η_b decay into charmonium in association with $c\bar{c}$ pair

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Abstract We calculate the inclusive decay rates of η_b into charmonium via double $c\bar{c}$ pairs for S- and P-wave states η_c, J/ψ and χ_{cJ} within the framework of non-relativistic QCD (NRQCD) factorization at leading order in α_s. Besides calculating the contributions of the color-singlet channels $\eta_b \to c\bar{c}[^{2S+1}S_L^{(1)}]+c\bar{c}$, the effects of $c\bar{c}$ pair in the color-octet configurations are also considered. We find that $\eta_b \to c\bar{c}[^3S_1^{(8)}]+c\bar{c}$ make a small contribution to $Br(\eta_b \to J/\psi(\eta_c)+c\bar{c})$. While in the $\eta_b \to \chi_{cJ}+c\bar{c}$ case, the color octet contributions are significant, for they are of the same $\alpha_s^4 v_c^5$ order as the color-singlet processes. We predict $Br(\eta_b \to J/\psi(\eta_c)+c\bar{c}) = 2.99(2.75)\times10^{-5}$ for S-wave states J/ψ and η_c, and $Br(\eta_b \to \chi_{cJ}+c\bar{c}) = (4.37, 3.40, 2.83)\times10^{-5}$ (for $J = 0,1,2$) for P-wave states χ_{cJ}. In the end, we also find $Br(\eta_b \to c\bar{c}c\bar{c})$ is almost saturated by η_b decay into charmonium in association with $c\bar{c}$ pair from the point of view of duality.

Key words η_b decay, associate production, non-relativistic QCD

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1 Introduction

Heavy quarkonium state constituted by $Q\bar{Q}$ pair is one of the simplest quark-antiquark composite particles. The studies of their production, decay and spectrum have long been interesting topics from both theoretical and experimental points of view ever since the first charmonium J/ψ and bottomonium Υ were discovered more than thirty years ago [1].

In the past year, the long-awaited $^1S_0$ bottomonium ground state $\eta_b(1S)$, hereafter referred to as the η_b, was finally found by BaBar collaboration in the photon energy spectrum of $\Upsilon(3S)$ [2], with a mass of $9388.9^{+3.1}_{-2.3}(\text{stat}) \pm 2.7(\text{syst})$ MeV. Soon after, it was also observed in $\Upsilon(2S) \to \gamma\eta_b$ decay at BaBar [3]. The existence of η_b is a solid prediction of QCD. The mass splitting between η_b and its spin-triplet partner $\Upsilon(1S)$ has been calculated in a potential model, lattice and perturbative QCD, and the recent theoretical predictions of the splitting energy range from 40 to 60 MeV [4–7].

Before the BaBar's experimental results came out, people have tried to search for η_b in various experiments. Following the original idea of Godfrey and Rosner [8], the CLEO collaboration [9] had also analyzed hundreds of $\Upsilon(2S)$ and $\Upsilon(3S)$ M1 transitions and the cascade decay $\Upsilon(3S) \to h_b\pi^0, h_b\pi^+\pi^-$ followed by $h_b \to \eta_b\gamma$, but found no evidence. Since η_b can directly couple to $\gamma\gamma$, at LEP II, the ALEPH [10], L3 [11] and DELPHI [12] collaborations managed to find η_b through $\gamma\gamma \to \eta_b$ +X, unfortunately, no obvious signal was seen in the four, six and eight-charged particle channels, and they only set the upper limits on the branching fractions. The η_b produced in hadron colliders are not as clean as those in e^+e^- environment, however its production rate is expected to be very large. This pushes people to think that they may search for η_b in its exclusive decay modes. Noticing the large branching ratios of η_c decay into two light vector mesons, Braaten, Fleming and Leibovich [13] estimated the branching ratio of η_b decay into double J/ψ and found $Br[\eta_b \to J/\psi\, J/\psi] = 7\times10^{-4\pm1}$. They suggested to look for η_b through its decay to $J/\psi\ J/\psi$, with the subsequent four μ's decay of the double J/ψ. Maltoni and Polosa [14] calculated the cross section for η_b production at the Tevatron up to

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the next-to-leading order (NLO) in α_s. They found that $\sigma(p\bar{p}\to\eta_b+X)\approx 2.5$ μb. Corresponding to more than two millions of event per pb, there will be 10^{12} η_b samples. The CDF collaboration [15] adopted this suggestion and found some events but did not arrive at a conclusive result. In the non-relativistic limit, the branching ratio of η_b decay to double J/ψ is 0 at tree level. And the first order relativistic corrections predicted [16] $Br[\eta_b\to J/\psi J/\psi]=2.4^{+4.2}_{-1.9}\times10^{-8}$, after including the NLO QCD corrections [17], it was obtained that $Br[\eta_b\to J/\psi J/\psi]=(2.1\text{–}18.6)\times10^{-8}$. However, Santorelli [18] argues that the final-state interaction may enhance the NRQCD prediction by about two orders of magnitude. With more accumulated experimental data, the theoretical uncertainty about η_b decay to double J/ψ will be clarified in the future. Maltoni [14] et al. also suggested to find η_b in hadron colliders through $\eta_b\to D^*D^{(*)}$. And it was also suggested to hunt η_b via its radiative decay into J/ψ with $Br[\eta_b\to J\psi\gamma]=(1.5\pm0.8)\times10^{-7}$ [19, 20].

