Viscosities in chiral symmetry breaking phase*

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Abstract In the chiral symmetry breaking phase described by the NJL model at quark level, along with the chiral symmetry restoration the ratio of shear viscosity to entropy density η/s drops down monotonously and reaches the minimum at the critical point, while the ratio of bulk viscosity to entropy density ζ/s behaves oppositely.

Key words shear viscosity, bulk viscosity, chiral symmetry breaking

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1 Introduction

Transport coefficients are often used to describe the properties of hot and dense medium. From recent application of the AdS/CFT method [1] to the strongly interacting quark matter [2, 3], the ratio of shear viscosity to entropy density η/s is considered as a probe to detect the deconfinement phase transition, it reaches the minimum $1/4\pi$ at the critical point. Considering a real QCD system [4, 5], however, we need to consider the quark contribution to the thermodynamics and especially the non-perturbative calculation at moderate temperature and density around the phase transition. Since it is hard to calculate transport coefficients with lattice for a QCD system with dynamic quarks, effective models [6, 7] are usually used to investigate the transport properties at finite temperature and density.

Another important phase transition for a QCD system is the chiral symmetry restoration. From the lattice calculation at finite temperature, the spontaneously broken chiral symmetry is restored at the critical temperature of the deconfinement. The chiral phase transition can happen in either a hadron system or a quark system. At quark level, the Nambu–Jona-Lasinio model (NJL) [8] describes well the chiral properties in the vacuum and at finite temperature and density. Recently, the model is used to calculate viscosities by using the Boltzmann transport equation [9]. Taking into account the fact that the Boltzmann equation is in principle for a weakly coupled dilute gas, it is not clear if the equation can be used in the strongly coupled region.

Unlike the Boltzmann equation, the Kubo formulas are valid for both weakly and strongly correlated matter in the frame of linear response. Therefore, they can be applied to calculate the transport coefficients in hydrodynamics up to the first order in derivative expansion of velocity. We study in this paper the viscosity around the chiral phase transition with the Kubo formulas in the NJL model.

2 Viscosities

From the Kubo formulas, the shear viscosity η and bulk viscosity ζ are defined through the imaginary part of the propagator in the medium [10–12],

$$\eta = -\frac{1}{6}\frac{\partial}{\partial\omega}\mathrm{Im}G_{\mathrm{R}}^{\eta}(\omega,0)\Big|_{\omega\to 0},$$

$$\zeta = -\frac{1}{9}\frac{\partial}{\partial\omega}\mathrm{Im}G_{\mathrm{R}}^{\zeta}(\omega,0)\Big|_{\omega\to 0}, \tag{1}$$

where the retarded Green functions in coordinate space are defined as

$$G_{\mathrm{R}}^{\eta}(x) = -\mathrm{i}\theta(t)\langle[\hat{T}_{ij}(x),\hat{T}_{ij}(0)]\rangle, i\neq j,$$

$$G_{\mathrm{R}}^{\zeta}(x) = -\mathrm{i}\theta(t)\langle[\hat{T}_{ii}(x),\hat{T}_{jj}(0)]\rangle, \tag{2}$$

and $\hat{T}_{\mu,\nu}$ is the energy-momentum operator which controls the dynamics of the medium.

With the quark spectra function $\rho(\epsilon,\boldsymbol{p})$ which is

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defined through the quark propagator $S(p_0,\boldsymbol{p})$,

$$S(p_0,\boldsymbol{p})=\int_{-\infty}^{\infty}\frac{\mathrm{d}\epsilon}{2\pi}\frac{\rho(\epsilon,\boldsymbol{p})}{p_0-\epsilon}, \tag{3}$$

the shear and bulk viscosities cab be expressed as

$$\begin{aligned}\eta &= \frac{1}{2}\int\frac{\mathrm{d}^4p}{(2\pi)^4}p_x^2 n_{\mathrm{F}}'(\epsilon-\mu)\mathrm{Tr}[\gamma_2\rho(p)\gamma_2\rho(p)],\\ \zeta &= \frac{1}{18}\int\frac{\mathrm{d}^4p}{(2\pi)^4}p_ip_j n_{\mathrm{F}}'(\epsilon-\mu)\mathrm{Tr}[\gamma_j\rho(p)\gamma_i\rho(p)],\end{aligned} \tag{4}$$

where $n_{\mathrm{F}}(x)=1/(1+\mathrm{e}^{\beta x})$ is the Fermi-Dirac distribution function, $n_{\mathrm{F}}'(x)$ stands for $n_{\mathrm{F}}'(x)=\partial_x n_{\mathrm{F}}(x)$, μ is the quark chemical potential, and the trace operator runs over color, flavor and Dirac spaces.

