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Semileptonic bc to cc and bb to bc baryon decays and heavy quark spin symmetry *

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Abstract We study the semileptonic decays of the lowest-lying bc baryons to the lowest-lying cc baryons ($\Xi_{bc}^{(\prime *)} \to \Xi_{cc}^{(*)}$ and $\Omega_{bc}^{(\prime *)} \to \Omega_{cc}^{(*)}$) , in the limit m_b, $m_c \gg \Lambda_{QCD}$ and close to the zero recoil point. The separate heavy quark spin symmetries make it possible to describe all these decays using a single form factor. We also show how these constraints can be used to test the validity of different quark model calculations. bb to bc baryon decays are also discussed.

Key words heavy quark spin symmetry, semileptonic decays, constituent quark models

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1 Introduction

The static theory for a system with two heavy quarks has infra-red divergences which can be regulated by the kinetic energy term $\bar{h}_Q(D^2/2m_Q)h_Q$. This term breaks the heavy quark flavour symmetry, but not the spin symmetry for each heavy quark flavour. The spin symmetry is sufficient to derive relations between form factors for decays of hadrons containing two heavy quarks in the heavy quark limit, as was first shown in Ref. [1]. The consequences of the separate spin symmetries of each of the heavy quarks for semileptonic decays of B_c mesons were worked out in Ref. [2]. The formalism was extended in Ref. [3] to describe semileptonic decays of bc (bb) baryons to cc (bc) baryons, and its predictions were confronted with different constituent quark model calculations in Ref. [4]. Here, we will review the main findings of Refs. [3] and [4] on the semileptonic decays of baryons containing two heavy quarks and a light quark.

According to heavy quark spin symmetry (HQSS) [2], in the infinite heavy quark mass limit, one can select the heavy quark subsystem of a doubly heavy baryon to have a well defined total spin $S_h = 0, 1$. In Table 1 we show the ground state

$$J^\pi = \frac{1}{2}^+, \ \frac{3}{2}^+$$

doubly heavy baryons classified so that S_h is well defined. Being ground states for the given quantum numbers, a total orbital angular momentum $L = 0$ is naturally assumed. HQSS guarantees that, in the infinite heavy quark mass limit, all baryons with the same flavour content listed in Table 1 are degenerate, and that a unique function describes the entire family of decays of cascade bc baryons Ξ_{bc}, Ξ'_{bc} and Ξ^*_{bc} to cascade cc baryons Ξ_{cc} and Ξ^*_{cc} near the zero recoil point. In this latter kinematical region, the velocities of the initial and final baryons are approximately the same. If the momenta of the initial bc and final cc baryons are $p_\mu = m_{bc}v_\mu$ and $p'_\mu = m_{cc}v'_\mu = m_{cc}v_\mu + k_\mu$ respectively, then k will be a small residual momentum near the zero-recoil point, and since the final baryon is on-shell, $k\cdot v = \mathcal{O}(1/m_{cc})$ will be suppressed. Moreover, this unique function, which describes all the decays, satisfies a normalization condition (a consequence of vector current conservation) at zero-recoil if the heavy quarks are degenerate. These results can

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straightforwardly be applied to the corresponding decays involving Ω baryons and also to the decays of bb baryons to bc baryons. Some of these decays have also been studied in various quark model approaches [5–10], and we will critically review to what extent these calculations are consistent with HQSS.

Table 1. Quantum numbers of the baryons analyzed in this study. J^π is the baryon spin parity, and S_h is the spin of the heavy degrees of freedom, well-defined in the infinite heavy mass limit. l denotes a u or d quark.

baryon	quark content	S_h	J^π	baryon	quark content	S_h	J^π
Ξ_cc	c c l	1	$1/2^+$	Ω_cc	c c s	1	$1/2^+$
Ξ^*_cc	c c l	1	$3/2^+$	Ω^*_cc	c c s	1	$3/2^+$
Ξ_bb	b b l	1	$1/2^+$	Ω_bb	b b s	1	$1/2^+$
Ξ^*_bb	b b l	1	$3/2^+$	Ω^*_bb	b b s	1	$3/2^+$
Ξ_bc	b c l	1	$1/2^+$	Ω_bc	b c s	1	$1/2^+$
Ξ'_bc	b c l	0	$1/2^+$	Ω'_bc	b c s	0	$1/2^+$
Ξ^*_bc	b c l	1	$3/2^+$	Ω^*_bc	b c s	1	$3/2^+$