Aside from giving suggestions to search for η_b in different environments, theorists also have done some work to study its decay properties. The inclusive decay of η_b to $c\bar{c}c\bar{c}$ was calculated in Ref. [14]. The exclusive decays of η_b into S- and P-wave double charmonia were considered within the light-cone formalism at leading-twist by Braguta, Likhoded and Luchinsky [21]. In addition, they also studied these processes in the framework of non-relativistic QCD factorization (NRQCD) approach. Until now, our knowledge about η_b decay is still lacking. It is necessary to do some further work. Although some decay channels may not be easily seen now, due to their small branching fractions, they may be measured in the forthcoming LHCb and Super-B experiments with high luminosity.

Among all the decay channels, η_b decay into charmonia are very special ones, for they include both the heavy quarkonium annihilation decay process (η_b decay) and the heavy quarkonium production process (charmonium production), which will not happen in the $c\bar{c}$ system. In this paper, we will systemically study the decay widths of $\eta_b\to J/\psi(\eta_c,\chi_{cJ})+c\bar{c}$. In recent years the charmoium production in association with $c\bar{c}$ pair is a very interesting production mechanism on both experimental and theoretical sides. Belle Collaboration [22] found that in e^+e^- annihilation the inclusive J/ψ production is dominated by $\sigma(e^+e^-\to J/\psi+c\bar{c})$. Theoretically, the NLO QCD corrections [23, 24] for J/ψ production via double $c\bar{c}$ pairs in e^+e^- annihilation enhance the LO results [25–28] significantly. And in J/ψ hadroproduction the associated process $p\bar{p}(p)\to J/\psi+c\bar{c}$ [29, 30] is also found to be important. Moreover, in Ref. [31], the color-transfer enhancement was introduced in double-heavy-quark-pair production. Likewise, it would be interesting to see what will happen on J/ψ as well as η_c and P-wave states χ_{cJ} production together with $c\bar{c}$ in η_b decay case.

Due to the large mass of heavy quark, heavy quarkonia are inherently non-relativistic. We will adopt the NRQCD effective field theory [32] to describe the $b\bar{b}$ bound state η_b and the $c\bar{c}$ mesons, and calculate the decay width of $\eta_b\to J/\psi(\eta_c,\chi_{cJ})+c\bar{c}$ based on NRQCD factorization formula.

The rest of this paper is organized as follows: In Section 2, we will briefly describe how we calculate the decay widths within the framework of NRQCD. In Section 3, we will calculate the contributions of the color-singlet processes for $\eta_b\to J/\psi\ (\eta_c,\chi_{cJ})c\bar{c}$. The impact of color-octet channel $\eta_b\to c\bar{c}([^3S_1,\underline{8}])+c\bar{c}$ will be investigated in Section 4. In the end, we will discuss our results and present a summary in Section 5.

2 Description of the basic factorization formula

In NRQCD, heavy quarkonium is decomposed by the $Q\bar{Q}[^{2S+1}L_J,\underline{a}]$ Fock states, where S is the definite spin of $Q\bar{Q}$, L is their orbital angular momentum, J is the total angular momentum, and $a=1,8$ denotes the color state. Each Fock state will contribute to the production or decay rates of heavy quarkonium, which are factorized into the product of short-distance coefficients and long-distance matrix elements. The short-distance part involves the production or annihilation of $Q\bar{Q}$ pair in certain state, which could be calculated perturbatively through the expansion of QCD coupling constant α_s. And the long-distance behavior of the transition of $Q\bar{Q}[^{2S+1}L_J,\underline{a}]$ to heavy quarkonium through the emission of soft gluons is parameterized by a non-perturbative matrix element, which is weighted by the power of heavy quark velocity v. One crucial feature of NRQCD is that it allows the contribution of heavy quark pair in color-octet configuration over short distance.

