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Parton distribution functions and nuclear EMC effect in a statistical model*

SHAO Li-Jing(邵立晶) ZHANG Yun-Hua(张运华) MA Bo-Qiang(马伯强)1)

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

Abstract A new and simple statistical approach is performed to calculate the parton distribution functions (PDFs) of the nucleon in terms of light-front kinematic variables. Analytic expressions of x-dependent PDFs are obtained in the whole x region. And thereafter, we treat the temperature T as a parameter of the atomic number A to explain the nuclear EMC effect in the region $x\in[0.2,0.7]$. We give the predictions of PDF ratios, and they are very different from those by other models, thus experiments aiming at measuring PDF ratios are suggested to provide a discrimination of different models.

Key words statistical model, parton distribution functions, EMC effect

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1 Introduction

The nucleon structure functions, in terms of the parton distribution functions (PDFs), are badly desired in hadronic study. However, due to the complicated non-perturbative effect, we still have difficulty to calculate them absolutely from the first principal theory of quantum chromodynamics (QCD).

Various models according to the spirit of QCD have been brought forward, therein statistical ones, providing intuitive appeal and physical simplicity, have made amazing success [1–29]. Actually, as can be speculated, with partons bound in the wee volume of the nucleon, not only the dynamic, but also statistical properties, for example, the Pauli exclusion principle, should have important effect on the PDFs.

In order to avoid tough problems risen in the infinite-momentum frame (IMF) [30–32], we start with instant-form statistical expressions in the nucleon rest frame, then perform transformation in terms of light-front kinematic variables. The analytic expressions of the PDFs we get are somehow different from those attained in other statistical models implemented in the IMF [4–8], and ours perform better with non-vanishing PDFs when $x\to 0$.

On the other hand, the nucleons in a nucleus were initially thought to be highly insensitive to their surroundings, and the only nuclear effect in deep inelastic scattering (DIS) was believed to be Fermi motion at large x. However, in 1982, it was discovered that nucleons inside a nucleus have a remarkably different momentum configuration as expected, which was named nuclear EMC effect [33–36]. In order to account for the EMC effect, there have been many efforts and insights implemented in various models, e.g., the cluster model [37–40], the pion excess model [38, 41–43], the x-rescaling model [44, 45], the Q^2-rescaling model [46–48], and the nucleon swelling model [49]. The statistical idea is also applied to the EMC effect [50–52]. However, in some sense, most of these available models provide a fairly good description, instead of an explanation, to the phenomena.

Worthy to note that, our intention of this work is only to illustrate whether the statistical effect is important to nucleon structure, not how well it matches experimental results. So we do not make any effort to fit the experimental data intentionally. There is no arbitrary parameter put by hand in our model, and all parameters are basic statistical quantities. Some of other statistical models can fit the experimental

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1) E-mail: mabq@phy.pku.edu.cn

data better by introducing many free parameters, however, it weakens the stringency at a cost.

2 Statistical approach

We assume that the nucleon is a thermal system in equilibrium, made up of free partons. Quarks and anti-quarks satisfy the Fermi-Dirac distribution, while the gluons obey the Bose-Einstein distribution,

$$f(k^0)=\frac{g_{\mathrm{f}}V}{(2\pi)^3}\frac{1}{\mathrm{e}^{\frac{k^0-\mu_{\mathrm{f}}}{T}}\pm 1}\,, \tag{1}$$

with the upper sign for Fermion, and nether sign for Boson; g_{f} is the degree of color-spin degeneracy, hence $g_{\mathrm{f}}=6$ for quark and $g_{\mathrm{f}}=16$ for gluon; μ_{f} is its chemical potential, hence $\mu_{\bar{\mathrm{q}}}=-\mu_{\mathrm{q}}$ and $\mu_{\mathrm{g}}=0$.

We introduce the light-front 4-momentum of the parton $k=(k^+,k^-,\boldsymbol{k}_\perp)$, where $k^+=k^0+k^3$, $k^-=k^0-k^3$, $\boldsymbol{k}_\perp=(k^1,k^2)$, and $k^+=P^+x=Mx$, where x is the light-front momentum fraction of the nucleon carried by the parton. On the trivial assumption that $\boldsymbol{k}_\perp$ is transversely isotropic, we can get PDFs analytically [29]

$$\begin{aligned}f(x)\;=\;&\pm\frac{g_{\mathrm{f}}\mathrm{MTV}}{8\pi^2}\left\{\left(Mx+\frac{m_{\mathrm{f}}^2}{Mx}\right)\ln\left[1\pm\mathrm{e}^{-\frac{\frac{1}{2}\left(Mx+\frac{m_{\mathrm{f}}^2}{Mx}\right)-\mu_{\mathrm{f}}}{T}}\right]-\right.\\&\left.2T\mathrm{Li}_2\left(\mp\mathrm{e}^{-\frac{\frac{1}{2}\left(Mx+\frac{m_{\mathrm{f}}^2}{Mx}\right)-\mu_{\mathrm{f}}}{T}}\right)\right\}\theta\left(x-\frac{m_{\mathrm{f}}^2}{M^2}\right),\end{aligned} \tag{2}$$

where $\mathrm{Li}_2(z)$ is defined as

$$\mathrm{Li}_2(z)=\sum\nolimits_{k=1}^{\infty}z^k/k^2,$$

and the step-function $\theta(x-m_{\mathrm{f}}^2/M^2)$ originates from the constraint $x\geqslant m_{\mathrm{f}}^2/M^2$ [53].

