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Structure of the even-even $^{78-84}$Kr isotopes within *SD*-pair shell model*

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Abstract The collective properties in the even-even $^{78-84}$Kr isotopes have been studied within the framework of the *SD*-pair shell model. It is found that the collectivity of low-lying states in the even-even Kr isotopes can be described very well.

Key words *SD*-pair shell model, spectrum, E2 transition

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1 Introduction

In the past two decades even-even Kr isotopes have been the objects of considerable experimental and theoretical attention due to the evolution of our understanding of nuclear structure [1–6]. Detailed theoretical investigations [2–4] have mainly been performed in the framework of the interacting boson model (IBM) [7]. The results show that the excitation energy, E2 and M1 transitions can be reproduced very well within the IBM. Especially the 0_2^+ state, which is particularly interesting to clarify the contribution of the different excitation mechanisms at low energy [2, 3, 5], can also be fitted very well.

Since the *SD*-pair shell model (SDPSM) [8–10] can describe the properties of low-lying states in even-even Xe, Ba,Ce and Mo isotopes [9, 11–18], it is interesting to see whether the properties in the Kr isotopes can be reproduced within the SDPSM, and this is the aim of this paper.

2 A brief review of the model

To this end, a Hamiltonian as in Refs. [15, 17, 18] is used, which is

$$H = H_\nu + H_\pi + \kappa Q_\pi^{(2)} \cdot Q_\nu^{(2)}, \tag{1}$$

$$H_\sigma = H_0 - G_{0\sigma}\mathcal{S}_\sigma{}^\dagger \mathcal{S}_\sigma - G_{2\sigma} P_\sigma^{(2)\dagger} P_\sigma^{(2)} - \kappa_\sigma Q_\sigma^{(2)} \cdot Q_\sigma^{(2)}, \tag{2}$$

$$H_0 = \sum_{a\sigma} \epsilon_{a\sigma} n_{a\sigma}, \tag{3}$$

$$\mathcal{S}_\sigma^\dagger = \sum_a \frac{\sqrt{2j+1}}{2} (C_{a\sigma}^\dagger \times C_{a\sigma}^\dagger)^{(0)},$$

$$P_\mu^{(2)\dagger} = \sum_{ab} Q_{ab} (C_{a\sigma}^\dagger \times C_{b\sigma}^\dagger)_\mu^{(2)},$$

$$Q_\mu^{(2)} = \sum_{i=1}^{n} r_i^2 Y_{2\mu}(\theta_i \phi_i), \tag{4}$$

and its second quantized form is given by

$$Q_\mu^{(2)} = \sum_{cd} q(cd2) P_\mu^2(cd), \tag{5}$$

$$q(cd2) = (-)^{c-\frac{1}{2}} \frac{\widehat{c}\widehat{d}}{\sqrt{20\pi}} C^{2\ 0}_{c\frac{1}{2},d-\frac{1}{2}} \varDelta_{cd2} \langle Nl_c | r^2 | Nl_d \rangle, \tag{6}$$

$$P_\mu^t(cd) = \left(C_c^\dagger \times \tilde{C}_d\right)_\mu^{\mathrm{t}}, \tag{7}$$

where N is the principal quantum number of the harmonic oscillator wave function, such that the energy is $(N+3/2)\hbar\omega_0$. The matrix elements for r^2 are given as

$$\langle Nl_c | r^2 | Nl_d \rangle = \begin{cases} (N+3/2) r_0^2, & l_c = l_d, \\ \varphi[(N+l_d+2\pm1)(N-l_d+1\mp1)]^{1/2} r_0^2, & l_c = l_d \pm 2, \end{cases} \tag{8}$$

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where the phase factor φ can be taken either as -1 or $+1$, and $r_0^2 = \dfrac{\hbar}{m_{\rm p}\omega_0} = 1.012 A^{1/3}$ fm^2.