In this work, the desired factorization formula for the processes under consideration is:

$$d\Gamma(\eta_b\to H+c\bar{c})=\sum_{m,n}d\hat{\Gamma}(m,n)\langle\eta_b|\mathcal{O}(m)|\eta_b\rangle\langle\mathcal{O}^H(n)\rangle, \tag{1}$$

where H represents the charmonium J/ψ, η_c or χ_{cJ}, $d\hat{\Gamma}(m,n)$ is the short-distance factor for $b\bar{b}$ pair in m state to decay into $c\bar{c}$ pair in n state in association with an open $c\bar{c}$ pair, where m,n denote collectively the total spin, orbital angular momentum and color of the heavy quark pair. The relative importance of each term in the NRQCD factorization formula Eq. (1) depends on the orders of the double expansion parameters α_s and v.

In the associated production of charmonia in η_b decay, both the contributions of $c\bar{c}$ in the color-singlet configuration and those in the color-octet configuration start at α_s^4. For J/ψ and η_c cases, though the color-octet matrix elements $\langle\mathcal{O}_8^{\psi}(^3S_1)\rangle$ and $\langle\mathcal{O}_8^{\eta_c}(^3S_1)\rangle$ are suppressed by v_c^4 relative to the color-singlet $\langle\mathcal{O}_1^{\psi}(^3S_1)\rangle$ and $\langle\mathcal{O}_1^{\eta_c}(^1S_0)\rangle$ accordingly, the kinematic enhancement of the $c\bar{c}[^3S_1,\underline{8}]$ produced via g^* fragmentation, which subsequently evolves into physical state J/ψ and η_c through double E1 transitions and M1 transition respectively may compensate for the suppression of matrix elements in part. And for P-wave states χ_{cJ}, the contributions of color octet $c\bar{c}[^3S_1,\underline{8}]$ are in the same order as those of color-singlet $c\bar{c}[^3P_J,\underline{1}]$. So we will also consider the contribution of $c\bar{c}[^3S_1,\underline{8}]$ to each decay channel $\eta_b\to J/\psi(\eta_c,\chi_{cJ})+c\bar{c}$ through $b\bar{b}[^1S_0,\underline{1}]\to c\bar{c}[^3S_1,\underline{8}]+c\bar{c}$. In principle, the $b\bar{b}$ in a color-octet configurations also have contributions at order $\alpha_s^4 v_b^4$. However, such contributions are suppressed by a factor of $(v_b/v_c)^4$ compared with the contributions of color-octet $c\bar{c}$, and will be ignored in this work.

3 Color-singlet contributions to $\eta_b\to J/\psi(\eta_c,\chi_{cJ})+c\bar{c}$

There are two Feynman diagrams shown in Fig. 1 for the color singlet processes. The short-distance factors $d\hat{\Gamma}(m,n)$ in the factorization formula Eq. (1) can be obtained by embodying spinor projection method [33] to project the parton level Feynman amplitude $\mathcal{M}$ for $b(p_b)\bar{b}(p_{\bar{b}})\to c(p_c)\bar{c}(p_{\bar{c}})+c(p_2)\bar{c}(p_3)$ onto certain channels with $b(p_b)\bar{b}(p_{\bar{b}})$ pair in m state and $c(p_c)\bar{c}(p_{\bar{c}})$ pair in n state. For $b\bar{b}[^1S_0,1](p_{\eta_b})\to c\bar{c}[^{2S+1}L_J,\underline{1}](p_1)+c(p_2)\bar{c}(p_3)$, where

$$p_b=\frac{1}{2}p_{\eta_b}+q_b,\ p_{\bar{b}}=\frac{1}{2}p_{\eta_b}-q_b, \tag{2}$$

$$p_c=\frac{1}{2}p_1+q_c,\ p_{\bar{c}}=\frac{1}{2}p_1-q_c, \tag{3}$$

and q_Q is the relative momentum, the projected Feynman amplitude is:

$$\mathcal{M}(b\bar{b}[^1S_0,1](p_{\eta_b})\to c\bar{c}[^{2S+1}L_J,\underline{1}](p_1)+c(p_2)\bar{c}(p_3))=$$
$$\sum_{s_1,s_2}\sum_{i,l}\sum_{L_z,S_z}\sum_{s_3,s_4}\sum_{k,l}\langle s_1;s_2\mid 00\rangle\langle 3i;\bar{3}j\mid 1\rangle\times\langle s_3;s_4\mid SS_z\rangle\langle LL_z;SS_z\mid JJ_z\rangle\langle 3k;\bar{3}l\mid 1\rangle\times$$
$$\begin{cases}\mathcal{M}(b_i(p_b,s_1)\bar{b}_j(p_{\bar{b}},s_2)\to c_k(p_c,s_3)\bar{c}_l(p_{\bar{c}},s_4)+c(p_2)\bar{c}(p_3)) & (L=S),\\ \epsilon_\alpha^*(L_Z)\mathcal{M}^\alpha(b_i(p_b,s_1)\bar{b}_j(p_{\bar{b}},s_2)\to c_k(p_c,s_3)\bar{c}_l(p_{\bar{c}},s_4)+c(p_2)\bar{c}(p_3)) & (L=P),\end{cases} \tag{4}$$

