

Testing Bell inequality at experiments of high energy physics^{*}

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Abstract Besides using the laser beam, it is very tempting to directly testify the Bell inequality at high energy experiments where the spin correlation is exactly what the original Bell inequality investigates. In this work, we follow the proposal raised in literature and use the successive decays $J/\psi \rightarrow \gamma\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$ to testify the Bell inequality. Our goal is twofold, namely, we first make a Monte-Carlo simulation of the processes based on the quantum field theory (QFT). Since the underlying theory is QFT, it implies that we pre-admit the validity of quantum picture. Even though the QFT is true, we need to find how big the database should be, so that we can clearly show deviations of the correlation from the Bell inequality determined by the local hidden variable theory. There have been some critiques on the proposed method, so in the second part, we suggest some improvements which may help to remedy the ambiguities indicated by the critiques. It may be realized at an updated facility of high energy physics, such as BES III.

Key words testing Bell inequality, experiments of high energy physics, new proposal

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1 Introduction

Since the birth of quantum mechanics (QM), dispute about its essence has not ceased. Indeed, nowadays nobody still doubts the validity of QM because of its great success in all fields. However, one can ask if the theory of QM is complete and the relevant principles, such as the wave-particle duality, the superposition principle and the probability interpretation are fundamental in nature or just an effective representation of other underlying principles.

Around what the essence of QM is, Bohr and Einstein conducted a sharp debate over many years. In 1935, Einstein, Podolsky and Rosen published a collaboration paper where they made a challenge to the completeness of QM, that is the famous EPR paradox [1]. It helps to develop the local hidden variable theory (LHVT), especially Bohm resurrected this field [2], and then based on this new understand-

ing, Bell raised a theorem which proves that a local hidden variable theory cannot repeat all predictions of QM [3]. Because of the local hidden variables, definite correlations of the involved states would be retained, even though they are separated by a space-like distance. By contrast, the quantum entanglement is supposed to be a fundamental character of nature and it manifests the difference of quantum mechanics from classical physics. To search a testable scheme which can distinguish between QM and the LHVT, Bell established the Bell inequality for correlation among spin polarizations of various states [3]. By the Bell inequality, if the correlation is due to a set of local hidden variables, the inequality must hold, however, one can easily show that within a certain parameter range the quantum superposition of different states would spoil this inequality. Explicitly, in literature, it is suggested that the correlation of polarizations of two spin-1/2 particles in a singlet state of the total

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spin, i.e. $|0,0\rangle$, the Bell inequality holds if the mechanism of local hidden variable theory (LHVT) applies, whereas it may be explicitly violated as QM is the dominating mechanism. That is an exclusive and direct test of LHVT. For almost half a century, many experimental schemes have been designed to realize such a measurement [4–7]. However, it was found that such experiments demand high accuracy and statistics, so that becomes very difficult. Just because of the difficulties, so far even though the importance of the test is obvious, people are still unable to carry out experiments with high precision to make a definite conclusion yet. No wonder, the first success was achieved in optical experiments, where the correlation between photon polarizations is studied thanks to the application of high quality laser beams and the high precision of optical apparatus as well as advanced techniques. Over half a century, many experiments have been carried out to testify, among them, the polarization entanglement experiments of two-photons and multi-photons have attracted the widest attention from the physics society [6]. All photon experimental data indicate that the Bell inequality and its extension forms are violated, and the results are fully consistent with the prediction of QM. The consistency can reach as high as 30 standard deviations. On the other aspect, however, as indicated in literature, the detection efficiency in optical experiments is rather low, therefore when analyzing the data, one needs to introduce additional assumptions, so that the requirement of LHVT cannot be completely satisfied. That is why as generally considered, so far, the Bell inequality has not undergone serious testing.

It would be interesting to return to the original formulism where the correlation between the polarizations of two spin-1/2 particles is discussed. However, because of the existence of Lorentz force, the spin polarization of a charged fermion cannot be directly measured even though an inhomogenous magnetic field is employed in experiment [8] (We will discuss this problem again at the end of this paper). Some authors alternatively suggested to directly measure the polarization correlation at experiments of high energy physics [4]. An ideal candidate is the successive decay mode of $\eta_c \rightarrow \Lambda \bar{\Lambda} \rightarrow p\pi^- + \bar{p}\pi^+$, because η_c is a 0^{-+} meson and thus $\Lambda \bar{\Lambda}$ would be in the spin $|0,0\rangle$ state. That is an entangled state of two spin-1/2 fermions. An obvious advantage is that the information of the spin polarization of $\Lambda(\bar{\Lambda})$ can be obtained by measuring the directions of the emitted $\pi^-(\pi^+)$, which would leave clear tracks in the detector.