We adopt the $SU(2)$ NJL model as a microscopic theory to describe the two flavor quark matter [8],

$$\mathcal{L}=\bar\psi(\mathrm{i}\gamma^\mu\partial_\mu-m_0+\mu\gamma_0)\psi+g[(\bar\psi\psi)^2+(\bar\psi\mathrm{i}\gamma_5\tau\psi)^2], \tag{5}$$

where ψ and $\bar\psi$ are the quark fields, τ is the Pauli matrix, m_0 is the current quark mass which explicitly breaks down the chiral symmetry, and g is the coupling constant in scalar and pseudoscalar channels with dimension GeV^{-2}. Since the model is nonrenormalizable, it is necessary to introduce a regulator Λ to avoid divergence. The three parameters, m_0, g and Λ are fixed by the vacuum values of pion mass, pion decay constant and constituent quark mass. The mean field approximation to quarks plus random phase approximation (RPA) to mesons in this model describe successfully the chiral dynamics in the vacuum and at finite temperature and density [8].

We first review the mean field propagator S_{mf} of quarks which is at $O(1)$ level of $1/N_{\mathrm{c}}$ expansion and the meson propagator D_{M} constructed with S_{mf} in RPA which is at $O(1/N_{\mathrm{c}})$ level. Then we consider the feedback of mesons on the quark motion. We derive the dressed quark propagator at $O(1/N_{\mathrm{c}})$ level by including the quark-meson interaction. Finally we calculate the transport coefficients with the known quark propagator or quark spectral function.

In mean field approximation, quarks propagate in the medium like quasi-particles,

$$S_{\mathrm{mf}}(p)=\frac{1}{\gamma^\mu p_\mu-m}=\int\frac{\mathrm{d}\epsilon}{2\pi}\frac{\rho_{\mathrm{mf}}(\epsilon,\boldsymbol{p})}{p_0-\epsilon} \tag{6}$$

with the mean field spectral function

$$\rho_{\mathrm{mf}}(p)=2\pi\frac{\gamma^\mu p_\mu+m}{2E_{\mathrm{p}}}[\delta(p_0-E_{\mathrm{p}})-\delta(p_0+E_{\mathrm{p}})], \tag{7}$$

where $E_{\mathrm{p}}=\sqrt{\boldsymbol{p}^2+m^2}$ is the quark energy. The mean field interaction only changes the quark mass m which is self-consistently determined by the gap equation,

$$m=m_0+2\mathrm{i}g\int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{Tr}\frac{1}{\gamma^\mu p_\mu-m_0-m}. \tag{8}$$

In the NJL model, the meson modes are regarded as quantum fluctuations above the mean field. The meson modes can be calculated in the frame of RPA. In the current case with only chiral condensation, the mean field quark propagator is diagonal in flavor space, and the summation of bubbles in RPA selects its specific channel by choosing at each stage the same proper polarization function. After the bubble summation one obtains the meson propagator [8],

$$D_{\mathrm{M}}(q)=\frac{2g}{1-2g\Pi_{\mathrm{M}}(q)} \tag{9}$$

with the polarization function

$$\Pi_{\mathrm{M}}(q)=\mathrm{i}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{Tr}\left[V_{\mathrm{M}}^*S_{\mathrm{mf}}(p+q)V_{\mathrm{M}}S_{\mathrm{mf}}(p)\right], \tag{10}$$

where V_{m} is the meson vertices

$$V_{\mathrm{M}}=\begin{cases}1 & \mathrm{M}=\sigma\\ \mathrm{i}\tau_+\gamma_5 & \mathrm{M}=\pi_+\\ \mathrm{i}\tau_-\gamma_5 & \mathrm{M}=\pi_-\\ \mathrm{i}\tau_3\gamma_5 & \mathrm{M}=\pi_0\end{cases},\quad V_{\mathrm{M}}^*=\begin{cases}1 & \mathrm{M}=\sigma\\ \mathrm{i}\tau_-\gamma_5 & \mathrm{M}=\pi_+\\ \mathrm{i}\tau_+\gamma_5 & \mathrm{M}=\pi_-\\ \mathrm{i}\tau_3\gamma_5 & \mathrm{M}=\pi_0\end{cases} \tag{11}$$

with the definition $\tau_\pm=(\tau_1\pm\mathrm{i}\tau_2)/\sqrt{2}$.

To the order $O(1/N_{\mathrm{c}})$, the quark self-energy contains a Hartree term and a meson exchange term, the former is a constant and the latter is momentum dependent and divided into a scalar and a vector part,

$$\begin{aligned}\Sigma(p) &= \Sigma_{\mathrm{mf}}+\Sigma_{\mathrm{M}}(p),\quad \Sigma_{\mathrm{mf}}=m,\\ \Sigma_{\mathrm{M}}(p) &= \gamma_\mu\Sigma_{\mathrm{M}}^\mu(p)+\Sigma_{\mathrm{M}}^{\mathrm{s}}(p)=\\ & -\mathrm{i}\int\frac{\mathrm{d}^4q}{(2\pi)^4}V_{\mathrm{M}}^*S_{\mathrm{mf}}(p+q)V_{\mathrm{M}}D_{\mathrm{M}}(q).\end{aligned} \tag{12}$$

