}(\tau^3X_{2,\mu})\mathrm{tr}(X_{2,\nu}\overline{W}_1^{\mu\nu})+$$
$$2\tilde{\alpha}_{12,5}\mathrm{tr}(\tau^3X_{1,\mu})\mathrm{tr}(X_{2,\nu}\overline{W}_1^{\mu\nu})+2\tilde{\alpha}_{12,6}\mathrm{tr}(\tau^3X_{2,\mu})\mathrm{tr}(X_{1,\nu}\overline{W}_2^{\mu\nu})+$$
$$\frac{1}{8}\tilde{\alpha}_{14}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}(\tau^3\overline{W}_{1,\mu\nu})\mathrm{tr}(\tau^3\overline{W}_{2,\rho\sigma})+\tilde{\alpha}_{15}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}(\overline{W}_{1,\mu\nu}\overline{W}_{2,\rho\sigma})+O(p^6)\,, \qquad (9)$$

where $\overline{W}_{i,\mu\nu}\equiv U_i^\dagger g_iW_{i,\mu\nu}U_i$ and $X_i^\mu=U_i^\dagger(D^\mu U_i)$. This Lagrangian combined with a neutral higgs field has been derived in Ref.[9].

5 $SU(2)_1\otimes SU(2)_2\otimes U(1)$ chiral Lagrangian – fermion part

With these new EWCLs (6) and (9), naive applications, such as γ-Z-Z′ mixing, W-W′ mixing and $W^\pm$,Z anomalous couplings, etc. are possible, howerver will not be discussed here. Another direction of further research is to investigate the constraints from low energy experiments. Considering that low energy experiments mainly involve quarks and leptons of the SM, we need to consider the fermionic part of the EWCL for Z′ or $W^{\pm\prime}$. In the following of this work we only discuss the case of $SU(2)_1\otimes SU(2)_2\otimes U(1)$ in which $W'^\pm$,Z′ couple to ordinary quarks and leptons. This can be seen as a generalization of the work presented in Ref. [7]. In the literature various models provide at least the following different arrangements for the fermion representations [10]:

1) Left-right symmetric (LR) [11, 12]: Left handed fermions belong to the doublet of $SU(2)_1$ and the singlet of $SU(2)_2$; Right handed fermions belong to the doublet of $SU(2)_2$ and the singlet of $SU(2)_1$.

2) Leptophobic (LP): Left handed fermions belong to the doublet of $SU(2)_1$ and the singlet of $SU(2)_2$; Right handed quarks belong to the doublet of $SU(2)_2$ and the singlet of $SU(2)_1$; Right handed leptons belong to the singlets of both $SU(2)$'s.

3) Hadrophobic (HP): Left handed fermions belong to the doublet of $SU(2)_1$ and the singlet of $SU(2)_2$; Right handed leptons belong to the doublet of $SU(2)_2$ and the singlet of $SU(2)_1$; Right handed quarks belong to the singlets of both $SU(2)$'s.

4) Fermionphobic (FP) [10, 13, 14]: Left handed fermions belong to the doublet of $SU(2)_1$ and the singlet of $SU(2)_2$; Right handed fermions belong to the singlets of both $SU(2)$'s.

5) Ununified (UN) [15]: Left handed leptons belong to the doublet of $SU(2)_1$ and the singlet of $SU(2)_2$; Left handed quarks belong to the doublet of $SU(2)_2$ and the singlet of $SU(2)_1$; Right handed fermions belong to the singlet of $SU(2)_1\otimes SU(2)_2$.

6) Non-universal (NU) [16]: One or two special families of left handed fermions (typical cases are the first two light families) belong to the doublet of $SU(2)_1$ and the singlet of $SU(2)_2$; Remaining left handed fermions belong to the doublet of $SU(2)_2$ and the singlet of $SU(2)_1$; Right handed fermions belong to the singlet of $SU(2)_1\otimes SU(2)_2$.

Table 1. Fermion transformation properties for the different models considered in the text. The numbers in brackets refer to $SU(2)_1$, $SU(2)_2$ and $U(1)$, respectively. Color indices are implicit. The right hand neutrinos are not present in some of the original models LP, FP, UN and NU and are labeled by $-$. Including them in this work is harmless to these models and their representation is $(1,1,0)$.