In the original proposal for testing the Bell inequality, there should be three independent directions and the polarizations along all the three directions must be measured and the formulation is

$$|E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{b}')| \leq 1 + E(\mathbf{b}, \mathbf{b}'), \quad (1)$$

where $E(\mathbf{n}_1, \mathbf{n}_2)$ is the correlation of two polarizations of the two particles which exist in an entangled state and are separated by a space-like distance. As suggested in the original literature, the inequality with three independent directions is reduced into a form with only one continuous parameter as

$$E(\theta_{ab}) \leq 1 - \theta_{ab}/\pi, \quad (2)$$

where θ_{ab} is the angle between the linear momenta of the two emitted pions in the respective CM frames of Λ and $\bar{\Lambda}$. The authors of Ref. [4] showed that the QM result would upset this inequality. Obviously, it is a very difficult experiment which demands a precise measurement. Thanks to the improvements of detection facility and technique, experimentalists may be able to measure the decay widths of small-probability processes. Moreover, the BES possesses a large database of J/ψ which provides a possibility to test the Bell inequality at high energy experiments.

On another aspect, this scheme has received some critiques. We will try to remedy those problems in the last section of this paper.

In this work, it is worth noticing that we simulate the successive processes $J/\psi \rightarrow \eta_c + \gamma$ and $\eta_c \rightarrow \Lambda \bar{\Lambda} \rightarrow p\pi^- + \bar{p}\pi^+$ based on the quantum field theory and the aim of such a work is to check if the inequality (2) could be satisfied. A natural question would be raised that the Bell inequality is to testify which one of the local hidden variable theory and QM is valid, thus when we employ the quantum field theory, it implicitly suggests that we have already assumed quantum mechanics is applied, so that it is not a real test of the Bell inequality and cannot determine if the local hidden variable theory fails. In fact, in the first part of this work, we do not intend to eventually make the final conclusion about the inequality, but will determine how big the database must be to guarantee the precision for drawing a definite conclusion about the difference between the predictions of QM and the local hidden-variable theory. Since QM is based on statistics, namely all possible quantum states have certain probabilities to occur, we can only obtain the results with certain statistics. Of course, if the event number can be infinity, the QM would predict a smooth curve for the required relation (the polarization correlation), but it is definitely impos-

sible, thus a question emerges. Namely, if the QM is right and the quantum field theory is a valid theory, we need the number of events to find a clear distinction above the bound set by the Bell inequality with one, two or three standard deviations. We use the Monte Carlo method to simulate the successive processes and draw the graphs with error bars being explicitly marked out. Finally, we find that the necessary number of J/ψ must be as large as 10^9 and when the statistical and systematic errors of concrete experiments are taken into account, the database must be enlarged at least by a factor of 10.

Considering the critiques of this method, we propose an improvement scheme. A long time ago, the authors of Ref. [5] suggested to set two Stern-Gerlach apparatuses which provide inhomogeneous magnetic fields to carry out an experiment with two proton beams to test the Bell inequality. In this work, following their suggestions, we would set such apparatuses which can distinguish different spin polarizations of $\Lambda(\bar{\Lambda})$, namely, due to their anomalous magnetic moments, they undergo different forces and decline their original trajectories. Our sensitive detector can record their decay daughters' trajectories to determine their polarizations. Because we have the freedom to adjust the directions of the magnetic fields, we can have three real independent directions as the original Bell inequality requires. A detailed discussion will be given in the last section.

This paper is organized as follows. After this long introduction, we will present all formulae which are necessary for numerical computations and for readers' convenience, we include a subsection to discuss the Monte-Carlo simulation and χ^2 analysis. The third section is for our numerical results where the Monte-Carlo errors are given. In Section 4, we discuss our proposal which suggests a modified version for probing the Bell inequality, and its advantages and difficulties.