where $\langle 3i;\bar{3}j\mid 1\rangle=\delta_{ij}/\sqrt{N_c}$, $\langle 3k;\bar{3}l\mid 1\rangle=\delta_{kl}/\sqrt{N_c}$, $\langle s_1;s_2\mid 00\rangle$, $\langle s_3;s_4\mid SS_z\rangle$ and $\langle LL_z;SS_z\mid JJ_z\rangle$ are the $SU(3)$-color, $SU(2)$-spin and angular momentum Clebsch-Gordan (C-G) coefficients for $Q\bar{Q}$ projecting on certain appropriate configurations at short distance. And $\mathcal{M}^\alpha$ is the derivative of the amplitude with respect to the relative momentum q_c^α.

The projection operator of Dirac spinor can be written in the form of product of γ matrixes. For spin-singlet $b\bar{b}$ annihilation, up to all order of v_b, we have [34]:

$$\sum_{s_1s_2}\langle s_1;s_2\mid 00\rangle u(p_b;s_1)\bar{v}(p_{\bar{b}},s_2)=\frac{-1}{2\sqrt{2}(E_b+m_b)}\times$$
$$(\not{p}_b+m_b)\frac{\not{p}_{\eta_b}+2E_b}{2E_b}\gamma_5(\not{p}_{\bar{b}}-m_b). \tag{5}$$

where $E_b=\sqrt{p_{\eta_b}^2}/2\approx m_b$ in the non-relativistic limit. Similarly, for the production of $c\bar{c}$ pair in spin-singlet and spin-triblet, the expressions of the spinor projectors are given in Eq. (6a) and Eq. (6b) respectively:

$$\sum_{s_3s_4}\langle s_3;s_4\mid 00\rangle v(p_c;s_3)\bar{u}(p_{\bar{c}},s_4)=\frac{-1}{2\sqrt{2}(E_c+m_c)}(\not{p}_c-m_c)\gamma_5\frac{\not{p}_1+2E_c}{2E_c}(\not{p}_{\bar{c}}+m_c), \tag{6a}$$

$$\sum_{s_3s_4}\langle s_3;s_4\mid 1S_z\rangle v(p_c;s_3)\bar{u}(p_{\bar{c}},s_4)=\frac{-1}{2\sqrt{2}(E_c+m_c)}(\not{p}_c-m_c)\not{\epsilon}^*(S_z)\frac{\not{p}_1+2E_c}{2E_c}(\not{p}_{\bar{c}}+m_c) \tag{6b}$$

where $\sqrt{p_1^2}=2E_{\rm c}\approx 2m_{\rm c}$ in the non-relativistic limit.

For S-wave spin-triblet J/ψ, the sum over its spins is:

$$\Pi_{\mu\nu}=\sum_{S_z}\epsilon^*_\mu(S_z)\epsilon_\nu(S_z)=-g_{\mu\nu}+\frac{p_{1\mu}p_{1\nu}}{p_1^2}. \quad (7)$$

For P-wave state we introduce $\epsilon^{J*}_{\alpha\beta}(J_z)$ to describe the L-S coupling which is defined as:

$$\sum_{L_zS_z}\epsilon^*_\alpha(L_z)\epsilon^*_\beta(S_z)\langle 1L_z;1S_z|JJ_z\rangle=\epsilon^{J*}_{\alpha\beta}(J_z) \quad (8)$$

and the sums over their all possible polarization states for $J=0,1,2$ are:

$$\sum_{J_z}\epsilon^{0*}_{\alpha\beta}(J_z)\epsilon^0_{\alpha1\beta1}(J_z)=\frac{1}{3}\Pi_{\alpha\beta}\Pi_{\alpha1\beta1}, \quad (9a)$$

$$\sum_{J_z}\epsilon^{1*}_{\alpha\beta}(J_z)\epsilon^1_{\alpha1\beta1}(J_z)=\frac{1}{2}(\Pi_{\alpha\alpha1}\Pi_{\beta\beta1}-\Pi_{\alpha\beta1}\Pi_{\beta\alpha1}), \quad (9b)$$

$$\sum_{J_z}\epsilon^{2*}_{\alpha\beta}(J_z)\epsilon^2_{\alpha1\beta1}(J_z)=\frac{1}{2}(\Pi_{\alpha\alpha1}\Pi_{\beta\beta1}+\Pi_{\alpha\beta1}\Pi_{\beta\alpha1})-\frac{1}{3}\Pi_{\alpha\beta}\Pi_{\alpha1\beta1}. \quad (9c)$$

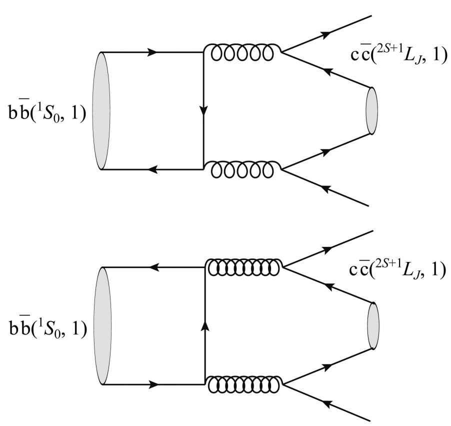

Fig. 1. Feynman diagrams for $b\bar{b}[^1S_0,1]\rightarrow c\bar{c}[^{2S+1}L_J,\underline{1}]+c+\bar{c}$.