fields/models	LR	LP	HP	FP	UN	NU
$q_{\alpha\mathrm{L}}=\begin{pmatrix}u_{\alpha\mathrm{L}}\\ d_{\alpha\mathrm{L}}\end{pmatrix}$	$\left(2,1,\frac{1}{6}\right)$	$\left(2,1,\frac{1}{6}\right)$	$\left(2,1,\frac{1}{6}\right)$	$\left(2,1,\frac{1}{6}\right)$	$\left(1,2,\frac{1}{6}\right)$	$\left(2,1,\frac{1}{6}\right)\delta_{\alpha\alpha_1}+\left(1,2,\frac{1}{6}\right)\delta_{\alpha\alpha_2}$
$q_{\alpha\mathrm{R}}=\begin{pmatrix}u_{\alpha\mathrm{R}}\\ d_{\alpha\mathrm{R}}\end{pmatrix}$	$\left(1,2,\frac{1}{6}\right)$	$\left(1,2,\frac{1}{6}\right)$	$\left(1,1,\frac{2}{3}\right)$ $\left(1,1,-\frac{1}{3}\right)$	$\left(1,1,\frac{2}{3}\right)$ $\left(1,1,-\frac{1}{3}\right)$	$\left(1,1,\frac{2}{3}\right)$ $\left(1,1,-\frac{1}{3}\right)$	$\left(1,1,\frac{2}{3}\right)$ $\left(1,1,-\frac{1}{3}\right)$
$l_{\alpha\mathrm{L}}=\begin{pmatrix}\nu_{\alpha\mathrm{L}}\\ e^-_{\alpha\mathrm{L}}\end{pmatrix}$	$\left(2,1,-\frac{1}{2}\right)$	$\left(2,1,-\frac{1}{2}\right)$	$\left(2,1,-\frac{1}{2}\right)$	$\left(2,1,-\frac{1}{2}\right)$	$\left(2,1,-\frac{1}{2}\right)$	$\left(2,1,-\frac{1}{2}\right)\delta_{\alpha\alpha_1}+\left(1,2,-\frac{1}{2}\right)\delta_{\alpha\alpha_2}$
$l_{\alpha\mathrm{R}}=\begin{pmatrix}\nu_{\alpha\mathrm{R}}\\ e^-_{\alpha\mathrm{R}}\end{pmatrix}$	$\left(1,2,-\frac{1}{2}\right)$	$-$ $(1,1,-1)$	$\left(1,2,-\frac{1}{2}\right)$	$-$ $(1,1,-1)$	$-$ $(1,1,-1)$	$-$ $(1,1,-1)$

The various models defined by the transformation properties of their fermion content with respect to the gauge group are summarized in Table 1. Since above fermions can belong to different representations for different underlying models, an universal expression to cover all these possible arrangements is needed. To reach this aim, we introduce two Goldstone operators $\hat{U}_\mathrm{L}$ and $\hat{U}_\mathrm{R}$ in the following way: If $f(\hat{U}_\mathrm{R},\hat{U}_\mathrm{L},D_\mu\hat{U}_\mathrm{R},D_\mu\hat{U}_\mathrm{L})$ is an arbitray function of $\hat{U}_\mathrm{R}$ and $\hat{U}_\mathrm{L}$ then define its acton on the fermion fields by

$$f(\hat{U}_\mathrm{R},\hat{U}_\mathrm{L},D_\mu\hat{U}_\mathrm{R},D_\mu\hat{U}_\mathrm{L})q_\alpha=\begin{cases}f(U_2,U_1,D_\mu U_2,D_\mu U_1)q_\alpha & \mathrm{LR}\\ f(U_2,U_1,D_\mu U_2,D_\mu U_1)q_\alpha & \mathrm{LP}\\ f(1,U_1,0,D_\mu U_1)q_\alpha & \mathrm{HP}\\ f(1,U_1,0,D_\mu U_1)q_\alpha & \mathrm{FP}\\ f(1,U_2,0,D_\mu U_2)q_\alpha & \mathrm{UN}\\ f(1,U_1,0,D_\mu U_1)q_\alpha\delta_{\alpha\alpha_1}+ & \\ f(1,U_2,0,D_\mu U_2)q_\alpha\delta_{\alpha\alpha_2} & \mathrm{NU}\end{cases}, \qquad (10)$$

$$f(\hat{U}_\mathrm{R},\hat{U}_\mathrm{L},D_\mu\hat{U}_\mathrm{R},D_\mu\hat{U}_\mathrm{L})l_\alpha=\begin{cases}f(U_2,U_1,D_\mu U_2,D_\mu U_1)l_\alpha & \mathrm{LR}\\ f(1,U_1,0,D_\mu U_1)l_\alpha & \mathrm{LP}\\ f(U_2,U_1,D_\mu U_2,D_\mu U_1)l_\alpha & \mathrm{HP}\\ f(1,U_1,0,D_\mu U_1)l_\alpha & \mathrm{FP}\\ f(1,U_1,0,D_\mu U_1)l_\alpha & \mathrm{UN}\\ f(1,U_1,0,D_\mu U_1)l_\alpha\delta_{\alpha\alpha_1}+ & \\ f(1,U_2,0,D_\mu U_2)l_\alpha\delta_{\alpha\alpha_2} & \mathrm{NU}\end{cases}.$$

In the case of the "Non-universality generation" mentioned above, α_1 denotes the specified generation (typically one of the first two generations) which acts as a doublet of $SU(2)_1$ and as a singlet of $SU(2)_2$ and α_2 denotes the remaining generation which acts as a doublet of $SU(2)_2$ and as a singlet of $SU(2)_1$.

With the help of above representation we can now write down the three dimensional universal Yukawa type interactions. For the lepton part we have

$$\mathcal{L}_{\mathrm{Y,lepton}}=\bar{l}^{\mathrm{I}}_{\alpha\mathrm{L}}[\hat{U}_\mathrm{L}(y^{\alpha\beta}+y_3^{\alpha\beta}\tau^3)\hat{U}^\dagger_\mathrm{R}]l^{\mathrm{I}}_{\beta\mathrm{R}}+$$

$$\frac{1}{2}[h_{\rm L}^{\alpha\beta}\overline{l_{\alpha\rm L}^{\rm Ic}}\hat{U}_{\rm L}^{*}(1+\tau^3)\hat{U}_{\rm L}^{\dagger}l_{\beta\rm L}^{\rm I}+ (L\rightarrow R)]+{\rm h.c.}\ , \quad (11)$$

where $h_{\rm L,R}^{\alpha\beta}$ are hermitian functions of the Higgs field h. $l^{\rm c}=C\bar{l}^{\rm T}$ is the charge conjugate field of l with C being the charge conjugation matrix. The symbol "I" indicates that they are gauge eigenstates. For quarks part we get

