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Analysis of the impacts of mechanical errors on the RF performance of a single spoke cavity

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Abstract Mechanical errors can not be avoided in fabrication. They will cause geometry errors and have impacts on the cavity performance. This paper systematically analyzes the impacts of mechanical errors on the RF performance of Peking University single spoke cavity. The various kinds of shape and size errors are considered, the influences on the resonation frequency and field flatness are studied by numerical simulation and the theoretical models are analyzed. The results show that the single spoke cavity is robust with respect to the mechanical tolerance. It also indicates the most essential factors for fabrication.

Key words single spoke cavity, mechanical errors, field flatness, perturbation

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1 Introduction

A 450 MHz β=0.2 single spoke cavity has been designed at Peking University. The major radio frequency (RF) parameters of the cavity are $B_{\mathrm{pk}}/E_{\mathrm{acc}}$= 5.22 mT/(MV/m) and $E_{\mathrm{pk}}/E_{\mathrm{acc}}$=2.65 [1]. Fabrication is in progress. Since all parts of the cavity are formed by machining or deep drawing and assembled by electron beam welding (EBW), mechanical errors cannot be avoided. These errors in the geometry may affect the RF performance. In this paper reasons for producing errors are analyzed, influences on major RF parameters are studied by a numerical simulation program, and also theoretical explanations are given for these impacts of mechanical errors.

2 Mechanical errors

Before studying and simulating obvious mechanical errors which cannot be ignored, one has to analyze them. The shape errors of deep drawing components are difficult to control and measure. On the other hand, the RF surface states and major sizes of the deep drawing parts are strengthened by dies. One requires a mechanical tolerance of the dies of under ±0.05 mm and a surface roughness of less than 0.8 μm. Under these conditions it is assumed that the deep drawing parts have no mechanical errors. Additional, the spoke bar part will be reshaped by moulds, which repairs the mechanical errors brought in by assembling and welding. This determines that welded components have no size errors. Under these assumptions there are no shape errors on the end-walls and the spoke bar in the center of the cavity. All mechanical errors, including the cavity total length, the asymmetry of both accelerating gaps, the parallelism of both end-walls, and the shrinkage due to the EBW, occur at the round edges of the last EBW. This study focuses on the two welded edges and gives the mechanical tolerances.

The major RF parameters affected by mechanical errors are the resonant frequency and the field flatness. If the resonant frequency has a deviation from the operation frequency, the power source will lose much RF power in order to drive the cavity to operate at the maximum accelerating gradient. If the field flatness has some errors, the accelerating gradient will be limited by the field enhanced gap, and the cavity cannot achieve a higher gradient. The influences of mechanical errors on both aspects are discussed in detail in this paper.

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3 Field flatness

The field flatness portrays the symmetry of both gaps between the spoke bar and irises on the end-walls. The most essential parameter is the length difference of both accelerating gaps. Its sensitivity to the field flatness can be studied by a numerical simulation program.

Based on the geometry of the single spoke cavity shown in Fig. 1, the simulation sweeps the coordinates of both end-walls to vary the length of each gap and the asymmetry. Assuming that the mechanical tolerance cannot be larger than 1 mm, the range of variations of the coordinates of each end-wall is limited by ±1 mm. It is also assumed that the length of the left gap is larger than, or equal to that of the right side. The coaxial line resonator model shows that the electromagnetic field in the right gap will be stronger.

$$\Delta X = L_l - L_r \geqslant 0,$$

$$E_r/E_l \geqslant 1,$$

Here, L_l and L_r are the distances from the cavity center to the left and right end walls; ΔX is the dimensional difference between both accelerating gaps; E_l and E_r are the peak electric fields along the beam line in the left and right gap, respectively.

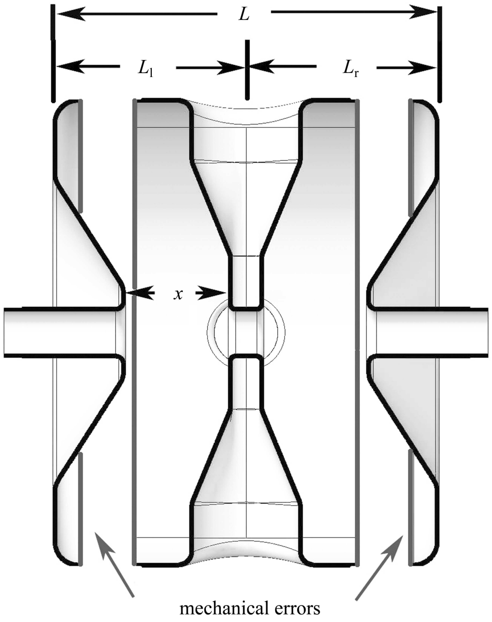

Fig. 1. Geometry of single spoke cavity, and the major mechanical errors in the considered region.

The electric field distribution on the beam line axis is obtained from simulation results. Fig. 2 gives partial results obtained for various conditions. The values of the peak fields are used in data processing and the calculation.