The three-body decay process $b\bar{b}[^1S_0,1](p_{\eta_b})\rightarrow c\bar{c}[^{2S+1}L_J,\underline{1}](p_1)+c(p_2)+\bar{c}(p_3)$ can be described in terms of the energy fraction x_i in the rest frame of $b\bar{b}$:

$$x_i=\frac{2p_{\eta_b}\cdot p_i}{m^2_{\eta_b}}, \quad \sum x_i=2\ . \quad (10)$$

The three-body phase space $d\Phi_3$ is given by

$$d\Phi_3=\prod_{i=1,2,3}\frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}\cdot(2\pi)^4\delta^4(p_{\eta_b}-p_1-p_2-p_3)=\frac{m^2_{\eta_b}}{2(4\pi)^3}dx_i\delta\left(2-\sum x_i\right)\ . \quad (11)$$

The variable x_3 can be integrated out by applying the delta function, and the phase space then becomes

$$d\Phi_3=\frac{m^2_{\eta_b}}{2(4\pi)^3}\int_{x_1^{\rm min}}^{x_1^{\rm max}}dx_1\int_{x_2^{\rm min}}^{x_2^{\rm max}}dx_2\ . \quad (12)$$

The integral's limits $x_1^{\rm min}=2r$, $x_1^{\rm max}=1$,

$$x_2^{\rm min}=\frac{1}{2}\left(2-x_1-\sqrt{\frac{(1-x_1)(x_1-2r)(2r+x_1)}{1+r^2-x_1}}\right),$$

$$x_2^{\rm max}=\frac{1}{2}\left(2-x_1+\sqrt{\frac{(1-x_1)(x_1-2r)(2r+x_1)}{1+r^2-x_1}}\right)$$

are determined by 4-momentum conservation, where $r=m_{\rm c}/m_{\rm b}$.

We compute the short-distance part $|\mathcal{M}|^2$ straightforwardly with the help of the spinor projection method, and at leading order in $v_{\rm b}$ and $v_{\rm c}$, $d\hat{\Gamma}(m,n)$ is given as

$$d\hat{\Gamma}(b\bar{b}[^1S_0,1](p_{\eta_b})\rightarrow c\bar{c}[^{2S+1}L_J,\underline{1}](p_1)+c(p_2)\bar{c}(p_3))=\frac{1}{2m_{\eta_b}}\frac{|\mathcal{M}|^2_J}{m_{\rm b}m_{\rm c}(2J+1)}d\Phi_3 \quad (13)$$

where the factors $m_{\rm b}$, $m_{\rm c}$ and $2J+1$ come from the normalization of the NRQCD four-fermion operators and J is the angular momentum of the $c\bar{c}$ pair. The analytic expressions of $|\mathcal{M}|^2$ for S-wave η_c and J/ψ are given by

$$|\mathcal{M}|^2_{\eta_c}=\frac{512C_F^2\pi^4\alpha_s^4}{C_A m_b^4(-2+x_1)^2(-1+x_2)^2(-1+x_1+x_2)^2}\{r^2\ (x_1{}^2+4x_1\ (-2+x_2)+4\ (2+(-2+x_2)\ x_2))-\left((-1+x_2)\ (-1+x_1+x_2)\ (-2+x_1+2x_2)^2\right)\}\ , \quad (14)$$

$$|\mathcal{M}|^2_{J/\psi}=\frac{512\alpha_s^4C_F^2\pi^4}{C_A m_b^4(-2+x_1)^2(-1+x_2)^2(-1+x_1+x_2)^2}\{r^2\ (-1+2x_1)(x_1{}^2+4x_1\ (-2+x_2)+4\ (2+(-2+x_2)\ x_2))-(-1+x_2)(-1+x_1+x_2)(9x_1{}^2+4x_1(-5+x_2)+4(3+(-2+x_2)x_2))\} \quad (15)$$

where $C_F=\dfrac{4}{3}$, $C_A=3$. For P-wave χ_{cJ}, the expressions are rather complicated, we will only give their numerical results here.