$$\mathcal{L}_{\rm Y,quark}=\overline{q}_{\alpha\rm L}^{\rm I}[\hat{U}_{\rm L}(\tau^{\rm u}y_{\rm u}^{\alpha\beta}+\tau^{\rm d}y_{\rm d}^{\alpha\beta})\hat{U}_{\rm R}^{\dagger}]q_{\beta\rm R}^{\rm I}+{\rm h.c.}\ , \quad (12)$$

where $\tau^{\rm u}=\dfrac{1+\tau^3}{2}$ and $\tau^{\rm d}=\dfrac{1-\tau^3}{2}$. The coefficients $y_{\rm u}^{\alpha\beta}$, $y_{\rm d}^{\alpha\beta}$ are functions of the Higgs field.

The next contribution is the four dimensional gauge interaction part of the Lagrangian

$$\begin{aligned}\mathcal{L}_{f-4}={}&{\rm i}\sum_{\alpha}\Big\{\overline{q}_{\alpha\rm L}^{\rm I}\not{D}q_{\alpha\rm L}^{\rm I}+\delta_{\rm L,1,\alpha}\overline{q}_{\alpha\rm L}^{\rm I}\hat{U}_{\rm L}(\not{D}\hat{U}_{\rm L})^{\dagger}q_{\alpha\rm L}^{\rm I}+\delta_{\rm L,2,\alpha}\overline{q}_{\alpha\rm R}^{\rm I}\hat{U}_{\rm R}\hat{U}_{\rm L}^{\dagger}(\not{D}\hat{U}_{\rm L})\hat{U}_{\rm R}^{\dagger}q_{\alpha\rm R}^{\rm I}+\\&\delta_{\rm L,3,\alpha}\overline{q}_{\alpha\rm L}^{\rm I}[(\not{D}\hat{U}_{\rm L})\tau^3\hat{U}_{\rm L}^{\dagger}-\hat{U}_{\rm L}\tau^3(\not{D}\hat{U}_{\rm L})^{\dagger}]q_{\alpha\rm L}^{\rm I}+\delta_{\rm L,4,\alpha}\overline{q}_{\alpha\rm L}^{\rm I}\hat{U}_{\rm L}\tau^3\hat{U}_{\rm L}^{\dagger}(\not{D}\hat{U}_{\rm L})\tau^3\hat{U}_{\rm L}^{\dagger}q_{\alpha\rm L}^{\rm I}+\\&\delta_{\rm L,5,\alpha}\overline{q}_{\alpha\rm R}^{\rm I}\hat{U}_{\rm R}[\tau^3\hat{U}_{\rm L}^{\dagger}(\not{D}\hat{U}_{\rm L})-(\not{D}\hat{U}_{\rm L})^{\dagger}\hat{U}_{\rm L}\tau^3]\hat{U}_{\rm R}^{\dagger}q_{\alpha\rm R}^{\rm I}+\delta_{\rm L,6,\alpha}\overline{q}_{\alpha\rm R}^{\rm I}\hat{U}_{\rm R}\tau^3\hat{U}_{\rm L}^{\dagger}(\not{D}\hat{U}_{\rm L})\tau^3\hat{U}_{\rm R}^{\dagger}q_{\alpha\rm R}^{\rm I}+\\&\delta_{\rm L,7,\alpha}[\overline{q}_{\alpha\rm L}^{\rm I}\hat{U}_{\rm L}\tau^3\hat{U}_{\rm L}^{\dagger}\not{D}q_{\alpha\rm L}^{\rm I}-(\overline{q}_{\alpha\rm L}^{\rm I}\not{D}^{\dagger})\hat{U}_{\rm L}\tau^3\hat{U}_{\rm L}^{\dagger}q_{\alpha\rm L}^{\rm I}]\Big\}+q^{\rm I}\rightarrow l^{\rm I},\delta\rightarrow\delta^{\rm l}+{\rm L}\leftrightarrow{\rm R}\,,\end{aligned}\quad (13)$$

in which

$$D_{\mu}q_{\alpha}=\begin{cases}(\partial_{\mu}+{\rm i}g_1\frac{\tau^{\rm a}}{2}W_{1,\mu}^{\rm a}P_{\rm L}+{\rm i}g_2\frac{\tau^{\rm a}}{2}W_{2,\mu}^{\rm a}P_{\rm R}+\frac{\rm i}{6}gB_{\mu})q_{\alpha} & \text{LR, LP}\\(\partial_{\mu}+{\rm i}g_1\frac{\tau^{\rm a}}{2}W_{1,\mu}^{\rm a}P_{\rm L}+{\rm i}g\frac{\tau^3}{2}B_{\mu}P_{\rm R}+\frac{\rm i}{6}gB_{\mu})q_{\alpha} & \text{HP, FP}\\(\partial_{\mu}+{\rm i}g_2\frac{\tau^{\rm a}}{2}W_{2,\mu}^{\rm a}P_{\rm L}+{\rm i}g\frac{\tau^3}{2}B_{\mu}P_{\rm R}+\frac{\rm i}{6}gB_{\mu})q_{\alpha} & \text{UN}\\(\partial_{\mu}+{\rm i}\delta_{\alpha\alpha_1}g_1\frac{\tau^{\rm a}}{2}W_{1,\mu}^{\rm a}P_{\rm L}+{\rm i}\delta_{\alpha\alpha_2}g_2\frac{\tau^{\rm a}}{2}W_{2,\mu}^{\rm a}P_{\rm L}+{\rm i}g\frac{\tau^3}{2}B_{\mu}P_{\rm R}+\frac{\rm i}{6}gB_{\mu})q_{\alpha} & \text{NU}\end{cases}, \quad (14)$$