2 The Bell inequality

As discussed in the introduction, one of the focuses of basic research of modern physics is to directly testify the Bell inequality which is derived based on the LHVT of quantum mechanics, and it is crucial to justify if QM is a complete local theory [9]. Starting from the LHVT of quantum mechanics, one can derive an inequality about the correlation between polarizations of two spin-1/2 fermions in a system of total-spin singlet, i.e. the Bell inequality. In the LHVT with the hidden variable being λ , the Bell in-

equality can be phrased as

$$E(\mathbf{a}, \mathbf{b}) = \int d\lambda A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) \rho(\lambda), \quad (3)$$

where $\rho(\lambda)$ is a distribution of the hidden variable(s) λ , $A(\mathbf{a}, \lambda)$ is the result of measurement of the projection $(\boldsymbol{\sigma}^{(A)} \cdot \mathbf{a})$ of spin $\boldsymbol{\sigma}^{(A)}$ of particle A along direction \mathbf{a} , whereas $B(\mathbf{b}, \lambda)$ is the corresponding measurement for the projection $(\boldsymbol{\sigma}^{(B)} \cdot \mathbf{b})$ of spin $\boldsymbol{\sigma}^{(B)}$ of another particle B along direction \mathbf{b} . The original expression of Bell inequality refers to three independent spatial directions \mathbf{a} , \mathbf{b} and \mathbf{c} as

$$|E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{c})| - E(\mathbf{b}, \mathbf{c}) \leq 1. \quad (4)$$

According to the theory of QM, this correlation function is the average value of operator $(\boldsymbol{\sigma}^{(A)} \cdot \mathbf{a}) \cdot (\boldsymbol{\sigma}^{(B)} \cdot \mathbf{b})$ over a spin singlet $|\psi\rangle$ (or written as $|\chi_{00}\rangle$). Namely:

$$E(\mathbf{a}, \mathbf{b})_{\text{QM}} = \langle \psi | (\boldsymbol{\sigma}^{(A)} \cdot \mathbf{a}) (\boldsymbol{\sigma}^{(B)} \cdot \mathbf{b}) | \psi \rangle = -\mathbf{a} \cdot \mathbf{b}. \quad (5)$$

It is easy to prove that the Bell inequality (4) contradicts to the QM result. If the QM result receives experimental support, one would quantitatively confirm that the local hidden variable theory cannot describe all predictions of quantum mechanics.

Besides the optical experiments, in recent years, some proposals of non-optical experiments have been raised to testify the LHVT of QM. Similar tests have also been realized at the proton-proton double-scattering [5] and other collider experiments [7], and their conclusions all support the QM and suggest that the Bell inequality is violated. Compared with optical experiments, high energy experiments deal with massive particles, especially the real spin-1/2 fermions. In such processes weak and strong interactions are involved while only EM interaction applies in optical experiments.

Among the proposals, we are especially interested in the successive decay processes $\eta_c \rightarrow \Lambda \bar{\Lambda} \rightarrow p\pi^- \bar{p}\pi^+$ at e^+e^- colliders, which was suggested by Törnqvist [4] a long while ago. The reason for our interest is not only because it provides a clear and realizable method, but also the Beijing Electron-Positron Collider (BEPC) will provide an incomparably large database which enables people to achieve high statistics for drawing a definite conclusion. The spin of η_c is zero and decays into an $\Lambda \bar{\Lambda}$ pair, that is exactly the Bohm's version about the entangled EPR state, therefore it should serve as an ideal probe for LHVT of QM. In the proposal, the polarizations of Λ and $\bar{\Lambda}$ can be determined respectively by the momenta directions of π^+ and π^- , and the relatively longer lifetime of $\Lambda(\bar{\Lambda})$ guarantees the space-like requirement. All of

these set a favorable condition for directly testing the Bell inequality.

In the standard calculations of the quantum field theory, the transition matrix element of $\Lambda \rightarrow p\pi^-$ reads[4]

$$M_\Lambda = (4\pi)^{1/2}(S + P\boldsymbol{\sigma}_\Lambda \cdot \mathbf{a}) \quad (6)$$

where \mathbf{a} is the unit vector of π^- in the CM frame of Λ : $\frac{\mathbf{P}_\pi^{\text{cm}}}{|\mathbf{P}_\pi^{\text{cm}}|}$, S and P are the S and P wave-amplitudes respectively. $\bar{\Lambda} \rightarrow \bar{p}\pi^+$ has a similar expression. Thus in the successive process $\Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$ the correlation between the momenta of the two pions can be converted into the correlation between polarizations of Λ and $\bar{\Lambda}$.

$$I(\mathbf{a}, \mathbf{b}) = \langle S|(M_\Lambda M_B)^\dagger(M_\Lambda M_B)|S\rangle = \left(\frac{|S|^2 + |P|^2}{4\pi}\right)^2 (1 + \alpha^2 \mathbf{a} \cdot \mathbf{b}) \quad (7)$$

where $\alpha \approx 0.642$.