The long-distance matrix elements $\langle\eta_b|\mathcal{O}(^1S_0)|\eta_b\rangle$, $\langle\mathcal{O}^{\eta_c}(^1S_0)\rangle$, $\langle\mathcal{O}^{J/\psi}(^3S_1)\rangle$, $\langle\mathcal{O}^{\chi_{cJ}}(^3P_J)\rangle$ are estimated through their relations with the non-relativistic wave functions:

$$\langle \eta_b | \mathcal{O}(^1S_0) | \eta_b \rangle = \frac{2N_c |R^b_{1S}(0)|^2}{4\pi} \ , \tag{16a}$$

$$\langle \mathcal{O}^{\eta_c}(^1S_0) \rangle = \frac{\langle \mathcal{O}^{J/\psi}(^3S_1) \rangle}{3} = \frac{2N_c |R^c_{1S}(0)|^2}{4\pi} \ , \tag{16b}$$

$$\frac{\langle \mathcal{O}^{\chi_{cJ}}(^3P_J) \rangle}{2J+1} = \frac{2N_c 3 |R'^c_{1P}(0)|^2}{4\pi} \ . \tag{16c}$$

We choose the following potential model results [35] as inputs

$$|R^b_{1S}(0)|^2 = 6.477 \text{ GeV}^3, \ |R^c_{1S}(0)|^2 = 0.81 \text{ GeV}^3,$$

$$|R'^c_{1P}(0)|^2 = 0.075 \text{ GeV}^5 \,. \tag{17}$$

Taking $m_b = 4.65$ GeV, $m_c = 1.5$ GeV, $\alpha_s(m_b) = 0.22$ and completing the phase space integrals numerically, we obtain:

$$\Gamma(\eta_b \to \eta_c + c\bar{c}) \approx 2.57 \times 10^{-1} \text{ keV}, \tag{18a}$$

$$\Gamma(\eta_b \to J/\psi + c\bar{c}) \approx 2.79 \times 10^{-1} \text{ keV}, \tag{18b}$$

$$\Gamma(\eta_b \to \chi_{cJ} + c\bar{c}) \approx (3.92, 2.37, 1.20) \times 10^{-2} \text{ keV}$$

$$(\text{for } J = 0, 1, 2). \tag{18c}$$

The color-singlet contributions to the re-scaled energy distribution $d\Gamma/dx_1$ of η_c and J/ψ are shown in Fig. 2, and the decay widths as functions of the energy ratios of χ_{cJ} to η_b are shown in Fig. 3.

The decay width of $\eta_b \to J/\psi + c\bar{c}$ has already been obtained in Ref. [36], and we find agreement after taking into account the difference of the values of the input parameters.

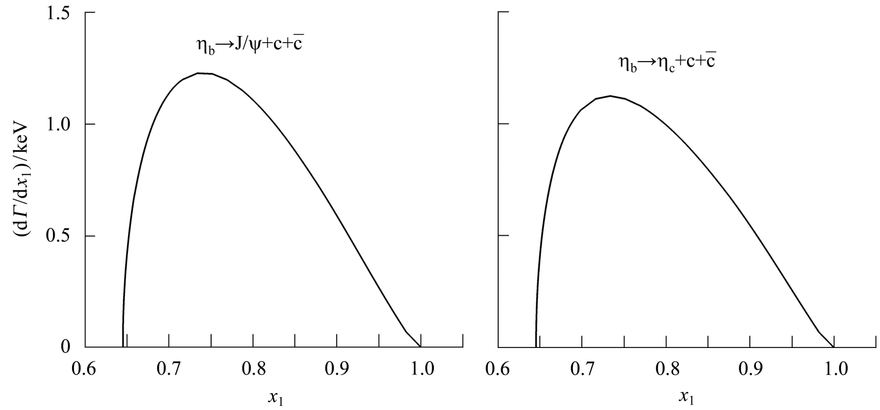

Fig. 2. The color-singlet contribution to the re-scaled J/ψ (left) and η_c (right) energy distributions in the processes of $\eta_b \to J/\psi + c + \bar{c}$ and $\eta_b \to \eta_c + c + \bar{c}$.