$$D_{\mu}l_{\alpha}=\begin{cases}(\partial_{\mu}+{\rm i}g_1\frac{\tau^{\rm a}}{2}W_{1,\mu}^{\rm a}P_{\rm L}+{\rm i}g_2\frac{\tau^{\rm a}}{2}W_{2,\mu}^{\rm a}P_{\rm R}-\frac{\rm i}{2}gB_{\mu})l_{\alpha} & \text{LR, HP}\\(\partial_{\mu}+{\rm i}g_1\frac{\tau^{\rm a}}{2}W_{1,\mu}^{\rm a}P_{\rm L}+{\rm i}g\frac{\tau^3}{2}B_{\mu}P_{\rm R}-\frac{\rm i}{2}gB_{\mu})l_{\alpha} & \text{LP, FP, UN}\\(\partial_{\mu}+{\rm i}\delta_{\alpha\alpha_1}g_1\frac{\tau^{\rm a}}{2}W_{1,\mu}^{\rm a}P_{\rm L}+{\rm i}\delta_{\alpha\alpha_2}g_2\frac{\tau^{\rm a}}{2}W_{2,\mu}^{\rm a}P_{\rm L}+{\rm i}g\frac{\tau^3}{2}B_{\mu}P_{\rm R}-\frac{\rm i}{2}gB_{\mu})l_{\alpha} & \text{NU}\end{cases},$$

and $P_{\rm \frac{R}{L}}=(1\pm\gamma_5)/2$. $(\not{D}\hat{U}_{\rm i})^{\dagger}\equiv\gamma^{\mu}(D_{\mu}\hat{U}_{\rm i})^{\dagger}$ for ${\rm i}={\rm L,R}$. The coefficients in Eq. (13) δ and $\delta^{\rm l}$ depend in general on the generation indices, which was not considered in the original LR case in Ref. [17].

6 Phenomenologies

Based on the Lagrangians Eqs. (12) and (13), we are able to search for some phenomenological constraints. As a preparation, we first discuss the mixing among quarks induced from Eq. (12) which in unitary gauge is

$$\mathcal{L}_{\rm Y,quark}\Big|_{\rm Unitary\ gauge}=\overline{q}_{\alpha\rm L}^{\rm I}(\tau^{\rm u}y_{\rm u}^{\alpha\beta}+\tau^{\rm d}y_{\rm d}^{\alpha\beta})q_{\beta\rm R}^{\rm I}+{\rm h.c.}\ . \quad (15)$$

The gauge eigenstates can be rotated into the mass eigenstates with unitary matrices $V_{\rm L,R}^{\rm u,d}$,

$$u_{\rm L,R}=V_{\rm L,R}^{\rm u}u_{\rm L,R}^{\rm I}\quad d_{\rm L,R}=V_{\rm L,R}^{\rm d}d_{\rm L,R}^{\rm I}\,. \quad (16)$$

The $y_{\rm u,d}$ matrices introduced in Eq. (15) are diagonalized as follows:

$$V_{\rm L}^{\rm u}y_{\rm u}V_{\rm R}^{\rm u\dagger}=M_{\rm diag}^{\rm u},\quad V_{\rm L}^{\rm d}y_{\rm d}V_{\rm R}^{\rm d\dagger}=M_{\rm diag}^{\rm d}, \quad (17)$$

where $M_{\rm diag}^{\rm u,d}$ represent the diagonal up- and down-quark mass matrices of physical quark masses.

$$q_{\alpha\rm L,R}=\begin{pmatrix}u_{\alpha\rm L,R}\\d_{\alpha\rm L,R}\end{pmatrix}= [(V_{\rm L,R}^{\rm u})_{\alpha\beta}\tau^{\rm u}+(V_{\rm L,R}^{\rm d})_{\alpha\beta}\tau^{\rm d}]\begin{pmatrix}u_{\beta\rm L,R}^{\rm I}\\d_{\beta\rm L,R}^{\rm I}\end{pmatrix}. \quad (18)$$

$$(V_{\rm L}^{\rm u}\tau^{\rm u}+V_{\rm L}^{\rm d}\tau^{\rm d})(\tau^{\rm u}y_{\rm u}^0+\tau^{\rm d}y_{\rm d}^0)(V_{\rm R}^{\rm u\dagger}\tau^{\rm u}+V_{\rm L,R}^{\rm d\dagger}\tau^{\rm d})= (\tau^{\rm u}M_{\rm diag}^{\rm u}+\tau^{\rm d}M_{\rm diag}^{\rm d}). \quad (19)$$

The usual Cabibbo-Kobayashi-Maskawa (CKM) matrix in the left sector, and the corresponding matrix in the right sector, are given by

$$V_{\rm L,R}^{\rm CKM}=V_{\rm L,R}^{\rm u}V_{\rm L,R}^{\rm d\dagger}. \quad (20)$$

Note that, a priori, there is no reason for $V_{\rm L}^{\rm CKM}$ to be equal to $V_{\rm R}^{\rm CKM}$.

