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A test of CPT symmetry in K^0 vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ decays*

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Abstract I show that the CP-violating asymmetry in K^0 vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ decays differs from that in $K_L \to \pi^+\pi^-$, $K_L \to \pi^0\pi^0$ or the semileptonic K_L transitions, if there exists CPT violation in K^0-$\bar{K}^0$ mixing. A delicate measurement of this difference at a super flavor factory (e.g., the ϕ factory) will provide us with a robust test of CPT symmetry in the neutral kaon system.

Key words K^0-$\bar{K}^0$ mixing, CPT violation

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1 The motivation

The CPT theorem claims that a Lorentz-invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry [1]. It is so far so good, because there is no convincing experimental hint at CPT violation [2]. The breaking of CPT symmetry, as expected in some "exotic" scenarios of new physics beyond the standard model (e.g., string theory) [3], would be a big deal. In any case, much more experimental tests of this theorem are desirable.

The K^0-$\bar{K}^0$ mixing system has been playing an important role in particle physics for testing fundamental symmetries (such as CP, T and CPT) and examining conservation laws (such as $\Delta S = \Delta Q$). The existing experimental evidence for CPT invariance in the mixing and decays of neutral kaon mesons remains rather poor [2]: it is not excluded that the strength of CPT-violating interactions could be as large as about ten percentage of that of CP-violating interactions. This unsatisfactory situation will be improved in the near future, in particular after a variety of more delicate measurements are carried out at a super flavor factory [4] (e.g., the ϕ factory [5]).

There are several possibilities of probing CPT violation in K^0-$\bar{K}^0$ mixing with the decays of K_S and K_L mesons into the two-pion and (or) the semileptonic states [2]. A different approach towards testing CPT symmetry, with the help of neutral kaon decays into the three-pion states, has also been pointed out in Ref. [6]. The idea is simply that the CP-violating effect induced by K^0-$\bar{K}^0$ mixing in K^0 vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ transitions should not be identical to that in $K_L \to \pi^+\pi^-$, $K_L \to \pi^0\pi^0$ or the semileptonic K_L decays, if CPT symmetry is broken. Thus a careful comparison between these two types of CP-violating effects may provide us with a robust test of CPT invariance in K^0-$\bar{K}^0$ mixing.

An unfortunate fact is that no attention has so far been paid to the method advocated in Ref. [6]. In this talk, which is more or less an advertisement, I shall explain why a test of CPT symmetry is possible by measuring the time-dependent CP-violating asymmetry between $K^0(t) \to \pi^+\pi^-\pi^0$ and $\bar{K}^0(t) \to \pi^+\pi^-\pi^0$ decays. My result is hopefully useful for the upcoming experiments of kaon physics.

2 The idea

Let me outline the main idea. The mass eigenstates of K^0 and $\bar{K}^0$ can in general be written as

$$
\begin{aligned}
|K_S\rangle &= \frac{1}{\sqrt{|p_1|^2+|q_1|^2}}\left(p_1|K^0\rangle + q_1|\bar{K}^0\rangle\right) , \\
|K_L\rangle &= \frac{1}{\sqrt{|p_2|^2+|q_2|^2}}\left(p_2|K^0\rangle - q_2|\bar{K}^0\rangle\right) , \qquad (1)
\end{aligned}
$$

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in which p_i and q_i (for $i=1,2$) are complex mixing parameters. Note that $p_1 = p_2$ and $q_1 = q_2$ follow from CPT invariance [7]. The traditional characteristic quantities of CP violation in the K^0-$\bar{K}^0$ mixing system [2], η_{+-}, η_{00} and δ_L, are all related to K_L decays and thus the (p_2, q_2) parameters. For example,

$$\delta_L \equiv \frac{|p_2|^2 - |q_2|^2}{|p_2|^2 + |q_2|^2} \tag{2}$$

in the absence of $\Delta S = -\Delta Q$ interactions. A measurement of CP violation associated with