Except for the coefficient α^2 and the normalization, this expression is the same as the correlation function for a system of two spin-1/2 particles given in the Bohm's version (5). If one can neglect the CP violation in the process, the expression can be understood as: polarization of Λ is $\alpha\mathbf{b}$, while polarization of $\bar{\Lambda}$ is $\alpha\mathbf{a}$. Taking appropriate normalization, the correlation function can be written as

$$R(\mathbf{a}, \mathbf{b}) = 1 + \alpha^2 \mathbf{a} \cdot \mathbf{b} \quad (8)$$

and the corresponding Bell inequality is rephrased as

$$|E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{c})| \leq 1 + E(\mathbf{b}, \mathbf{c}), \quad (9)$$

where $E = \frac{1-R}{\alpha^2}$. Törnqvist defined a quantity $\cos\Theta_{ab} \equiv \mathbf{a} \cdot \mathbf{b}$, and by it one notices that the Bell inequality can be expressed as within a continuous region, variable Θ_{ab} satisfies an alternative inequality:

$$|E(\Theta_{ab})| \leq 1 - 2\Theta_{ab}/\pi. \quad (10)$$

Obviously as shown above, by QFT, the correlation function $|E(\Theta_{ab})|$ is linearly proportional to $\cos\Theta_{ab}$. In Fig. 1, for explicitness and clarity, we do not normalize the data, but keep the event number density where we choose each bin in the figure covering a region of $0.05(\cos\Theta_{ab})$. According to Törnqvist's suggestion, the DM2 collaboration reported their preliminary result in 1985. Due to very poor statistics, they only presented the fitting results for $J/\psi \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$. Even though in this process Λ and $\bar{\Lambda}$ also reside in an entangled state and the correlation functions are similar, there is an obvious difference from the η_c case. Within error tolerance, their results do

not show any obvious discrepancy from the QM prediction, on another aspect, one still cannot draw any conclusion from the analysis.

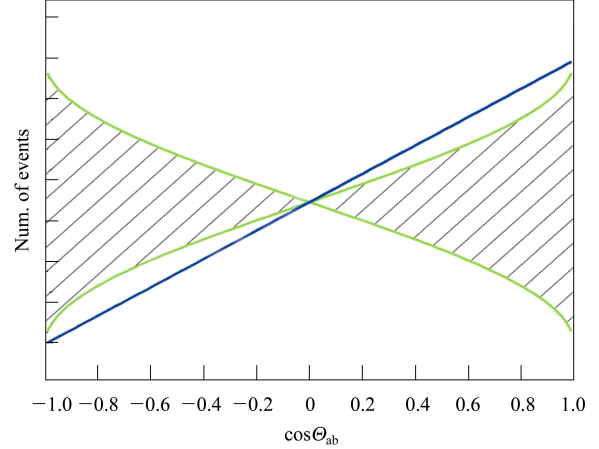


Fig. 1. The horizontal axis is $\cos\Theta_{ab}$ and the shadowed region is allowed by the Bell inequality, and the blue straightline is the QFT theoretical prediction.

The BES III has begun operation and a great amount of J/ψ and η_c data has been accumulating, so the statistics are incomparably improved and one is confronting a valuable chance to make precise measurements on $\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$, just based on this possibility, we will re-carry out a test on the LHVT in the charm energy range. We are going to simulate the successive decay processes $J/\psi \rightarrow \gamma\eta_c$, $\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$ in terms of the Monte-Carlo method and estimate the necessary event database for clearly distinguishing QM result from that which obeys the Bell inequality. Even though for a real experimental setting, this simulation is not accurate enough, it can still provide a theoretical reference which may guide the experimentalists to design the experiment, or make the decision if one can carry out the measurement with the considered luminosity and detection error tolerance at BES III.