Any $n\times n$ unitary matrix has n^2 real parameters among which $n(n-1)/2$ may be expressed in the form of $\sin\theta_{\alpha\beta}$, $\cos\theta_{\alpha\beta}$ with $n^2-n(n-1)/2=n(n+1)/2$ phases left. Since each quark field can be redefined through a phase transformation, $2n-1$ phases are not physical. If $V_{\rm L}^{\rm CKM}$ and $V_{\rm R}^{\rm CKM}$ are independent, the total number of physical phases is $2\times\frac{n(n+1)}{2}-(2n-1)=n^2-n+1$. In our case of 3 generations of fermions, $V_{\rm L}^{\rm CKM}$ can be taken as the standard form [18],

$$V_{\rm L}^{\rm CKM}=\begin{pmatrix} V_{\rm L}^{\rm ud} & V_{\rm L}^{\rm us} & V_{\rm L}^{\rm ub}\\ V_{\rm L}^{\rm cd} & V_{\rm L}^{\rm cs} & V_{\rm L}^{\rm cb}\\ V_{\rm L}^{\rm td} & V_{\rm L}^{\rm ts} & V_{\rm L}^{\rm tb}\end{pmatrix}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}{\rm e}^{-{\rm i}\delta}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}{\rm e}^{{\rm i}\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}{\rm e}^{{\rm i}\delta} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}{\rm e}^{{\rm i}\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}{\rm e}^{{\rm i}\delta} & c_{23}c_{13}\end{pmatrix}. \tag{21}$$

Then the most general $V_{\rm R}^{\rm CKM}$ may have the form of the standard CKM matrix with 5 phases added:

$$V_{\rm R}^{\rm CKM}=\begin{pmatrix} \bar V_{\rm R}^{\rm ud}{\rm e}^{2{\rm i}\alpha_1} & \bar V_{\rm R}^{\rm us}{\rm e}^{{\rm i}(\alpha_1+\alpha_2+\beta_1)} & \bar V_{\rm R}^{\rm ub}{\rm e}^{{\rm i}(\alpha_1+\alpha_3+\beta_1+\beta_2)}\\ \bar V_{\rm R}^{\rm cd}{\rm e}^{{\rm i}(\alpha_1+\alpha_2-\beta_1)} & \bar V_{\rm R}^{\rm cs}{\rm e}^{2{\rm i}\alpha_2} & \bar V_{\rm R}^{\rm cb}{\rm e}^{{\rm i}(\alpha_2+\alpha_3+\beta_2)}\\ \bar V_{\rm R}^{\rm td}{\rm e}^{{\rm i}(\alpha_1+\alpha_3-\beta_1-\beta_2)} & \bar V_{\rm R}^{\rm ts}{\rm e}^{{\rm i}(\alpha_2+\alpha_3-\beta_2)} & \bar V_{\rm R}^{\rm tb}{\rm e}^{2{\rm i}\alpha_3}\end{pmatrix}, \tag{22}$$

where

$$\begin{pmatrix} \bar V_{\rm R}^{\rm ud} & \bar V_{\rm R}^{\rm us} & \bar V_{\rm R}^{\rm ub}\\ \bar V_{\rm R}^{\rm cd} & \bar V_{\rm R}^{\rm cs} & \bar V_{\rm R}^{\rm cb}\\ \bar V_{\rm R}^{\rm td} & \bar V_{\rm R}^{\rm ts} & \bar V_{\rm R}^{\rm tb}\end{pmatrix}=\begin{pmatrix} \bar c_{12}\bar c_{13} & \bar s_{12}\bar c_{13} & \bar s_{13}{\rm e}^{-{\rm i}\bar\delta}\\ -\bar s_{12}\bar c_{23}-\bar c_{12}\bar s_{23}\bar s_{13}{\rm e}^{{\rm i}\bar\delta} & \bar c_{12}\bar c_{23}-\bar s_{12}\bar s_{23}\bar s_{13}{\rm e}^{{\rm i}\bar\delta} & \bar s_{23}\bar c_{13}\\ \bar s_{12}\bar s_{23}-\bar c_{12}\bar c_{23}\bar s_{13}{\rm e}^{{\rm i}\bar\delta} & -\bar c_{12}\bar s_{23}-\bar s_{12}\bar c_{23}\bar s_{13}{\rm e}^{{\rm i}\bar\delta} & \bar c_{23}\bar c_{13}\end{pmatrix}, \tag{23}$$

with $\bar c_{12}=\cos\bar\theta_{12}$, $\bar s_{12}=\sin\bar\theta_{12}$, etc. In general, $\bar\theta_{\alpha\beta}$ are not equal to those in $V_{\rm L}^{\rm CKM}$. If $\bar V_{\rm R}^{\alpha\beta}=(V_{\rm L}^{\alpha\beta})^*$ holds for $\alpha={\rm u,c,t}$ and $\beta={\rm d,s,b}$, then $V_{\rm R}^{\rm CKM}$ of Eq. (22) coincides with that given in Refs. [19, 20], which is called pseudo-manifest left-right symmetric, and is originally proposed to construct left-right symmetric models with spontaneously CP violation.