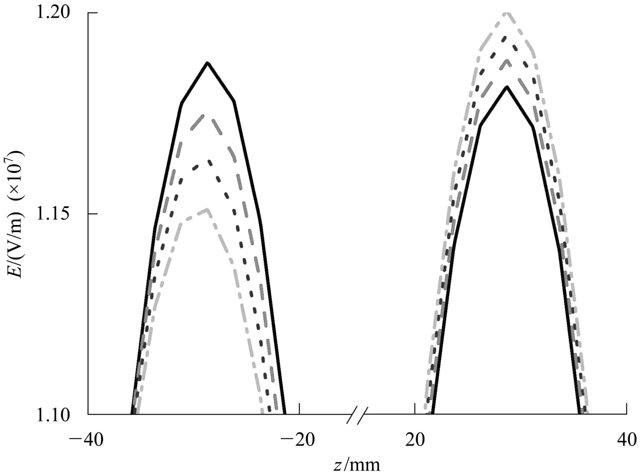

Fig. 2. Electric field distributions on the beam line axis. L_r=92 mm and L_l is swept from 92 mm to 93.5 mm. —— L_l=92 mm; – · – · – L_l=92.5 mm; – – – – L_l =93 mm; — — — L_l= 93.5 mm.

The relationship between ΔX and E_r/E_l is listed in Table 1, and fitted by a linear expression as follows.

$$E_r/E_l = 0.0321 \times \Delta X + 0.9996, \tag{1}$$

The correlation coefficient is 0.99983.

Table 1. Peak electric field ratios. The values of E_r/E_l are mean values for different conditions, and normalized to 1 at ΔX=0.

ΔX/mm	E_r/E_l	ΔX/mm	E_r/E_l
0	1.000	1.25	1.040
0.25	1.008	1.50	1.048
0.50	1.016	1.75	1.056
0.75	1.024	2.00	1.064
1.00	1.032		

The field flatness is usually defined as [2]

$$\text{flatness} = \Delta\varphi_{\text{average}}/\Delta\varphi_{\max}, \tag{2}$$

In the "bead pulling" method to measure the field profile, $\Delta\varphi$ is the phase shift of the microwave in case that perturbations exist in the accelerating gap. The amplitude difference in each gap can be expressed through the frequency shifts [3, 4]. In first order perturbation theory [5], Eq. (2) can be written as

$$\text{flatness} = 0.5 \times \left[(E_{\min}/E_{\max})^2 + 1\right], \tag{3}$$

So the field flatness is directly related to $(E_r/E_l)^2$.

If the spoke cavity is required to limit the field flatness above 95% after fabrication without flatness tuning, ΔX only needs to be less than 1.70 mm. It means that the asymmetry has little influence on the field flatness, and the spoke cavity is robust with respect to the mechanical tolerance. Tuning of the spoke cavity after fabrication in order to obtain a flat field profile is not necessary, since typical fabrication errors will not appreciably affect the field profile.

4 Resonant frequency

The resonant frequency of the spoke cavity is mainly determined by the length of the spoke bar and the distance from the end-wall to the spoke bar. The first factor is the inductance in the equivalent circuit, while the second one is the capacitance. However, all geometry errors will influence the resonant frequency based on shape perturbations. Every possible error brought in by machining or welding should be considered. The same model as for the front section is used in the numerical simulation.

First, the total length of the spoke cavity is varied. A symmetrical cavity is assumed,

$$L_\mathrm{l}=L_\mathrm{r}.$$

The results are listed in Table 2. The referenced frequency is 449.856 MHz for ΔL=0. ΔL is the difference between the actual and the design value of total length of the cavity, from the end wall to the other.

Table 2. Frequency shifts under symmetrical conditions.

ΔL/mm	Δf/kHz	ΔL/mm	Δf/kHz
−2.0	−1947	0.5	472
−1.5	−1460	1.0	942
−1.0	−981	1.5	1401
−0.5	−481	2.0	1865
0	0		

Table 3. Frequency shift under asymmetrical conditions and a cavity length of 185 mm.

ΔX/mm	Δf/kHz
0	0
0.5	−5.7
1.0	−28.3
1.5	−52.0
2.0	−83.2

A linear fit gives the following relationship

$$\Delta f=954\times\Delta L-21. \tag{4}$$

The correlation coefficient is 0.99983 and the frequency shift is about 95 kHz/0.1 mm.

Secondly, the asymmetry is studied. The total length of the cavity is assumed to be a constant. Due to the above results, the total length is controlled between 184.75 mm and 185.25 mm in the simulation, while the design dimension is 185 mm. The calculated frequency shifts are listed in Table 3 and Table 4. The three groups of data are fitted and a mean value of the frequency shift rate of −4.3 kHz/0.1 mm is obtained.

Table 4. Frequency shift under asymmetrical conditions. The cavity length of the second series is 184.75 mm and that of the third one is 185.25 mm.