In practice, the PDFs in a certain system should be constrained with some conversation laws. For example, in the proton, they are

$$u_{\mathrm{V}}=\int[u(x)-\bar{u}(x)]\,\mathrm{d}x=2\,, \tag{3}$$

$$d_{\mathrm{V}}=\int[d(x)-\bar{d}(x)]\,\mathrm{d}x=1\,, \tag{4}$$

$$\sum_f\int xf(x)\,\mathrm{d}x=1\,. \tag{5}$$

For free proton, we also introduce the Gottfried sum,

$$\begin{aligned}S_{\mathrm{G}}\;=\;&\int_0^1\frac{F_2^{\mathrm{p}}(x)-F_2^{\mathrm{n}}(x)}{x}\,\mathrm{d}x=\\&\frac{1}{3}+\frac{2}{3}\int_0^1\left[\overline{u}(x)-\overline{d}(x)\right]\mathrm{d}x\,,\end{aligned} \tag{6}$$

whose experimental value is 0.235 ± 0.026 [54, 55].

Now there are four unknown parameters T, V, μ_{u}, μ_{d} (M is taken as given) and four constraints, i.e., Eqs. (3)–(6), thus the parameters can be determined uniquely. The results for the proton are $T_0=47$ MeV, $r_0=(3V_0/4\pi)^{1/3}=2.8$ fm, $\mu_{\mathrm{u}}\approx64$ MeV, and $\mu_{\mathrm{d}}\approx36$ MeV. However, the radius r_0 seems a little larger than the realistic value, possibly due to the oversimplified assumption of the uniform distribution of partons and negligence of surface effect.

After the PDFs are addressed, the nucleon structure function

$$F_2(x)=2xF_1(x)=x\sum\nolimits_f e_f^2f(x)$$

can be attained directly. Further, with p-n isospin symmetry, i.e.,

$$u^{\mathrm{n}}(x)=d^{\mathrm{p}}(x),\ d^{\mathrm{n}}(x)=u^{\mathrm{p}}(x),\ \bar{u}^{\mathrm{n}}(x)=\bar{d}^{\mathrm{p}}(x),$$
$$\bar{d}^{\mathrm{n}}(x)=\bar{u}^{\mathrm{p}}(x),\ g^{\mathrm{n}}(x)=g^{\mathrm{p}}(x),$$

we can obtain the structure function of the neutron as well. Various results, together with discussions, for PDFs and structure functions are illustrated in Ref. [29].

In nucleon, the valence numbers of heavy flavors are zero, then the chemical potentials of them all vanish, hence no extra parameter is introduced after adding heavy flavors. However, we found that the contributions from them are rather small. Further, we identify that the s, $\bar{s}$ asymmetry in the nucleon [56] does not originate from the pure statistical effect.

3 Nuclear EMC effect

The reasonableness and simpleness of the model encourage us to apply it to nuclear EMC effect [52].

We mainly assume that a nucleon under a different nuclear circumstance is equivalent to at a different temperature, and subsequently along with different V, μ_u, and μ_d. Practically, we release the constraint from the Gottfried sum, i.e., Eq. (6), and remain Eqs. (3)–(5) for protons immersing in the nuclear environment. In other words, we introduce the temperature T as a parameter versus the atomic number A, then fit the theoretical ratios of structure functions to experimental data in the EMC region $x \in [0.2,\ 0.7]$.

The fit is rather impressive [52]. The temperature we get is about 1∼2 MeV lower in bound nucleons than in free ones, and jointly the volume is bigger about 5%∼10%. Our result is qualitatively consistent with other models, such as the Q^2-rescaling model and the nucleon swelling model. Worthy to note that, including the strange quark and taking different masses of it lead to some slightly difference in results, so the s flavor is considered as a modification. We also give explicitly the predictions of PDFs of the nucleons inside different nuclei. The ratios of the PDFs of iron to deuterium are depicted in Ref. [52]. They are quite different from the predictions of other models, i.e., the cluster model [39, 40], the pion excess model [41–43], and the Q^2-rescaling model [46–48]. And to distinguish various models and look into the immanent cause of the EMC effect, we suggest more experiments to identify the PDFs in nuclei, especially for anti-quarks, the strange quark, and the gluon. Dimuon yield in Drell-Yan process, semi-inclusive hadron productions in DIS, charmed quarks production in DIS via the photon-gluon fusion mechanism, and Λ-K process, are suggested.

4 Summary

We preform a new statistical approach and obtain analytic expressions of the parton distribution functions (PDFs) in terms of light-front kinematic variables in the whole x region. There is no arbitrary parameter or extra corrected term put by hand in our model, which guarantees the stringency of our conclusion. And then, we treat the nucleon temperature T as a parameter of the atomic number A to mimic the nuclear EMC effect, and find that the nuclear effect can be explained as a shift of T; the larger A, the more significant influence. Further, we present the predictions of PDF ratios for iron as an example. These predictions are rather different from those of other available models. Experiments are expected to provide more information of the PDFs in nuclei, especially for anti-quarks, the strange quark, and the gluon, then we can test various models better.

All of these show that although the statistical effect is not everything, it is very important to some aspects of the nucleon structure and nuclear EMC effect.

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