$$\delta_S = \frac{|p_1|^2 - |q_1|^2}{|p_1|^2 + |q_1|^2} \tag{3}$$

has been assumed to be extremely difficult, if not impossible, due to the rapid decay of the K_S meson to the two-pion state or the semileptonic state. Nevertheless, I shall show that δ_S can be measured from the rate asymmetry of K^0 and $\bar{K}^0$ mesons decaying into the three-pion state $\pi^+\pi^-\pi^0$. The difference between δ_S and δ_L signifies CPT violation in K^0-$\bar{K}^0$ mixing. This point can be seen more clearly if one adopts the popular (ϵ, δ) parameters to describe CP- and CPT-violating effects in the K^0-$\bar{K}^0$ mixing system [2]:

$$\begin{aligned} p_1 &= 1+\epsilon+\delta\,, \\ p_2 &= 1+\epsilon-\delta\,, \\ q_1 &= 1-\epsilon-\delta\,, \\ q_2 &= 1-\epsilon+\delta\,. \end{aligned} \tag{4}$$

Then

$$\begin{aligned} \delta_L &= 2\left(\mathrm{Re}\,\epsilon - \mathrm{Re}\,\delta\right)\,, \\ \delta_S &= 2\left(\mathrm{Re}\,\epsilon + \mathrm{Re}\,\delta\right)\,. \end{aligned} \tag{5}$$

It turns out that $\delta_S - \delta_L = 4\mathrm{Re}\,\delta$ is a clear signature of CPT violation [6].

Let me quote two typical experimental constraints on the CPT-violating parameter δ in K^0-$\bar{K}^0$ mixing: $\mathrm{Re}\delta = (2.9 \pm 2.6_{\rm stat} \pm 0.6_{\rm syst}) \times 10^{-4}$ obtained by the CPLEAR Collaboration [8] and $\mathrm{Im}\,\delta = (0.4 \pm 2.1) \times 10^{-5}$ obtained by the KLOE Collaboration [9]. A systematic analysis of the CP- and CPT-violating parameter space has already been done by the Particle Data Group in Ref. [2].

3 The approach

The CP eigenvalue for the $\pi^+\pi^-\pi^0$ final state is given by $(-1)^{l+1}$, where l is the relative angular momentum between π^+ and π^-. Since the sum of the masses of three pions is close to the kaon mass, the pions have quite low kinetic energy $E_{\rm CM}(\pi)$ in the kaon rest-frame, and the states with $l > 0$ are suppressed by the centrifugal barrier [10]. Thus the K_L meson decays dominantly into the kinematics-favored ($l=0$) and CP-allowed ($CP = -1$) $\pi^+\pi^-\pi^0$ state. The decay amplitude of $K_S \to \pi^+\pi^-\pi^0$ consists of both the kinematics-suppressed ($l = 1$) but CP-allowed (CP=+1) component, and the kinematics-favored ($l=0$) but CP-forbidden (CP=−1) component. This implies an interesting Dalitz-plot distribution for the $K_S \to \pi^+\pi^-\pi^0$ transition: it is symmetric with respect to π^+ and π^- for the CP-violating amplitude, but anti-symmetric for the CP-conserving amplitude. Let the ratio of K_S and K_L decay amplitudes be

$$\eta_{+-0} = \frac{A(K_S \to \pi^+\pi^-\pi^0)}{A(K_L \to \pi^+\pi^-\pi^0)}\,. \tag{6}$$

It is clear that η_{+-0} depends only upon the CP-violating component of $A(K_S \to \pi^+\pi^-\pi^0)$, if data are integrated over the whole Dalitz plot [10, 11]. The time-dependent rates for the initially pure K^0 and $\bar{K}^0$ states decaying into $\pi^+\pi^-\pi^0$, denoted by $\mathcal{R}(t)$ and $\bar{\mathcal{R}}(t)$ respectively, can be calculated with the help of Eqs. (1) and (6). I arrive at [6]

$$\begin{aligned} \mathcal{R}(t) &\propto \Big[|q_1|^2 + |q_2|^2|\eta_{+-0}|^2 \mathrm{e}^{-\Delta\Gamma t} + \\ &\quad 2\mathrm{Re}\left(q_1^* q_2 \eta_{+-0} \mathrm{e}^{\mathrm{i}\Delta m t}\right) \mathrm{e}^{-\Delta\Gamma t/2}\Big]\,, \\ \bar{\mathcal{R}}(t) &\propto \Big[|p_1|^2 + |p_2|^2|\eta_{+-0}|^2 \mathrm{e}^{-\Delta\Gamma t} - \\ &\quad 2\mathrm{Re}\left(p_1^* p_2 \eta_{+-0} \mathrm{e}^{\mathrm{i}\Delta m t}\right) \mathrm{e}^{-\Delta\Gamma t/2}\Big]\,, \end{aligned} \tag{7}$$