The dressed quark self energy becomes

$$S(p)=\frac{1}{\gamma^\mu p_\mu-\Sigma(p)}. \tag{13}$$

In the chiral symmetry breaking phase with $T<T_{\mathrm{c}}$, where T_{c} is the critical temperature for the chiral phase transition, the mesons are stable bound states and one can use pole approximation to simplify the meson propagator,

$$D_{\mathrm{M}}(q)\sim\frac{1}{q^2-m_{\mathrm{M}}^2} \tag{14}$$

with the meson mass m_{M} determined by

$$1-2g\Pi_{\mathrm{M}}(m_{\mathrm{M}},0)=0. \tag{15}$$

In chiral limit with $m_0=0$, from the comparison between the gap equation for the quark mass and pole equation for the meson mass, it is easy to see that pions are Goldstone modes with $m_\pi = 0$, corresponding to the spontaneously broken chiral symmetry, and sigma mass m_σ is two times the quark mass m.

To further simplify the numerical calculation, we take pole approximation for the dressed quarks too. Note that in the chiral breaking phase, quarks can decay into pions and their pole is a complex pole,

$$S(p) \sim \frac{1}{\gamma^\mu p_\mu - (M - \mathrm{i}\Gamma/2)}. \quad (16)$$

Under the approximation of $\Gamma \ll M$, the complex pole equation is divided into two equations to separately determine the quark mass M and width Γ,

$$M = m + [\mathrm{Re}\Sigma_0(M,0) + \mathrm{Re}\Sigma_\mathrm{s}(M,0)],$$

$$\Gamma = -2\mathrm{Im}[\Sigma_0(M,0) + \Sigma_\mathrm{s}(M,0)], \quad (17)$$

where we have employed the fact that the 3-vector part of the dressed quark self-energy vanishes when the quark 3-momentum is set to be zero.

We first show our numerical results Fig. 1 on the scaled shear viscosity η/s and bulk viscosity ζ/s as functions of temperature T at zero baryon density. The entropy density $s = -\partial\Omega/\partial T$ can be derived from the thermodynamic potential $\Omega = \Omega_\mathrm{mf} + \Omega_\mathrm{m}$, where Ω is the contribution from mesons. As is well know, η/s is divergent and ζ/s vanishes in free gas limit. In the chiral symmetry breaking phase, the ratio η/s drops down monotonously with increasing temperature and reaches the minimum at the critical temperature. This reflects the fact that the quarks are strongly coupled around the critical point. As for the bulk viscosity, it goes up monotonously with increasing temperature and the maximum is located at the critical point. While the bulk viscosity is much less than the shear viscosity, it plays a role at the critical point. Such properties of the scaled shear and bulk viscosities agree qualitatively with the previous theoretical calculations [5, 6, 9].

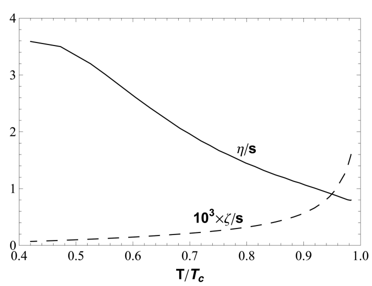

Fig. 1. The scaled shear and bulk viscosities η/s (solid line) and ζ/s (dashed line) as functions of scaled temperature T/T_c at zero chemical potential in the chiral symmetry breaking phase.

We calculate also the density dependence of the two viscosities at fixed temperature in the chiral symmetry breaking phase Fig. 2. Considering the fact that entropy is proportional to the number of dynamic degrees of freedom of the system, we use quark number density $n = -\partial\Omega/\partial\mu$ instead of the entropy density s. Similar to the temperature behavior, the ratio η/n decreases with increasing quark chemical potential μ and approaches to the minimum value at the critical point, and the ratio ζ/n increases with μ and reaches the maximum at μ_c.

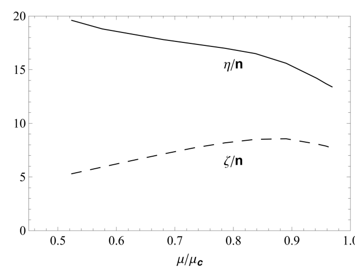

Fig. 2. The scaled shear and bulk viscosities η/n (solid line) and ζ/n (dashed line) as functions of scaled quark chemical potential μ/μ_c at fixed temperature $T = 150$ MeV in the chiral symmetry breaking phase.

3 Conclusion

In summary, we investigated the transport coefficients in a chiral symmetry breaking phase in the frame of the NJL model. Like what previously found in the study of deconfinement phase transition, the scaled shear viscosity monotonously decreases with increasing temperature and baryon density in the chiral breaking phase, and oppositely, the scaled bulk viscosity increases with temperature and density.

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