With the above given preparations we are ready to discuss the phenomenology. Notice that once there exists a W′ boson, there may be low energy phenomenological constraints from the $\rm K^0-\bar K^0$, $\rm B_d^0-\bar B_d^0$ and $\rm B_s^0-\bar B_s^0$ systems. In most cases W′ will generate extra Feynman box diagrams which contribute to the mass differences in the $\rm K^0-\bar K^0$, $\rm B_d^0-\bar B_d^0$, $\rm B_s^0-\bar B_s^0$ systems and the corresponding CP violation parameters. We will concentrate on the constraints on our EWCL coming from the mass differences in the $\rm K^0-\bar K^0$, $\rm B_d^0-\bar B_d^0$, $\rm B_s^0-\bar B_s^0$ systems, $\Delta m_{\rm K},\Delta m_{\rm B_d},\Delta m_{\rm B_s}$ and the indirect influence on the CP violation parameter $|\epsilon_{\rm K}|$, mainly for the LR and LP models. The Feynman diagrams responsible for these processes are drawn in Fig. 2. We can explicitly decompose the contributions into those related to W and its corresponding Goldstone boson ϕ_1 and those related to W′ and its corresponding Goldstone boson ϕ_2 as given in the following equation.

$$\Delta m_{\rm K}=\Delta m_{\rm K}^{\rm WW}+\Delta m_{\rm K}^{\rm WW'},$$

$$|\epsilon_{\rm K}|=|\epsilon_{\rm K}|^{\rm WW}+|\epsilon_{\rm K}|^{\rm WW'},$$

$$\Delta m_{\rm B_d}=\Delta m_{\rm B_d}^{\rm WW}+\Delta m_{\rm B_d}^{\rm WW'},$$

$$\Delta m_{\rm B_s}=\Delta m_{\rm B_s}^{\rm WW}+\Delta m_{\rm B_s}^{\rm WW'}.$$

Ignoring the details of the calculation which is already given in Ref. [21], the results for the conventional electroweak gauge boson W and the corresponding Goldstone boson ϕ_1 contributions $\Delta m_{\rm K}^{\rm WW}$, $\Delta m_{\rm B_d}^{\rm WW}$, $\Delta m_{\rm B_s}^{\rm WW}$ and $|\epsilon_{\rm K}|^{\rm WW}$ are drawn in Fig. 3. In Fig. 3 $\varDelta_{1,1}$ is the anomalous coupling for the charged current which is related to the anomalous couplings introduced in Eq. (13) by $\varDelta_{1,1,\alpha}=1-\delta_{{\rm L},1,\alpha}-\delta_{{\rm L},4,\alpha}$ for the case of LR, LP, HP and FP. The SM theoretical results correspond to $\varDelta_{1,1}=1$. We find that except for $\Delta m_{\rm K}$, the SM results match the experimental values for $|\epsilon_{\rm K}|$, $\Delta m_{\rm B_d}$ and $\Delta m_{\rm B_s}$ within 23%, with errors being expected from the uncertainty of the matrix elements. For $\Delta m_{\rm K}$ the error is roughly 33%, which comes mainly from the long distance contributions [22].

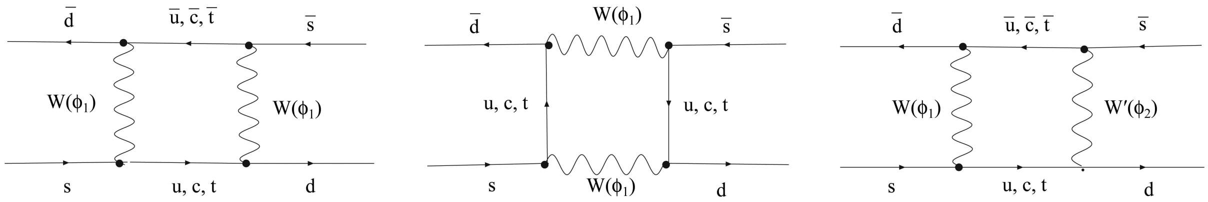

Fig. 2. Feynman diagrams for $\Delta m_{\mathrm{K}}, \Delta m_{\mathrm{B_d}}, \Delta m_{\mathrm{B_s}}$ and $|\epsilon_{\mathrm{K}}|$.

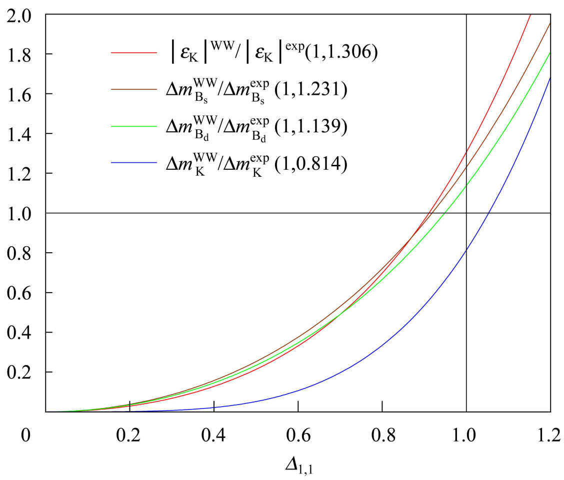

Fig. 3. Pure W contributions to the $\mathrm{K}^0-\bar{\mathrm{K}}^0$, $\mathrm{B}^0_{\mathrm{d}}-\bar{\mathrm{B}}^0_{\mathrm{d}}$, $\mathrm{B}^0_{\mathrm{s}}-\bar{\mathrm{B}}^0_{\mathrm{s}}$ systems and indirect contribution to *CP* violation in K mesons. The anomalous coupling $\Delta_{1,1}$, correspongding to the SM result, $\Delta_{1,1}=1$ is explicitly quoted in brackets.