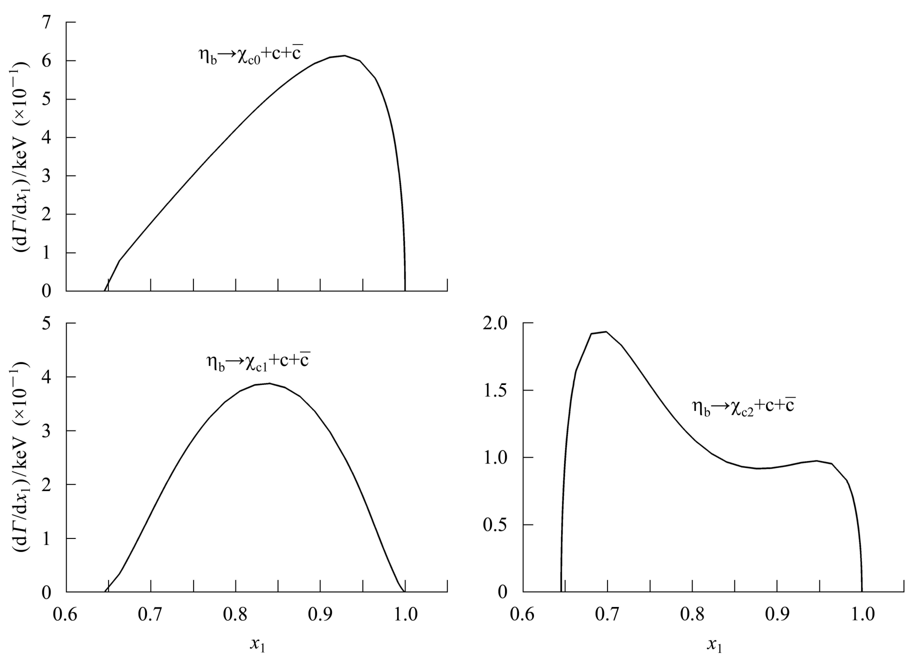

Fig. 3. The color-singlet contribution to the re-scaled χ_{cJ} (right) energy distributions in the processes of $\eta_b \to \chi_{cJ} + c + \bar{c}$. The upper curve is for J=0, and the lower in the left (right) is for J=1 (2).

4 Color-octet contribution to $\eta_b \to J/\psi(\eta_c, \chi_{cJ}) + c\bar{c}$

Now we proceed to estimate the color-octet $b\bar{b}[^1S_0,1](p_{\eta_b}) \to c\bar{c}[^3S_1,\underline{8}](p_1)+c(p_2)\bar{c}(p_3)$ contribution to the decay widths of $\eta_b \to J/\psi(\eta_c,\chi_{cJ})+c\bar{c}$. There are four Feynman diagrams. Two of them have the same topologies as the color-singlet ones shown in Fig. 1, and the two $g^* \to c\bar{c}[^3S_1,\underline{8}]$ fragmentation diagrams are shown in Fig. 4.

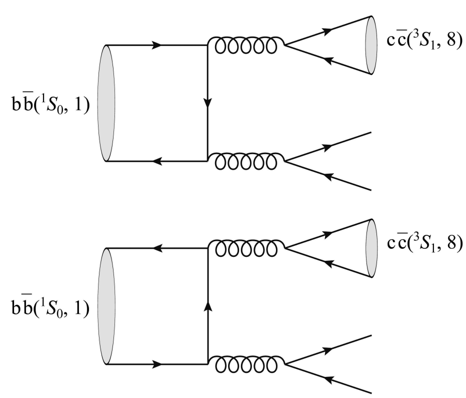

Fig. 4. Feynman diagrams for $b\bar{b}[^1S_0,1] \to c\bar{c}[^3S_1,\underline{8}]+c+\bar{c}$.

We calculate the Feynman amplitude in the same way as Section 3, except for replacing the color-singlet projector $\delta_{kl}/\sqrt{N_c}$ by the color-octet one $\sqrt{2}(T^a)_{kl}$. The color octet-matrix elements can be extracted from J/ψ and χ_{cJ} production phenomenologically. How to determine the values of color octet matrix elements have been discussed in lots of works (for examples, see Ref. [37]). In our numerical studies, we set

$$\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle = 1.06\times10^{-2}\,\mathrm{GeV}^3\,, \tag{19a}$$

$$\langle \mathcal{O}_8^{J/\psi}(^1S_0)\rangle = 1.0\times10^{-2}\,\mathrm{GeV}^3\,, \tag{19b}$$

$$\langle \mathcal{O}_8^{\chi_{c1}}(^3S_1)\rangle/m_c^2 = 1.0\times10^{-2}\,\mathrm{GeV}^3\,. \tag{19c}$$

In the non-relativistic limit, the ratios of color-octet matrix elements $\langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1)\rangle$ satisfy 1:3:5 for J=0,1,2 respectively. Because of heavy quark spin symmetry, we can derive the relation $\langle \mathcal{O}_8^{\eta_c}(^3S_1)\rangle = \langle \mathcal{O}_8^{J/\psi}(^1S_0)\rangle$. Then the color octet contributions are:

$$\Gamma_{\mathrm{octet}}(\eta_b \to \eta_c + c\bar{c}) = 9.2\times10^{-3}\,\mathrm{keV}\,, \tag{20a}$$

$$\Gamma_{\mathrm{octet}}(\eta_b \to J/\psi + c\bar{c}) = 9.8\times10^{-3}\,\mathrm{keV}\,, \tag{20b}$$

$$\Gamma_{\mathrm{octet}}(\eta_b \to \chi_{cJ} + c\bar{c}) = (0.31, 0.92, 1.54)\times 10^{-2}\,\mathrm{keV}\quad (\text{for } J=0,1,2)\,. \tag{20c}$$

From the numerical results in Eq. (13) we find that the color-octet contributions to η_b decays into η_c and J/ψ in association with a charm-quark pair are only about 3%−4% of the color-singlet results. In the case of P-wave states χ_{cJ}, since the color-octet matrix elements are of the same order in v as the color-singlet matrix elements, the color-octet contributions are as important as the color singlet ones. The ratios of the color-octet contributions to the color-singlet ones are 8%, 39%, 130% for $J=0,1,2$ respectively.