To end this introduction, we devote a few words to the effects arising from the mixing of the Ξ and Ξ′ bc−states [11, 12] (see also the talk by E. Henńandez [13]). Owing to the finite value of the heavy quark masses, the hyperfine interaction between the light quark and any of the heavy quarks can admix both $S_\mathrm{h}=0$ and $S_\mathrm{h}=1$ spin components into the wave function. This mixing should be negligible for bb and cc doubly heavy baryons as the antisymmetry of the wave function would require radial excitations and/or higher orbital angular momentum in the $S_\mathrm{h}=0$ component. However, in the bc sector, the mass eigenstate Ξ (Ω) particles are mixtures of the Ξ_bc, Ξ'_bc (Ω_bc, Ω'_bc) states listed in Table 1. Indeed, the mixing angle is large, around 30° ([11, 12]). This hyperfine mixing greatly affects the decay widths of doubly heavy baryons involving Ξ_bc−baryons. This was firstly established by Roberts and Pervin [10] and later on confirmed in Ref. [12]. Nevertheless, the HQSS predictions for the weak matrix elements of the unmixed states derived in Ref. [3] can be used to predict those of the mixed states, and moreover they might be used in the future to experimentally extract information on the mixtures in the actual physical bc−baryon states [12].

2 Spin Symmetry

The invariance under separate spin rotations of the b and c quarks leads to relations between the form factors for vector and axial-vector current decays of cascade bc baryons to cascade cc baryons. These decays are induced by the weak $\mathrm{b}\to\mathrm{c}\,\mathrm{l}^-\bar\nu_\mathrm{l}$ ($\mathrm{l}=\mathrm{e},\mu$) transition. To represent the lowest-lying $L=0$ bcq baryons we will use wavefunctions comprising tensor products of Dirac matrices and spinors, namely:

$$B'_\mathrm{bc} = -\left[\frac{(1+\not v)}{2}\gamma_5\right]_{\alpha\beta} u_\gamma(v,r), \quad (1)$$

$$B_\mathrm{bc} = \left[\frac{(1+\not v)}{2}\gamma_\mu\right]_{\alpha\beta}\left[\frac{1}{\sqrt{3}}(v^\mu+\gamma^\mu)\gamma_5 u(v,r)\right]_\gamma, \quad (2)$$

$$B^*_\mathrm{bc} = \Xi^*_\mathrm{bc} = \left[\frac{(1+\not v)}{2}\gamma_\mu\right]_{\alpha\beta} u^\mu_\gamma(v,r), \quad (3)$$

where we have indicated Dirac indices α, β and γ explicitly on the right-hand sides and r is a helicity label for the baryon[1)]. For the B^*_bc, $u^\mu_\gamma(v,r)$ is a Rarita-Schwinger spinor. These wavefunctions can be considered as matrix elements of the form $\langle 0|\mathrm{c}_\alpha \bar{\mathrm{q}}^\mathrm{c}{}_\beta \mathrm{b}_\gamma|B^{(\prime *)}_\mathrm{bc}\rangle$ where $\bar q^\mathrm{c}=q^\mathrm{T}C$ with C the charge-conjugation matrix. We couple the c quark and light quark to spin 0 for the B'_bc or 1 for the B_bc and B^*_bc states. Under a Lorentz transformation, Λ, and b and c quark spin transformations S_b and S_c, a wavefunction of the form $\Gamma_{\alpha\beta}\,u_\gamma$ transforms as:

$$\Gamma u \to S(\Lambda)\Gamma S^{-1}(\Lambda)\,S(\Lambda)u, \quad \Gamma u \to S_\mathrm{c}\Gamma\, S_\mathrm{b} u. \quad (4)$$

The states in Eqs. (1), (2) and (3) have a common normalization $\bar u u \mathrm{Tr}(\Gamma\overline{\Gamma})$ and are mutually orthogonal. To build states where the b and c quarks are coupled to definite spin, we need the linear combinations

$$|0;1/2,M\rangle_\mathrm{bc} = -\frac{1}{2}|0;1/2,M\rangle_\mathrm{cq} + \frac{\sqrt{3}}{2}|1;1/2,M\rangle_\mathrm{cq},$$