For $\Delta m_{\mathrm{K}}^{\mathrm{WW'}}$, $\Delta m_{\mathrm{B_d}}^{\mathrm{WW'}}$, $\Delta m_{\mathrm{B_s}}^{\mathrm{WW'}}$ and $|\epsilon_{\mathrm{K}}|^{\mathrm{WW'}}$, the contributions from the new gauge boson W′ and the corresponding Goldstone boson ϕ_2, involve loops of the top and charm quarks with the CKM matrix elements as effective couplings. These couplings are

$$\lambda_{\mathrm{x}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{x}}^{\mathrm{RL}}(\mathrm{K})= |V_{\mathrm{L}}^{\mathrm{xs}}V_{\mathrm{L}}^{\mathrm{xd}*}\bar{V}_{\mathrm{R}}^{\mathrm{xs}}\bar{V}_{\mathrm{R}}^{\mathrm{xd}*}|\mathrm{e}^{-\mathrm{i}(\alpha_1-\alpha_2-\beta_1-\phi_{\mathrm{xs}}-\bar{\phi}_{\mathrm{xs}}+\phi_{\mathrm{xd}}+\bar{\phi}_{\mathrm{xd}})} \quad \mathrm{x}=\mathrm{c},\mathrm{t},$$

$$\lambda_{\mathrm{c}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{t}}^{\mathrm{RL}}(\mathrm{K})= |V_{\mathrm{L}}^{\mathrm{cs}}\bar{V}_{\mathrm{R}}^{\mathrm{cd}*}\bar{V}_{\mathrm{R}}^{\mathrm{ts}}V_{\mathrm{L}}^{\mathrm{td}*}|\mathrm{e}^{-\mathrm{i}(\alpha_1-\alpha_3-\beta_1+\beta_2-\phi_{\mathrm{cs}}+\bar{\phi}_{\mathrm{cd}}-\bar{\phi}_{\mathrm{ts}}+\phi_{\mathrm{td}})} \quad \arg(V_{\mathrm{L}}^{\alpha\beta})=\phi_{\alpha\beta},$$

$$\lambda_{\mathrm{t}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{c}}^{\mathrm{RL}}(\mathrm{K})= |V_{\mathrm{L}}^{\mathrm{ts}}\bar{V}_{\mathrm{R}}^{\mathrm{td}*}\bar{V}_{\mathrm{R}}^{\mathrm{cs}}V_{\mathrm{L}}^{\mathrm{cd}*}|\mathrm{e}^{-\mathrm{i}(\alpha_1-2\alpha_2+\alpha_3-\beta_1-\beta_2-\phi_{\mathrm{ts}}+\bar{\phi}_{\mathrm{td}}-\bar{\phi}_{\mathrm{cs}}+\phi_{\mathrm{cd}})} \quad \arg(\bar{V}_{\mathrm{R}}^{\alpha\beta})=\bar{\phi}_{\alpha\beta}. \quad (24)$$

In order to make quantitative estimations, we limit ourselves to the case of a special pseudo-manifest left-right symmetric situation. In this situation, $\bar{V}_{\mathrm{R}}^{\alpha\beta}=(V_{\mathrm{L}}^{\alpha\beta})^*$, which implies the relations $\phi_{\alpha\beta}=-\bar{\phi}_{\alpha\beta}$ between the phases defined in Eq. (24). Then the CKM factors can be simplified to

$$\lambda_{\mathrm{c}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{c}}^{\mathrm{RL}}(\mathrm{K})=|V_{\mathrm{L}}^{\mathrm{cs}}V_{\mathrm{L}}^{\mathrm{cd}}|^2\mathrm{e}^{-\mathrm{i}(\alpha_1-\alpha_2-\beta_1)}$$
$$\lambda_{\mathrm{t}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{t}}^{\mathrm{RL}}(\mathrm{K})=|V_{\mathrm{L}}^{\mathrm{ts}}V_{\mathrm{L}}^{\mathrm{td}}|^2\mathrm{e}^{-\mathrm{i}(\alpha_1-\alpha_2-\beta_1)}\ , \quad (25)$$

$$\lambda_{\mathrm{c}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{t}}^{\mathrm{RL}}(\mathrm{K})+\lambda_{\mathrm{t}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{c}}^{\mathrm{RL}}(\mathrm{K})= 2|V_{\mathrm{L}}^{\mathrm{cs}}V_{\mathrm{L}}^{\mathrm{cd}}V_{\mathrm{L}}^{\mathrm{ts}}V_{\mathrm{L}}^{\mathrm{td}}|[\cos(\alpha_1-\alpha_2-\beta_1)\cos(\alpha_2-\alpha_3+\beta_2)- \mathrm{i}\sin(\alpha_1-\alpha_2-\beta_1)\cos(\alpha_2-\alpha_3+\beta_2)].$$

A further constraint on $|\epsilon_{\mathrm{K}}|$ leads to the choice of phase angles $\alpha_1-\alpha_2-\beta_1=0$ which results in

$$\mathrm{Im}[\lambda_{\mathrm{c}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{c}}^{\mathrm{RL}}(\mathrm{K})]=\mathrm{Im}[\lambda_{\mathrm{t}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{t}}^{\mathrm{RL}}(\mathrm{K})]= \mathrm{Im}[\lambda_{\mathrm{c}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{t}}^{\mathrm{RL}}(\mathrm{K})+\lambda_{\mathrm{t}}^{\mathrm{LR}}(\mathrm{K})\lambda_{\mathrm{c}}^{\mathrm{RL}}(\mathrm{K})]=0, \quad (26)$$

so that the cc loop, tt loop and ct loop do not have individual *CP* violations. One has then