Piecing the color-singlet and color-octet contributions together, we then get the partial widths

$$\Gamma_{\mathrm{total}}(\eta_b \to \eta_c + c\bar{c}) = 2.66\times10^{-1}\,\mathrm{keV}\,, \tag{21a}$$

$$\Gamma_{\mathrm{total}}(\eta_b \to J/\psi + c\bar{c}) = 2.89\times10^{-1}\,\mathrm{keV}\,, \tag{21b}$$

$$\Gamma_{\mathrm{total}}(\eta_b \to \chi_{cJ} + c\bar{c}) = (4.23, 3.29, 2.74)\times 10^{-2}\,\mathrm{keV}\quad (\text{for } J=0,1,2)\,. \tag{21c}$$

At LO in α_s and v_b, the total width of η_b could be estimated using its decay into two gluons:

$$\Gamma(\eta_b \to \text{anything}) \approx \Gamma(\eta_b \to gg) = \frac{2\alpha_s^2}{3m_b^2}|R_{1S}^b(0)|^2 = 9.67\ \mathrm{MeV}\,. \tag{22}$$

Finally, we obtain the branching ratios of η_b decay into charmonium in association with a charm-quark pair, which are:

$$Br(\eta_b \to \eta_c + c\bar{c}) = 2.75\times10^{-5}\,, \tag{23a}$$

$$Br(\eta_b \to J/\psi + c\bar{c}) = 2.99\times10^{-5}\,, \tag{23b}$$

$$Br(\eta_b \to \chi_{cJ} + c\bar{c}) = (4.37, 3.40, 2.83)\times 10^{-6}\ (\text{for } J=0,1,2)\,. \tag{23c}$$

5 Conclusion

In summary, we have calculated the partial widths of η_b decays into S- and P-wave charmonium states J/ψ, η_c and χ_{cJ} together with $c\bar{c}$-quark pair at leading order in α_s. In addition to calculating the contributions of the color-singlet channels, we also consider the color-octet contribution of $b\bar{b}[^1S_0,1] \to c\bar{c}[^3S_1,\underline{8}]+c\bar{c}$. We find that for the S-wave charmonium states, the color-octet contribution is so tiny and could be neglected, while for P-wave charmonium states the color-octet contribution is significant and can not be ignored. This result is very similar to the associated production of J/ψ, η_c and χ_{cJ} in e^+e^- annihilation at BaBar and Belle [27]. It is because in η_b decay or e^+e^- annihilation at B-factories, the energy scale is only about 10 GeV, in which the fragmentation effect is not prominent. As was found in Ref. [30], only in the large p_t region in $p\bar{p}$ collisions at Tevatron, do the color-octet effects for the associated production dominate.

From the perspective of duality, the decay of $\eta_b \to c\bar{c}c\bar{c}$ contains all the processes with at least four charm quarks in the final states. In this work, we have considered η_c+$c\bar{c}$, J/ψ+$c\bar{c}$ and χ_{cJ}+$c\bar{c}$ production in η_b decay. If we add all these contributions together and compare the branching ratios with the upper limit[1)] of $Br(\eta_b \to c\bar{c}c\bar{c}) = 8.7\times10^{-5}$ given in Ref. [14], we find $Br(\eta_b \to \text{charmonia} + c\bar{c}) = 6.8\times10^{-5}$, which almost saturates the four-charm decay of η_b.

In BaBar experiments, they used about 1×10^8 $\Upsilon(3S)$ and 0.92×10^8 $\Upsilon(2S)$ samples and measured $Br(\Upsilon(3S) \to \gamma\eta_b) \approx 4.8\times10^{-4}$ and $Br(\Upsilon(2S) \to \gamma\eta_b) \approx 4.2\times10^{-4}$. Then there are about 1×10^5 η_b events produced. So in the current stage it might not be feasible to find the decay of η_b to charmonium together with a charm-quark pair. However, the associated charmonium production is an interesting mechanism on both experimental and theoretical sides, and these measurements may be possible in the future Super-B project.

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1) We normalize their result by a naive factor of $(\frac{0.22}{0.182})^4$ owing to the different choice of coupling constant