where $\Delta m > 0$ and $\Delta\Gamma > 0$ denote the mass difference and the width difference of K_S and K_L mesons, respectively. To a good degree of accuracy, I obtain the following CP-violating asymmetry:

$$\begin{aligned} \mathcal{A}(t) &\equiv \frac{\bar{\mathcal{R}}(t) - \mathcal{R}(t)}{\bar{\mathcal{R}}(t) + \mathcal{R}(t)} = \delta_S - \\ & 2\mathrm{e}^{-\Delta\Gamma t/2}\left[\mathrm{Re}\,\eta_{+-0}\cos(\Delta m t) - \mathrm{Im}\,\eta_{+-0}\sin(\Delta m t)\right]\xi - \\ & 2\mathrm{e}^{-\Delta\Gamma t/2}\left[\mathrm{Re}\,\eta_{+-0}\sin(\Delta m t) + \mathrm{Im}\,\eta_{+-0}\cos(\Delta m t)\right]\zeta\,, \end{aligned} \tag{8}$$

in which

$$\begin{aligned} \xi &= \frac{\mathrm{Re}(p_1 p_2^* + q_1 q_2^*)}{|p_1|^2 + |q_1|^2} = \\ &\quad 1 + \mathcal{O}(|\epsilon|^2) + \mathcal{O}(|\delta|^2) + \mathcal{O}(\mathrm{Re}(\epsilon\delta^*))\,, \\ \zeta &= \frac{\mathrm{Im}(p_1 p_2^* + q_1 q_2^*)}{|p_1|^2 + |q_1|^2} = \mathcal{O}(\mathrm{Im}(\epsilon\delta^*))\,. \end{aligned} \tag{9}$$

It is obvious that δ_S can be determined through the measurement of $\mathcal{A}(t)$. In particular, the relationship $\lim\limits_{t\to\infty} \mathcal{A}(t) = \delta_S$ holds.

As I have emphasized, the difference between δ_S and δ_L hints at CPT violation in K^0-$\bar{K}^0$ mixing. If $|\mathrm{Re}\,\delta|/\mathrm{Re}\,\epsilon \sim 0.1$, then the difference $\delta_S - \delta_L = 4\mathrm{Re}\,\delta$ can be as large as $0.4\mathrm{Re}\,\epsilon \sim 6.6\times10^{-4}$ in magnitude, where the experimental value $\mathrm{Re}\,\epsilon \approx 1.65\times10^{-3}$ has been used [2]. Since both ϵ and δ are small quantities, it turns out that $\xi \approx 1$ and $\zeta \approx 0$ are good approximations. Eq. (8) is therefore simplified to

$$\mathcal{A}(t) = \delta_S - 2e^{-\Delta\Gamma t/2}\Big[\mathrm{Re}\,\eta_{+-0}\cos(\Delta m t) - \mathrm{Im}\,\eta_{+-0}\sin(\Delta m t)\Big] . \qquad (10)$$

In the neglect of CPT violation, namely, $\delta_S = 2\mathrm{Re}\,\epsilon$, Eq. (10) can simply reproduce the result obtained in Ref. [10]. For illustration, I plot the behavior of $\mathcal{A}(t)$ in Fig. 1, in which $\delta_S = 3\times10^{-3}$ and $|\eta_{+-0}| = 5\times10^{-3}$ have typically been input. One may observe that $\mathcal{A}(t)$ approaches δ_S for $t \geqslant 5\tau_S$ and reaches δ_S if $t \geqslant 10\tau_S$, where τ_S is the mean lifetime of the K_S meson. This implies a certain feasibility to determine δ_S from the time-dependent CP-violating asymmetry $\mathcal{A}(t)$.