$$|\epsilon_{\mathrm{K}}|^{\mathrm{WW'}}=0. \quad (27)$$

The qualitative estimations for $\Delta m_{\mathrm{K}}^{\mathrm{WW'}}$, $\Delta m_{\mathrm{B_d}}^{\mathrm{WW'}}$, $\Delta m_{\mathrm{B_s}}^{\mathrm{WW'}}$ are

$$\frac{\Delta m_{\mathrm{K}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{K}}^{\mathrm{exp}}}=\Delta_{2,1}^2\frac{g_2^2}{g_1^2}\underbrace{\frac{\Delta m_{\mathrm{K_{tt}}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{K}}^{\mathrm{exp}}}}_{-10^5}\Bigg[\mathrm{Re}(\underbrace{\lambda_{\mathrm{t}}^{\mathrm{LR}}\lambda_{\mathrm{t}}^{\mathrm{RL}}}_{5.9\times10^{-6}})+ \mathrm{Re}(\underbrace{\lambda_{\mathrm{c}}^{\mathrm{LR}}\lambda_{\mathrm{c}}^{\mathrm{RL}}}_{0.049})\underbrace{\frac{\Delta m_{\mathrm{K_{cc}}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{K_{tt}}}^{\mathrm{WW'}}}}_{10^{-3}}+ \mathrm{Re}(\underbrace{\lambda_{\mathrm{c}}^{\mathrm{LR}}\lambda_{\mathrm{t}}^{\mathrm{RL}}+\lambda_{\mathrm{t}}^{\mathrm{LR}}\lambda_{\mathrm{c}}^{\mathrm{RL}}}_{0.0011\times\cos(\alpha_2-\alpha_3+\beta_2)})\underbrace{\frac{\Delta m_{\mathrm{K_{ct}}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{K_{tt}}}^{\mathrm{WW'}}}}_{10^{-2}}\Bigg], \quad (28)$$

$$\frac{\Delta m_{\mathrm{B_q}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{B_q}}^{\mathrm{exp}}}=\Delta_{2,1}^2\frac{g_2^2}{g_1^2}\underbrace{\frac{\Delta m_{\mathrm{B_{q_{tt}}}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{B_q}}^{\mathrm{exp}}}}_{-10^5\,(\mathrm{q=d});-10^4(\mathrm{q=s})}\left|\underbrace{\lambda_{\mathrm{t}}^{\mathrm{LR}}\lambda_{\mathrm{t}}^{\mathrm{RL}}}_{5.9\times10^{-6}}+\underbrace{\lambda_{\mathrm{c}}^{\mathrm{LR}}\lambda_{\mathrm{c}}^{\mathrm{RL}}}_{0.049}\underbrace{\frac{\Delta m_{\mathrm{B_{q_{cc}}}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{B_{q_{tt}}}}^{\mathrm{WW'}}}}_{10^{-3}}+\underbrace{\lambda_{\mathrm{c}}^{\mathrm{LR}}\lambda_{\mathrm{t}}^{\mathrm{RL}}+\lambda_{\mathrm{t}}^{\mathrm{LR}}\lambda_{\mathrm{c}}^{\mathrm{RL}}}_{0.0011\times\cos(\alpha_2-\alpha_3+\beta_2)}\underbrace{\frac{\Delta m_{\mathrm{B_{q_{ct}}}}^{\mathrm{WW'}}}{\Delta m_{\mathrm{B_{q_{tt}}}}^{\mathrm{WW'}}}}_{10^{-2}}\right|, \quad \mathrm{q}=\mathrm{d},\mathrm{s} \quad (29)$$

where $\varDelta_{2,1}$ is related to the anomalous couplings introduced in Eq. (13) by $\varDelta_{2,1,\alpha}=1-\delta_{\mathrm{R},1,\alpha}-\delta_{\mathrm{R},4,\alpha}$ for the case of LR, LP and $\varDelta_{2,1,\alpha}=\delta_{\mathrm{L},2,\alpha}-\delta_{\mathrm{L},6,\alpha}$ for the case of HP, FP.

7 Summary

To summarize, minimal and next to minimal coupling new force carriers beyond the SM are the Z′ and $\mathrm{W}^{\pm,\prime}$,Z′ particles. We have constructed the corresponding EWCLs. For the phenomenological observables Δm_{K}, $|\epsilon_{\mathrm{K}}|$, $\Delta m_{\mathrm{B_d}}$, $\Delta m_{\mathrm{B_s}}$, we find that

1) For Δm_{K}, the contribution from W′ increases the difference between the theoretical result and experiment.

2) For $|\epsilon_{\mathrm{K}}|$, $\Delta m_{\mathrm{B_d}}$, $\Delta m_{\mathrm{B_s}}$, the contributions from W′ reduce the difference between the theoretical result and experiment.

3) For $|\epsilon_{\mathrm{K}}|$, in the case of a pseudo-manifest LR symmetry, W′ makes no contribution.

In order that the contributions from W′ to Δm_{K}, $|\epsilon_{\mathrm{K}}|$, $\Delta m_{\mathrm{B_d}}$, $\Delta m_{\mathrm{B_s}}$ do not conflict with the experimental data, we are left with several possible options:

1) $M_{\mathrm{W}'}\gg M_{\mathrm{W}}$.

2) $g_2\ll g_1$.

3) $\varDelta_{2,1}\ll 1$.

4) A special choice of a right handed CKM matrix

These possibilities need further experimental inspirations and future investigations.

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