1) We use the standard relativistic normalization for hadronic states and our spinors satisfy $\bar u u = 2\,m$, $\bar u^\mu u_\mu = -2\,m$ where m is the mass of the state

$$
\begin{aligned}
|1;1/2,M\rangle_{\rm bc} &= \frac{\sqrt{3}}{2}|0;1/2,M\rangle_{\rm cq}+\frac{1}{2}|1;1/2,M\rangle_{\rm cq},\\
|1;3/2,M\rangle_{\rm bc} &= |1;3/2,M\rangle_{\rm cq}, \qquad (5)
\end{aligned}
$$

where the second and third arguments are the total spin quantum numbers of the baryon and the first argument denotes the total spin of the bc or cq subsystem. For the cc baryons there are some differences because we have two identical quarks. In this case the states are:

$$B'_{\rm cc} = -\sqrt{\frac{2}{3}}\left[\frac{(1+\not v)}{2}\gamma_5\right]_{\alpha\beta} u_\gamma(v,r), \qquad (6)$$

$$B_{\rm cc} = \left[\frac{(1+\not v)}{\sqrt{2}}\gamma_\mu\right]_{\alpha\beta}\left[\frac{1}{\sqrt{3}}(v^\mu+\gamma^\mu)\gamma_5 u(v,r)\right]_\gamma, \qquad (7)$$

$$B^*_{\rm cc} = \Xi^*_{\rm cc}=\sqrt{\frac{1}{2}}\left[\frac{(1+\not v)}{2}\gamma_\mu\right]_{\alpha\beta} u^\mu_\gamma(v,r). \qquad (8)$$

The two charm quarks can only be in a symmetric spin-1 state and therefore $B'_{\rm cc}$ and $B_{\rm cc}$ correspond to the same baryon state $\Xi_{\rm cc}$ (or $\Omega_{\rm cc}$ if the light quark is s). We can now construct amplitudes for semileptonic cascade bc to cascade cc baryon decays, determined by matrix elements of the weak current $J^\mu=\bar{c}\gamma^\mu(1-\gamma_5)b$. We first build transition amplitudes between the $B^{(\prime *)}_{\rm bc}$ and $\Xi^{(*)}_{\rm cc}$ states and subsequently take linear combinations to obtain transitions from $\Xi^{(\prime *)}_{\rm bc}$ states. The most general form for the matrix element respecting the heavy quark spin symmetry is

$$
\begin{aligned}
&\langle \Xi^{(*)}_{\rm cc},v,k,M'|J^\mu(0)|B^{(\prime *)}_{\rm bc},v,M\rangle=\\
&\bar{u}_{\rm cc}(v,k,M')\gamma^\mu(1-\gamma_5)u_{\rm bc}(v,M)\mathrm{Tr}[\Gamma_{\rm bc}\Omega\overline{\Gamma}_{\rm cc}]+\\
&\bar{u}_{\rm cc}(v,k,M')\Gamma_{\rm bc}\Omega\overline{\Gamma}_{\rm cc}\gamma^\mu(1-\gamma_5)u_{\rm bc}(v,M), \qquad (9)
\end{aligned}
$$

where M and M' are the helicities of the initial and final states and $\Omega=-\eta(\omega)/2$, with $\omega=v\cdot v'$. To simplify, we use the equations of motion ($\not v u=u$, $\not v\Gamma=\Gamma$, $\gamma_\mu u^\mu=0$, $v_\mu u^\mu=0$), while terms with $\not k$ will always lead to contributions proportional to $v\cdot k$ which is set to 0 at the order we are working. We also make use of the relations $\bar{u}\gamma_\mu u=\bar{u}v_\mu u$, $\bar{u}\gamma_5 u=0$, $\bar{u}\not k u=0$ and $\bar{u}\not k\gamma_\mu\gamma_5 u=-\bar{u}\not k v_\mu\gamma_5 u$. Our results for cascade bc to cascade cc transition matrix elements are [3]:

$$\Xi_{\rm bc}\to\Xi_{\rm cc} \quad \eta\frac{1}{\sqrt{2}}\bar{u}_{\rm cc}\left(2\gamma^\mu-\frac{4}{3}\gamma^\mu\gamma_5\right)u_{\rm bc}, \qquad (10)$$

$$\Xi'_{\rm bc}\to\Xi_{\rm cc} \quad -\sqrt{\frac{2}{3}}\eta\,\bar{u}_{\rm cc}\left(-\gamma^\mu\gamma_5\right)u_{\rm bc}, \qquad (11)$$

$$\Xi_{\rm bc}\to\Xi^*_{\rm cc} \quad -\sqrt{\frac{2}{3}}\eta\,\bar{u}^\mu_{\rm cc}u_{\rm bc}, \qquad (12)$$

$$\Xi'_{\rm bc}\to\Xi^*_{\rm cc} \quad -\sqrt{2}\eta\,\bar{u}^\mu_{\rm cc}u_{\rm bc}, \qquad (13)$$

$$\Xi^*_{\rm bc}\to\Xi_{\rm cc} \quad -\sqrt{\frac{2}{3}}\eta\,\bar{u}_{\rm cc}u^\mu_{\rm bc}, \qquad (14)$$

$$\Xi^*_{\rm bc}\to\Xi^*_{\rm cc} \quad -\sqrt{2}\eta\,\bar{u}^\lambda_{\rm cc}\left(\gamma^\mu-\gamma^\mu\gamma_5\right)u_{{\rm bc}\,\lambda}. \qquad (15)$$

If the b and c quarks become degenerate, then vector current conservation ensures that $\eta(1)=1$. Similarly, relations for the decays of bb baryons to bc baryons can be obtained [12].

3 Results and conclusions

All hadronic matrix elements of the $J=V-A$ current implicit in the left hand sides of Eqs. (10)–(15), near zero recoil, are given in terms of a unique function, $\eta(\omega)$, of the product of four velocities, up to corrections suppressed by the mass of the charm and bottom quarks. These matrix elements are usually parameterized in terms of form factors, whose number

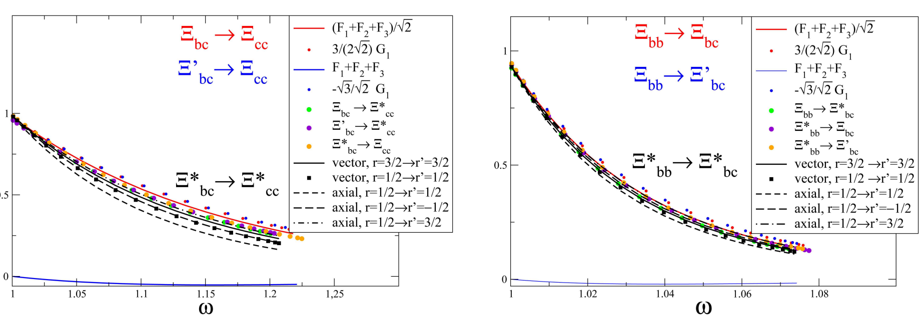

Fig. 1. (color online). Left panel: Different η functions obtained for $\Xi^*_{\rm bc}\to\Xi^*_{\rm cc}$ transitions (black curves) using the vector or the axial part of the weak transition current, and for different spin configurations. We also show the corresponding results obtained for $1/2\to1/2$ and $1/2\longleftrightarrow3/2$ transitions. Form factors are taken from Refs. [4, 8]. Baryon wave functions are obtained by means of a variational approach [8, 14, 15], while the semileptonic decay widths are computed in coordinate space [8] by using a scheme derived in Ref. [16]. Right panel: same as left panel for $\rm bb\to bc$ transitions.

is restricted by Lorentz covariance and the discrete C, P, T symmetries. There are six form factors to describe $\Xi_{\rm bc} \to \Xi_{\rm cc}$, another six for $\Xi'_{\rm bc} \to \Xi_{\rm cc}$, eight each for $\Xi_{\rm bc} \to \Xi^*_{\rm cc}$, $\Xi'_{\rm bc} \to \Xi^*_{\rm cc}$ and $\Xi^*_{\rm bc} \to \Xi_{\rm cc}$, and even more [9] for $\Xi^*_{\rm bc} \to \Xi^*_{\rm cc}$. In Fig. 1, we show constituent quark model results for the various form factors [4, 8]. We see, that to a good approximation, better in the $\rm bb \to bc$ case as one is closer to the infinite heavy quark mass limit, all $1/2 \to 1/2$, $1/2 \longleftrightarrow 3/2$ and $3/2 \to 3/2$ transitions are governed in terms of just one function, as deduced in Eqs. (10)–(15) for the $\rm bc \to cc$ transitions. This function is different for the $\rm bc \to cc$ and $\rm bb \to bc$ cases due to heavy flavour symmetry breaking.