The building blocks of the SDPSM are "realistic" collective pairs $A_\mu^{r\dagger}$ of angular momentum $r = 0, 2$ with projection μ, built from many non-collective pairs $\left(C_a^\dagger \times C_b^\dagger\right)_\mu^r$ in the single-particle levels a and b,

$$A_\mu^{r\dagger} = \sum_{ab} y(abr) \left(C_a^\dagger \times C_b^\dagger\right)_\mu^r, \tag{9}$$

$$y(abr) = -\theta(abr) y(bar), \quad \theta(abr) = (-)^{a+b+r},$$

where $y(abr)$ are the distribution coefficients. How to determine these "realistic" S-D pairs is an important question. The structures of the S and D pairs depend on the Hamiltonian. Different S-D pairs represent different truncations. In this paper the S-pair is fixed by the BCS method [19, 20]. Namely, for fixed $G_{0\nu}$ and $G_{0\pi}$, $u_{\rm a}$ and $\upsilon_{\rm a}$ are obtained by

$$H_\sigma = H_{0\sigma} + G_{0\sigma} \mathcal{S}^\dagger(\sigma) \mathcal{S}(\sigma), \tag{10}$$

and then the S-pair structure is fixed as

$$S^\dagger = \sum_a y(aa0)(C_a^\dagger \times C_a^\dagger)^0, \quad y(aa0) = \hat{a} \frac{\upsilon_a}{u_a} .$$

The D-pair is obtained by using the commutator [21],

$$D^\dagger = \frac{1}{2}[Q^2, S^\dagger] = \sum_{ab} y(ab2)(C_a^\dagger \times C_b^\dagger)^2, \tag{11}$$

$$y(ab2) = -\frac{1}{2} q(ab2) \left[\frac{y(aa0)}{\hat{a}} + \frac{y(bb0)}{\hat{b}}\right]. \tag{12}$$

The E2 and M1 transition operators are

$$T({\rm E2}) = e_\pi Q_\pi^{(2)} + e_\nu Q_\nu^{(2)},$$
$$T({\rm M1}) = T({\rm M1})_\pi + T({\rm M1})_\nu,$$
$$T({\rm M1})_\sigma = \sqrt{\frac{3}{4\pi}} (g_{l\sigma} L + g_{s\sigma} S), \tag{13}$$

where e_π and e_ν are effective charges of the protons and neutrons, $g_{l\sigma}$ and $g_{s\sigma}$ are the orbital and spin effective gyro-magnetic ratios.

In this work $Z_{\rm core} = 28$ and $N_{\rm core} = 50$ are taken as the cores, and protons(neutrons) are treated as particles(holes) with respect to those cores. The single particle energy levels for protons are set to be 1.11, 0, 0.77 and 2.5 (in units of MeV) for $p_{1/2}$, $p_{3/2}$, $f_{5/2}$ and $g_{9/2}$, respectively. Since the single-particle energies for neutron holes can not be determined experimentally, the same set of energy levels as those of the protons are used for simplicity.