ΔX/mm	Δf_1/kHz	Δf_2/kHz
0.25	−240.2	236.1
0.75	−258.6	226.2
1.25	−274.3	195.5
1.75	−304.7	171.1

Finally, the parallelism is considered. According to the simulation results, the angle variation between the spoke bar and the end-walls will cause a frequency shift of less than 2.5 kHz/0.1° and of that between both end-walls of less than 8 kHz/0.1°.

The calculation results of the influences on the resonant frequency show that the cavity's total length is the only key factor to control the resonant frequency in the fabrication process. The effects of other factors are not dramatic. The mechanical tolerance of cavity's total length should be less than 0.2 mm, including machining errors and welding shrinkage.

5 Analysis of results

The analysis with the physics model of the spoke cavity shows that the field flatness is not sensitive to the symmetry. The spoke cavity can be simplified by a coaxial line resonator model, shorted at both ends with the capacitance in the equivalent circuit in the middle, while the inductance is placed at both ends of the inner conductor. The inner and outer conductor can be considered as counter electrodes in a capacitance, and the surface of each one is an equipotential plane at any time. The voltage between the inner and outer conductors around the periphery of the spoke bar is a constant. The electric field intensity can be expressed as

$$E=V/L_{\text{inner–outer}}.$$

Considering the geometry of the spoke cavity and the field profile of the accelerating field, the formula can be written as

$$E_{\mathrm{acc}}=V/L_{\mathrm{gap}}. \tag{5}$$

Substituting Eq. (5) into Eq. (3), one obtains

$$\text{flatness}=\frac{1}{2}\left[\left(\frac{2L_{\text{gap}}-\Delta X}{2L_{\text{gap}}+\Delta X}\right)^2+1\right]. \qquad (6)$$

ΔX can also be calculated from Eq. (6). For the flatness lager than 95%, ΔX must be less than 1.66 mm. It agrees with the numerical simulation results and proves that the method of analysis is feasible.

For the spoke cavity at Peking University, the designed L_{gap} is 31.5 mm, a little larger than 1/3 $L_{\text{iris–iris}}$. But ΔX is of the order of 1 mm or less, which is much smaller than $2L_{\text{gap}}$. It is obvious that a typical mechanical error ΔX will not affect the field flatness dramatically. It also gives an inference that if the geometric β is higher, or the resonant frequency is lower, the stability of the mechanical tolerance will be enhanced, because of the larger L_{gap}.

Resonant frequency shifts can be explained by the shape perturbations theory. The fractional change of the resonant frequency is then expressed [5] as

$$\frac{f-f_0}{f_0}=\frac{\int_{\Delta V}(\mu|H|^2-\varepsilon|E|^2)\mathrm{d}v}{\int_{V_0}(\mu|H|^2+\varepsilon|E|^2)\mathrm{d}v}. \qquad (7)$$

The electromagnetic field in the spoke cavity is assumed unchanged whether there exists a perturbation or not. The perturbation volume (ΔV) is much smaller than the total cavity volume (V_0). So Δf is a function of ΔV, and only the interval of integration in the numerator of the right side varies. Each condition of mechanical tolerance is considered by Eq. (7).

Assuming that the cavity total length reduces, it is equivalent to increasing the perturbation volume (ΔV). Since the electric field on the surface of the end-walls is more sensitive than the magnetic field, the effect of the electric field will play a major role for the perturbation. Therefore the resonator frequency will decrease with decreasing total length of the cavity. The approximate calculation with Eq. (7) was also attempted. The field energy density can be obtained by the numeral simulation software. The integration of the electric field energy density on the end wall surface gave 8.77 J/m and that of magnetic field gave 7.32 J/m, while the cavity storage energy was 1 J. If the cavity total length is 0.1 mm longer than the design, Eq. (7) gives the result

$$\frac{\Delta f}{f_0}=\frac{(7.32-8.77)\times(-10^{-4})}{1}=1.45\times10^{-4}.$$

The frequency shift is about 65 kHz/0.1 mm. This integration result is of the same order of magnitude as the pervious simulation result.

Considering the asymmetry of the accelerating gaps, the perturbation volumes of each side are converse and equal in value. For the whole cavity, the effects of the perturbation on both sides are counteracted, and it is equivalent to no perturbation. On the other hand, the field will become weaker if the position is farther off the spoke bar. The effect of the increasing perturbation volume is a little greater than that of the decreasing one. So the resonant frequency will marginally decrease if asymmetry exists.

Parallelism is analyzed in the same way as asymmetry. The influences of mechanical errors, except the cavity total length, are insignificant.

6 Conclusion

This paper systematically studies the influences of mechanical errors on the cavity RF performance. Only the cavity total length has an obvious effect on the frequency shift. Other factors can be ignored for typical fabrication errors. Also the single spoke cavity is robust, even tuning in order to obtain a flat field profile is not necessary. This study describes a research and analysis method for mechanical tolerances, and an experimental method of monitoring the quality of products will be developed based on it. It also gives some directions on the cavity fabrication, and focuses on the essential factors influencing mechanical tolerances.

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