To the extent that one is close enough to the infinite heavy quark mass limit and near zero recoil, we can make use of the HQSS results in Eqs. (10)–(15) and the similar ones for $\rm bb \to bc$ transitions, to approximate the hadron tensor that governs these decays. Thus, it is possible to construct ratios of widths where the dependence on the universal $\eta(\omega)$ function will cancel out, in the strict near zero recoil approximation (for details, see Ref. [4]). In Table 2 we show different model predictions for several ratios that should be one in the infinitely heavy quark limit. We see that the calculations by Hernández et al. [4], Ebert et al. [7] and Faessler et al. [9] turn out to be in reasonable agreement with HQSS predictions. Only the second of the ratios can be computed from the results of Roberts and Pervin in [10], and we find a value of 0.80 (0.88) for Ξ (Ω) type baryons. The results in Ref. [5] are also not inconsistent with HQSS constraints. However, HQSS predictions turn out to be incompatible with the results of Ref. [6], hinting at problems in the model or the calculation in that work.

Table 2. Decay width ratios for semileptonic $\rm bb \to bc$ decay of doubly heavy Ξ and Ω baryons. In all cases the approximate result obtained using HQSS is 1.

	[4]		[7]		[6]		[9]	
$\rm bb \to bc$	Ξ	Ω	Ξ	Ω	Ξ	Ω	Ξ	Ω
$\dfrac{\Gamma(\mathrm{B}^*_{\rm bb} \to \mathrm{B}'_{\rm bc}\, l\bar\nu_l)}{3\Gamma(\mathrm{B}^*_{\rm bb} \to \mathrm{B}_{\rm bc}\, l\bar\nu_l)}$	$1.00^{+0.01}_{-0.04}$	$1.00^{+0.03}_{-0.01}$	0.99	0.99	0.05	–	$0.9^{+0.5}_{-0.3}$	$0.9^{+0.6}_{-0.4}$
$\dfrac{\Gamma(\mathrm{B}_{\rm bb} \to \mathrm{B}^*_{\rm bc}\, l\bar\nu_l)}{\frac{2}{3}\Gamma(\mathrm{B}_{\rm bb} \to \mathrm{B}'_{\rm bc}\, l\bar\nu_l)}$	$0.86^{+0.08}_{-0.06}$	$0.86^{+0.05}$	0.96	0.99	9.53	–	$0.9^{+0.5}_{-0.3}$	$0.9^{+0.5}_{-0.3}$
$\dfrac{\Gamma(\mathrm{B}^*_{\rm bb} \to \mathrm{B}_{\rm bc}\, l\bar\nu_l)}{\frac{1}{3}\Gamma(\mathrm{B}_{\rm bb} \to \mathrm{B}'_{\rm bc}\, l\bar\nu_l)}$	$0.98^{+0.09}_{-0.03}$	$0.97^{+0.06}_{-0.14}$	1.01	1.03	36.4	–	$1.0^{+0.5}_{-0.3}$	$0.9^{+0.5}_{-0.4}$
$\dfrac{\Gamma(\mathrm{B}^*_{\rm bb} \to \mathrm{B}^*_{\rm bc}\, l\bar\nu_l)}{\Gamma(\mathrm{B}_{\rm bb} \to \mathrm{B}_{\rm bc}\, l\bar\nu_l) + \frac{1}{2}\Gamma(\mathrm{B}_{\rm bb} \to \mathrm{B}^*_{\rm bc}\, l\bar\nu_l)}$	$0.94^{+0.07}_{-0.06}$	$0.93^{+0.11}_{-0.10}$	1.01	1.01	0.31	–	$1.1^{+0.8}_{-0.5}$	$1.1^{+0.8}_{-0.5}$

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