3 Our method

3.1 Random sampling

Generally, a complete Monte-Carlo simulation consists of 1. constructing the concerned random process; 2. taking random numbers which coincide with the probability function; and 3. estimating the errors and determining the confidence level of the result.

3.2 Goodness-of-fit tests of χ^2

In this scheme, the quantity to be measured is the cosine of the angle between the two emitted pions. The statistical quantity

$$\chi_{\text{obs}}^2 = \sum_{i=1}^N \frac{(n_i - n_{\text{theory}})^2}{n_i} = \sum_{i=1}^N \frac{(n_i - np_{0i})^2}{n_i}, \quad (11)$$

should approximately obey the $\chi^2(N-1)$ distribution. In the expression N is the total number of the bins within the concerned physical region, n_i is the experimental or M-C estimated event number in the i -th bin, n is the total event numbers (normalization: $\sum n_i = n$), np_{0i} is the theoretical event number and p_{0i} is the probability of the event appearing in the i -th bin.

For a given Confidence Level (C.L.), $1 - \alpha$, one can apply the $\chi^2(N-1)$ accumulation integration to determine the critical value of $\chi_{\alpha}^2(N-1)$:

$$\alpha = \int_{\chi_{\alpha}^2(N-1)}^{\infty} f(y; N-1) dy. \quad (12)$$

In the above expression, $f(y; N-1)$ is the probability density of $\chi^2(N-1)$ and the critical range is $\chi^2(N-1) > \chi_{\alpha}^2(N-1)$. Generally, if the theoretically evaluated value χ_{obs}^2 is greater than $\chi_{\alpha}^2(N-1)$, it indicates that the data are not consistent with the theory at the confidence level $1 - \alpha$; whereas if the obtained χ_{obs}^2 is smaller than $\chi_{\alpha}^2(N-1)$, at $1 - \alpha$ C.L., the data can be described by the theory to be tested.

3.3 Simulation and results

We are simulating the successive reactions $J/\psi \rightarrow \gamma\eta_c$, $\eta_c \rightarrow \Lambda\bar{\Lambda}$, $\Lambda \rightarrow p\pi^-$, $\bar{\Lambda} \rightarrow \bar{p}\pi^+$ step by step. Then taking into account the Lorentz boost effects, we calculate the angle between the momenta of the two emitted pions which are measured in the center-of-mass reference frames of Λ and $\bar{\Lambda}$ respectively.

In Table 1, one can note that $BR(J/\psi \rightarrow \eta_c\gamma) \times BR(\eta_c \rightarrow \Lambda\bar{\Lambda}) \times BR(\Lambda \rightarrow p\pi^-)^2 = 5.5 \times 10^{-6}$, namely, for almost every 10^6 J/ψ events, there would be 5.5 events corresponding to the expected reaction $J/\psi \rightarrow \eta_c\gamma \rightarrow \Lambda\bar{\Lambda}\gamma \rightarrow p\pi^-\bar{p}\pi^+\gamma$. Therefore we first carry out our M-C simulation with 10^7 J/ψ events, and then gradually increase the number of the produced J/ψ events. According to the statistics, we would determine the necessary J/ψ event number by which we can clearly distinguish the QFT result from the Bell inequality. In the following figures, the proportional errors are the reciprocal of the square-root of the event number in the corresponding bin, i.e. one standard deviation.

Table 1. BR of decay [10].

channel	BR
$J/\psi \rightarrow \eta_c\gamma$	$(1.3 \pm 0.4)\%$
$\eta_c \rightarrow \Lambda\bar{\Lambda}$	$(1.04 \pm 0.31) \times 10^{-3}$
$\Lambda \rightarrow p\pi^-$	$(63.9 \pm 0.5)\%$

The results of the Monte-Carlo simulation are shown in Figs. 2, 3 and 4. Then we employ the χ^2 analysis to make the goodness-of-fit test to the boundary values of the QM prediction in comparison with the Bell inequality. The corresponding χ^2 values are listed in Table 2. The employed equation is

$$\chi_{\text{Q.M.}}^2 = \sum_{i=1}^n (n_i - n_{\text{Q.M.}})^2 / n_i, \quad \chi_{\text{B.I.}}^2 = \sum_{i=1}^n (n_i - n_{\text{B.I.}})^2 / n_i.$$