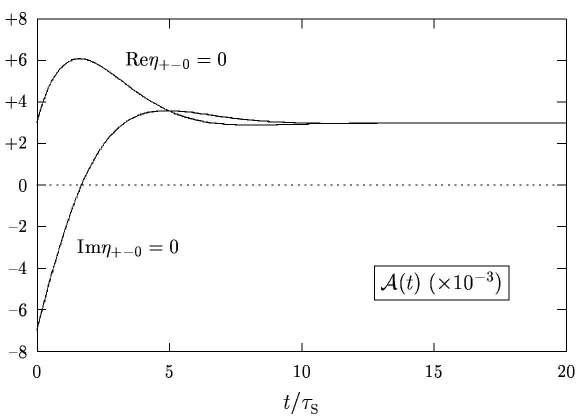

Fig. 1. An illustrative plot for the CP-violating asymmetry $\mathcal{A}(t)$ with the typical inputs $\delta_S = 3\times10^{-3}$ and $|\eta_{+-0}| = 5\times10^{-3}$ [6].

4 The discussion

In the above analysis I have taken an integration over the whole Dalitz plot, such that η_{+-0} solely contains the CP-violating part of $A(K_S \to \pi^+\pi^-\pi^0)$. To look at the CP-conserving component of $A(K_S \to \pi^+\pi^-\pi^0)$, one may study the phase-space regions $E_{CM}(\pi^+) > E_{CM}(\pi^-)$ and $E_{CM}(\pi^+) < E_{CM}(\pi^-)$ separately [10]. In this case the corresponding CP-violating asymmetries between $\bar{\mathcal{R}}(t)$ and $\mathcal{R}(t)$ take the same form as $\mathcal{A}(t)$ in Eq. (8) or Eq. (10), but η_{+-0} should be replaced by $(\eta_{+-0} \pm \lambda)$, where λ denotes the CP-conserving contribution to the ratio of K_S and K_L decay amplitudes [10]. Certainly, the CP-violating parameter δ_S can still be determined from measuring the time dependence of the relevant decay rate asymmetries.

An accurate measurement of δ_S from K^0 vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ should be feasible at the ϕ factory, where a huge amount of $K^0\bar{K}^0$ events can be coherently produced [5]. Choosing the semileptonic decay of one kaon to tag the flavor of the other kaon decaying into $\pi^+\pi^-\pi^0$ on the ϕ resonance, one should be able to construct the time-dependent rate asymmetry between $K^0(t) \to \pi^+\pi^-\pi^0$ and $\bar{K}^0(t) \to \pi^+\pi^-\pi^0$ decays in a way similar to Eq. (8). It is also expected that other super flavor factories may measure δ_S and δ_L to a good degree of accuracy.

Note that Lorentz invariance has been taken for granted in what I have discussed. As pointed out by Greenberg [12], "If CPT invariance is violated in an interacting quantum field theory, then that theory also violates Lorentz invariance". In my discussions, the dependence of the CPT-violating parameter δ on the sidereal time should in general be considered, since CPT violation may simultaneously imply a violation of Lorentz symmetry in the neutral kaon system. For simplicity, here I take δ to be a constant by assuming that the boost parameters of both K^0 and $\bar{K}^0$ are small and the corresponding Lorentz-violating effect is rotationally invariant in the laboratory frame [13]. In this approximation, my results are essentially valid as the averages over the sidereal time, such that the effect of Lorentz violation due to the direction of motion is negligible.

Finally, I like to mention that different approaches have been discussed to test CPT symmetry in D^0-$\bar{D}^0$, B^0_d-$\bar{B}^0_d$ or B^0_s-$\bar{B}^0_s$ mixing [14]. The idea presented here cannot directly be applied to those heavy neutral-meson systems. In this sense, it represents a unique way applicable in the K^0-$\bar{K}^0$ mixing system to test the CPT theorem.

5 The conclusion

To conclude, the CP-violating effect induced by K^0-$\bar{K}^0$ mixing in K^0 vs $\bar{K}^0 \to \pi^+\pi^-\pi^0$ decays is possible to deviate to some extent from that in $K_L \to \pi\pi$ or the semileptonic K_L transitions due to the violation of CPT symmetry. Measuring or constraining this tiny difference may serve as a robust test of CPT invariance in the neutral kaon system.

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