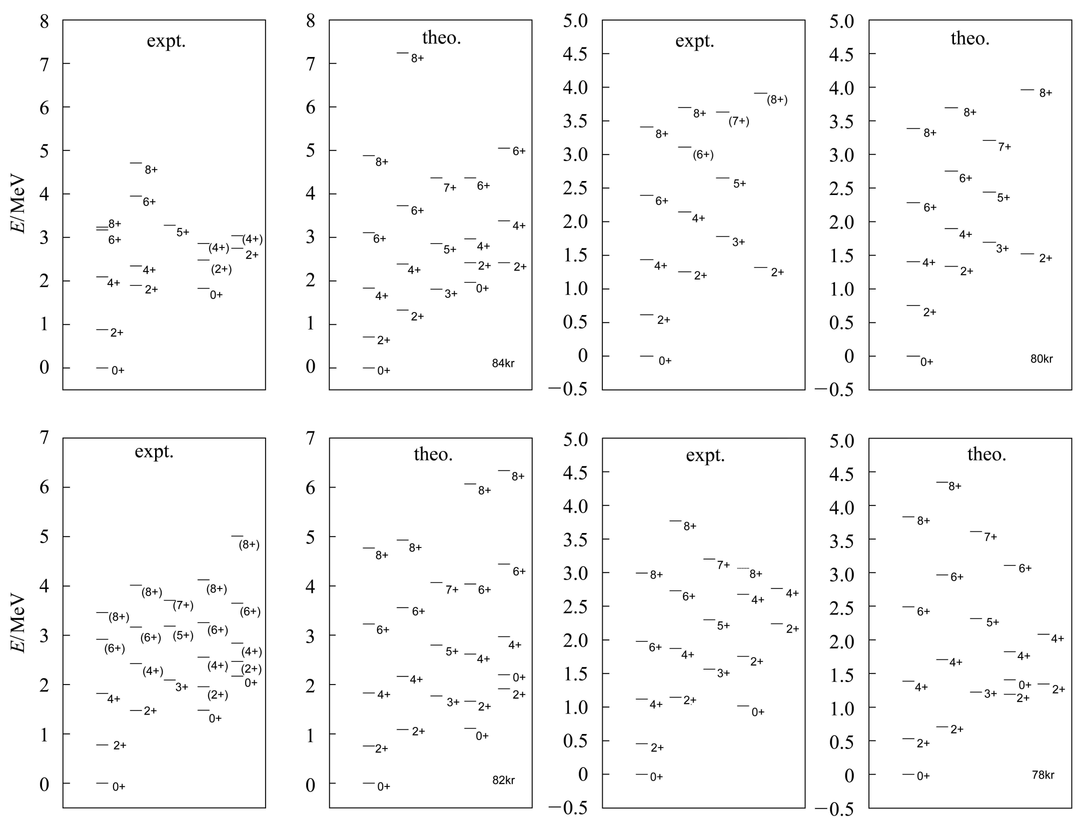

Fig. 1. The excitation energies for the even-even Kr nuclei. The experimental data are taken from Ref. [22].

3 Results

By fitting the excitation energies of $^{78-84}$Kr, the parameters $G_{0\pi}$, $G_{0\nu}$, and κ are fixed and given in Table 1. The other parameters $G_{2\sigma}$ and κ_σ are fixed to be $G_{2\pi} = G_{2\nu} = 0.052$ MeV/r_0^4 and $\kappa_\pi = \kappa_\nu = 0.01$ MeV/r_0^4 for all the nuclei.

Table 1. The parameters fixed by fitting the excitation energies for $^{78-84}$Kr.

	^{84}Kr	^{82}Kr	^{80}Kr	^{78}Kr
G_π/MeV	0.180	0.14	0.14	0.21
G_ν/MeV	0.180	0.13	0.12	0.20
$\kappa/(\text{MeV}/r_0^4)$	0.010	0.042	0.032	0.01

The calculated and experimental spectra are shown in Fig. 1. One can see that a general agreement between the calculation and experiment is achieved. The prediction of the ground states and quasi-γ band can be considered nearly satisfactory.

The calculated E2 transition values are given in Table 2 with the label BCS. The effective charges are set to be $e_\pi = 2$e and $e_\nu = -1.9$e since neutrons are treated as holes in this paper. From Table 2 one can see that in comparison with those of the IBM and experimental data, the B(E2) values are fitted very well in the SDPSM.

The nature of the low-lying 0_2^+ states in even-even nuclei of the $A \approx 70-80$ mass region is still an open question. Detailed theoretical work has mainly been performed in the framework of the IBM. The 0_2^+ states were interpreted as "intruder" states and can be fitted rather well in the IBM. Table 2 shows that although the calculated $B(\text{E2}; 0_2^+ \to 2_1^+)$ are still smaller than those of the experiment, they are considered to be reasonably well reproduced, and most of them are close to those of the IBM results.

Table 2. The E2 transitions. The experimental data are taken from Refs. [1, 2].