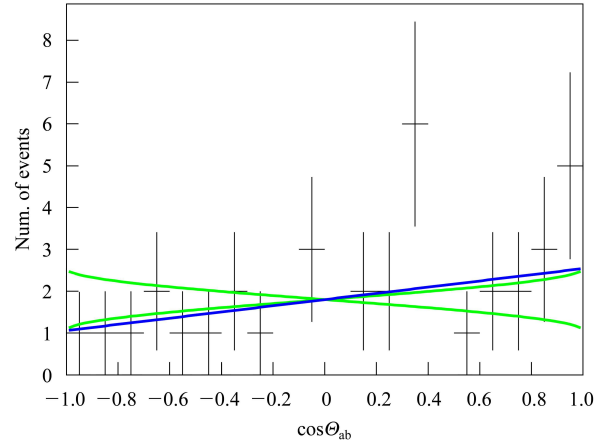


Fig. 2. 10^7 J/ψ events, Θ_{ab} is the angle between π^- and π^+ which are measured respectively in the center-of-mass reference frames of Λ and $\bar{\Lambda}$. We divide the region of $\cos\theta$ $[-1, +1]$ into 20 bins.

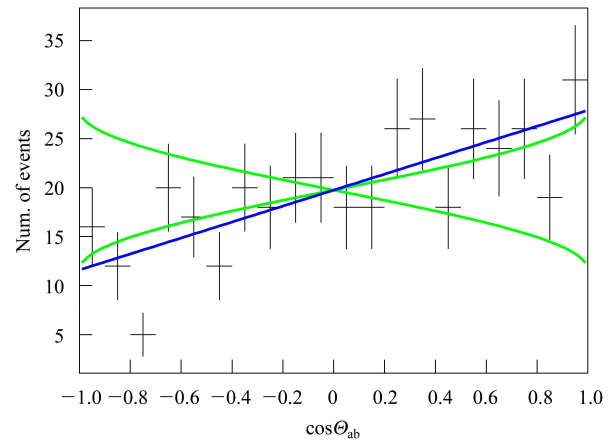


Fig. 3. 10^8 J/ψ events, the other notations are the same as for Fig. 2.

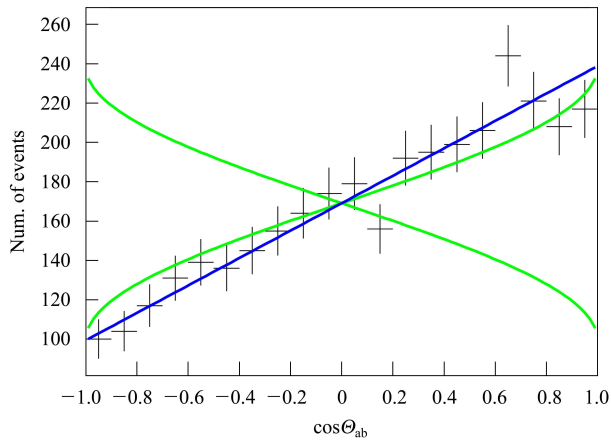


Fig. 4. 10^9 J/ψ events, the other notations are the same as for Fig. 2.

In Fig. 2, one can observe that as 10^7 J/ψ events are generated, there would only be 36 pion pairs, thus we can note a left-right asymmetry, but the Monte-Carlo results are not sufficient to test if QM are different from the Bell inequality. When the event number is increased to 10^8 , we obtain 395 pion pairs, and the results obviously incline into the prediction of QM (see Fig. 3). Even though the error bars are still too large to clearly distinguish between the Bell inequality and the QM, by the χ^2 best-fitting test, it is shown that $\chi_{Q.M.}^2 < \chi_{B.I.}^2$ and the χ^2 of the boundary value of the Bell inequality is larger than the QM value by 20%. When the J/ψ event number reaches 10^9 , we can generate 3382 pion pairs, the central values of the Monte-Carlo simulation distribute at the vicinity of the straightline predicted by the QM, Fig. 4, and the error bars also are shortened to the length which is comparable with the deviation between the QM and Bell inequality. Then the goodness-of-fit is $\chi_{B.I.}^2/\chi_{Q.M.}^2 = 1.83$, i.e. the two theories result in remarkable distinction. It corresponds to 80% C.L. Numerical results are shown in the following table. For higher statistics (smaller statistical errors, or higher C.L.) much more J/ψ events are needed.

Table 2. χ^2 value of fit with different statistics.

statistics (J/ψ)	10^7	10^8	10^9
$\chi_{Q.M.}^2$	7.20	27.44	13.07
$\chi_{B.I.}^2$	7.51	32.84	23.89

As a brief summary, it is possible to test the Bell inequality with a database of as large as 10^9 J/ψ events, but the accuracy is not sufficiently high as expected. Moreover, when the detection efficiency is taken into account, this number must be at least multiplied by a factor of 10. Thus, for drawing a convincing conclusion, 10^{11} J/ψ events seem to be necessary,

and it is probably beyond the reach of BES III for a few years unless its luminosity can be enhanced remarkably.

4 Discussion and our new proposal

In the above section, we carefully investigate the possibility of testing the Bell inequality at e^+e^- collider via successive reactions $J/\psi \rightarrow \gamma\eta_c$, $\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$. With our numerical simulation, we conclude that as the event number of J/ψ reaches 10^9 , one can test the validity of the Bell inequality at 80% C.L., but a real test needs 10^{11} J/ψ events at least.

It is worth noticing that our above work does not make a judgement about the validity of the LHVT, but assuming if Törnqvist's proposal really makes sense, how many J/ψ events we need to realize the test i.e. to effectively distinguish the QM prediction from the region allowed by the Bell inequality. The analysis can be a reference for our BES colleagues.

Now let us turn to discuss a different issue which originates from critiques to the above experiment. According to those critiques, the reasonability of testing the LHVT with the successive decays $J/\psi \rightarrow \gamma\eta_c$, $\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$ proposed by Törnqvist is questionable. Namely, what the experiment is supposed to do is not the same as the fundamental idea of Bohm et al. based on the analysis of LHVT.

The critiques are focusing on the key point: Can the aforementioned collider experiments testify if the Bell inequality holds? The critiques can be summarized into a few points.

1) The collider experiments are different from the optical experiments, namely they are not active measurements, but passive ones, in other words, we cannot control the momentum direction of the decay products, the pions, and then their polarizations.

2) Unlike proton and photon, in the collider experiments, the concerned particles are unstable and only leave tracks of a few cm, which are too short to let us make precise measurements on the spin polarizations of the involved particles.

3) The successive reaction such as $J/\psi \rightarrow \gamma\eta_c$, $\eta_c \rightarrow \Lambda\bar{\Lambda} \rightarrow p\pi^-\bar{p}\pi^+$, is very complicated, so the measurements may not realize a concise test of the original Bell inequality.

4) There may exist a concept substitution [11]. In QM, different spin projections are non-commutative, and cannot be simultaneously measured. In collider experiments, such non-commutative quantities are replaced by commutative momentum components. That is different from the LHVT, namely there does

not exist a subject to be measured which is related to the criterion of the EPR completeness. The reason is that as Λ decays, in QM, it corresponds to a measurement and the momentum represents the result of measurement on the spin polarizations. Because the measurement induces a de-coherence, part of the information is lost.

5) Finally, there is another logic problem. The spin-polarization correlation of Λ and $\bar{\Lambda}$ is converted into the correlation between the pion momenta which are directly measured at the collider as $1+\alpha^2\mathbf{a}\cdot\mathbf{b}$ where \mathbf{a} and \mathbf{b} are the directions of the π 's in the center-of-mass frames of their respective parent particles (Λ or $\bar{\Lambda}$)(compared with Eq. (8) where their spin polarizations are respectively $\alpha\mathbf{a}$ and $\alpha\mathbf{b}$ when the CP violation is ignored). However, in the original Bell inequality the quantity to be dealt with is $\mathbf{a}\cdot\mathbf{b}$ instead, thus the region covered by the Bell inequality is -1 to $+1$ while the direct measured momentum correlation is $1-\alpha^2$ to $1+\alpha^2$ which has an overlapping region with $[-1, +1]$. If one wants to check the distinction between the QM results and the Bell inequality, he must extract $\mathbf{a}\cdot\mathbf{b}$ from $1+\alpha^2\mathbf{a}\cdot\mathbf{b}$ which is derived from QFT (i.e. QM), thus he is confronting an embarrassing situation where he needs to admit the validity of QM and uses it to testify QM. Generally, the result loses persuasiveness.