$J_i \to J_f$	^{84}Kr	^{82}Kr			^{80}Kr			^{78}Kr		
	BCS	Expt.	BCS	IBM	Expt.	BCS	IBM	Expt.	BCS	IBM
$2_1^+ \to 0_1^+$	0.080	0.044(2)	0.068	0.051	0.076(6)	0.088	0.071	0.134(5)	0.1037	0.097
$4_1^+ \to 2_1^+$	0.087	0.065(24)	0.060	0.078	0.064(6)	0.085	0.078	0.180(14)	0.1465	0.152
$6_1^+ \to 4_1^+$	0.089		0.075		0.13(3)	0.114	0.12	0.20(2)	0.1672	0.17
$2_2^+ \to 2_1^+$	0.078	0.016(8)	0.07	0.030	0.05(1)	0.063	0.07	0.013(4)	0.1458	0.086
$2_2^+ \to 0_1^+$	0.001	0.0002(1)	0.005	0.001	0.0006(1)	0.003	0.001	0.005(1)	0.0001	0.002
$2_3^+ \to 2_1^+$	0.000	0.014(8)	0.002	0.036		0.033		0.049(3)	0.0019	
$3_1^+ \to 2_1^+$	0.001		0.002		0.0012(2)	0.000	0.0009	0.0006(1)	0.0003	
$4_2^+ \to 2_1^+$	0.007	0.024(6)	0.020	0.001	0.0007(4)	0.039	0.0014		0.0001	
$4_2^+ \to 2_2^+$	0.020	0.018(5)	0.025	0.028	0.10(5)	0.034	0.04	0.093(12)	0.0521	0.063
$4_2^+ \to 4_1^+$	0.086	0.08(2)	0.011	0.009	0.07(4)	0.007	0.02	0.040(5)	0.0362	0.04
$0_2^+ \to 2_1^+$	0.001	0.030(10)	0.018	0.055	0.07(3)	0.022	0.07	0.091(5)	0.0425	0.09

Table 3. The M1 transitions. The experimental data are taken from Ref. [4].

$J_i \to J_f$	^{78}Kr			^{80}Kr			^{82}Kr			^{84}Kr		
	Expt.	IBM	BCS	Expt.	IBM	BCS	Expt.	IBM	BCS	Expt.	IBM	BCS
$2_2^+ \to 2_1^+$	0.016(2)	0.006	0.0070	0.0004(2)	0.002	0.1743	0.001(1)	0.009	0.02	0.037(12)	0.021	0.0658
$2_3^+ \to 2_1^+$			0.0088			0.2275			0.007			0.2742
$2_4^+ \to 2_1^+$			0.2997			0.0002			0.152			0.1558
$2_5^+ \to 2_1^+$			0.00001			0.011			0.000			0.000
$5_1^+ \to 4_1^+$	0.001(1)	0.004										

In Table 3 the M1 transitional values are listed. Since it is difficult to determine the g-factor uniquely, the effective g-factors are fixed as $g_{l\pi} = 1.1$, $g_{l\nu} = -0.1$, $g_{s\pi} = 3.910$ and $g_{s\nu} = -2.678$ (all in units of μ_N^2) as in Ref. [23]. The experimental data and the IBM results are also given. The calculated results shows that the strongest M1 transitions occur in (2_3^+, 2_4^+), 2_4^+, (2_2^+, 2_3^+), and 2_4^+ for ^{84}Kr, ^{82}Kr, ^{80}Kr and ^{78}Kr, respectively. The lack of the experimental data did not allow for a definite conclusion.

4 A brief summary

In summary, within the framework of the SDPSM, the general properties of the low-lying states in even-even Kr nuclei have been studied. This analysis shows that, since the properties in the even-even Kr nuclei can be reproduced very well in the SDPSM, it implies that the IBM has a sound shell-model foundation and the truncation scheme adopted in the SD-pair shell model seems reasonable.

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