To overcome the problems, some authors [5] suggested that the test be carried out in terms of the proton-proton secondary-scattering experiment. They planned to install a Stern-Gerlach apparatus to measure the spin-polarizations of proton. However, as indicated in the textbook of QM [8], because $\nabla\cdot\mathbf{B}=0$, the Lorentz force would smear out any observable effect as a non-uniform magnetic field applies.

Inspired by their ideas, we suggest modifying the method proposed by Törnqvist. Our proposal is the following. One can install two Stern-Gerlach [12] apparatuses at two sides with flexible angles with respect to the electron-positron beams. The apparatus provides a non-uniform magnetic field which may decline the trajectory of the neutral $\Lambda(\bar{\Lambda})$ due to its non-zero anomalous magnetic moment i.e. the force is proportional to $\frac{d}{dn}(-\boldsymbol{\mu}\cdot\mathbf{B})$ where $\boldsymbol{\mu}$ is the anomalous magnetic moment of Λ , \mathbf{B} is a non-uniform external magnetic field and $\frac{d}{dn}$ is a directional derivative. Because Λ is neutral, the Lorentz force does not apply, therefore one may expect to use the apparatus to directly measure the polarization of $\Lambda(\bar{\Lambda})$. The declination of trajectory of Λ (here we use Λ as an illustration) depends on its spin polarization. But one

must first identify the particle flying into the Stern-Gerlach apparatus, i.e. to make sure it is Λ or $\bar{\Lambda}$. It can be determined by its decay product, i.e. if its decay product is $p^+\pi^-$, it is Λ , otherwise is $\bar{\Lambda}$. Here one only needs the decay product to tag the decaying particle, but does not use it to do kinematic measurements.

The advantages are obvious in that one can completely avoid the problems listed above. First, we turn a passive measurement into an active one, because we can independently adjust the direction of each Stern-Gerlach apparatus, we will obtain the spin-correlations among three independent directions as required by the original Bell inequality. This is in analog to the optical experiment and conducts a real test on the Bell inequality.

One can install such an apparatus as illustrated in Fig. 5, particles 1 and 2 are produced in a spin singlet and later are separated by a space-like distance after a time interval t and then fly into two Stern-Gerlach apparatuses with gradients of magnetic fields being \mathbf{a} and \mathbf{b} respectively. By their declinations, one can note their polarizations. It overcomes the flaws of the original experimental setting.

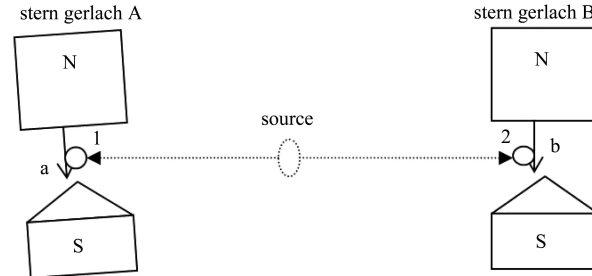


Fig. 5. The experimental setting with two Stern-Gerlach apparatuses.

There is no such thing as a free lunch in the world, i.e. advantages also bring up more difficulties. Let us list a few problems which we can figure out up to the present moment.

1) The lifetime of $\Lambda(\bar{\Lambda})$ is not very long, and as a product of η_c , it can probably travel a few cm before it decays, so it would be difficult to satisfy the space-like conditions. To enforce the space-like condition, we need to install the two Stern-Gerlach apparatuses far enough. It means that we only choose the Λ s which live longer, if so, the statistics would be greatly decreased.

2) The magnetic field is not easily adjusted.

3) The decay products of $\Lambda(\bar{\Lambda})$ are charged and they may leave very complicated trajectories in the non-uniform magnetic field and make the event re-

construction very difficult. In other words, it is hard to determine the location of the production vertex.

4) Besides the above difficulties, one still confronts another serious problem. The magnetic fields in the detector and especially in the Stern-Gerlach apparatuses may flip the polarizations of Λ and/or $\bar{\Lambda}$, thus the coherence between Λ and $\bar{\Lambda}$ is spoiled [13]. Therefore the magnetic fields in the detector should not be too strong to cause such decoherence, but on an other aspect they cannot be too weak, otherwise the declinations of the trajectories of Λ and $\bar{\Lambda}$ are not

detectable.

Even though all the problems are difficult, we believe that with the rapid development of high energy physics facilities and detection techniques they can be eventually solved and we will be able to testify the LHVT according to its original proposal.

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