Phenomenological study on the significance of the scalar potential and Lamb shift *

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Abstract: We indicated in our previous work that for QED the contributions of the scalar potential, which appears at the loop level, is much smaller than that of the vector potential, and in fact negligible. But the situation may be different for QCD, the reason being that the loop effects are more significant because α_s is much larger than α, and secondly the non-perturbative QCD effects may induce the scalar potential. In this work, we phenomenologically study the contribution of the scalar potential to the spectra of charmonia. Taking into account both vector and scalar potentials, by fitting the well measured charmonia spectra, we re-fix the relevant parameters and test them by calculating other states of the charmonia family. We also consider the role of the Lamb shift and present the numerical results with and without involving the Lamb shift.

Key words: charmonia, scalar potential, Lamb shift

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1 Introduction

By a symmetry consideration, Chen et al. suggested [1] that for the Coulomb interaction, to maintain the hidden symmetry $SO(4)$ in the Schrödinger equation, the scalar and vector potentials must have the same weight in the Dirac equation. The hidden symmetry is just the familiar Lenz symmetry, which also exists in classical physics. However, if so, the orbit-spin coupling would disappear. In fact, the scalar and vector potentials make opposite contributions to the orbit-spin coupling, thus if they have the same weight, their contributions would exactly cancel each other. This definitely contradicts the data. Therefore, one concludes that this symmetry does not exist in the relativistic extension. Usually, one is tempted to think that the relativistic Dirac equation should possess a higher symmetry than its non-relativistic approximation, but this is not the case we are confronting. A general theory that only considers the Lorentz structure of the vertices [2], there are five types of coupling. But which one dominates should be selected by the underlying physics. We turn to look at the deeper side, namely starting to investigate the problem in quantum field theory.

The basic theory that induces the electric Coulomb potential is QED, whose coupling is vector-type $\bar{\psi}\gamma_\mu\psi A^\mu$, thus at the tree-level, the induced potential is the vector one and the other types should be induced at higher order, i.e. loop level. In our earlier work [3], we showed explicitly that the scalar coupling $1\otimes 1$, which results in the scalar potential, appears at the loop level and its contribution is suppressed by a factor α/π. For QED, it is a small value and cannot make a sizable contribution. Thus the apparent $SO(4)$ symmetry at the classical level is almost fully violated. However, the situation would be different for the QCD case, because first α_s at the charm-mass-scale is much larger than α and secondly the non-perturbative QCD effects may also cause the scalar potential.

This case is noticed by Leviatan and some studies

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have been carried out [4–6]. In this work, we are not going to further discuss the origin of the scalar potential or try to derive it from the quantum field theory, but generally assume its existence and by fitting the spectra of the charmonia family, we obtain its fraction. Moreover, the QED theory predicts the Lamb shift, which is due to the vacuum effects. In QM, it only shifts the S-wave spectra because in the non-relativistic limit, it is proportional to $\delta(\boldsymbol{r})$, but by the quantum field theory, the other l-states are also affected. In other words, by considering the Lamb shift, the positions of the spectra would deviate from those obtained without the Lamb shift. In this work, we include its contribution and re-fit the charmonia spectra to obtain a new set of the model parameters. For a comparison, we will present the numerical results with and without taking the Lamb shift into account.

Unlike the hydrogen-like atoms where the nucleus is very heavy and approximated at rest, therefore only the motion of the electron is considered and the corresponding equation, either the relativistic Dirac equation or non-relativistic Schrödinger equation, is a one-body equation. However, for charmonia, the charm and anti-charm quarks are of the same mass and the equation that properly describes charmonia should be a two-body equation.

For simplicity but without losing the significant characters, we do not directly solve the two-body Dirac equation, which is very complicated. One can derive the effective potential between the two constituents (c and $\bar{\rm c}$) in terms of the perturbative theory where the effective Lorentz vertices are set according to the general Lorentz structures [7]. Because of the limitation of the perturbative theory, we can only obtain the Coulomb-type interaction and the corresponding spin-orbit, spin-tensor and relativistic correction pieces. It is noted that the fundamental QCD indeed provides only the vector potential at the tree level, but, as indicated above, the loop effect and even the non-perturbative effect may result in a scalar potential. Thus we just keep the potential forms and introduce two phenomenological constants in front of the scalar and vector potentials and the induced terms. For the confinement piece, we employ the linear confinement i.e. the Cornell-type. In fact, the exact form of the full potential including both scalar and vector pieces was given by Lucha et al. [7], and we just re-check their results and then substitute the potential into the Schrödinger equation.

Now we can reduce the two-body Schrödinger equation into one particle equation where the kinetic term is $\dfrac{1}{2\mu}\boldsymbol{p}^2$ where μ is the reduced mass and is $m_{\rm c}/2$ in our case. Solving the differential equation, we obtain the spectra. Since there exist several phenomenological parameters that so far cannot be derived from the underlying theory, we can fix them by fitting a few well measured charmonia states.

Moreover, as is well known, the vacuum fluctuation induces the Lamb shift. The basic Lagrangian of the Lamb shift has been derived by some authors, and for interaction, we have $H_{\rm int}=-L_{\rm int}$ [8, 9]. Thus we substitute the expression into our data fitting process to re-derive the phenomenological parameters. Indeed, the Lamb shift only occurs at the loop level, but the Coulomb-type $-\alpha_{\rm s}/r$ appears at the tree level of QCD. It seems that they belong to different levels, but as we introduce the phenomenological parameters that include the loop and non-perturbative QCD effects, we cannot distinguish between the tree level contribution and that of higher orders. However, for the Lamb shift, we do not introduce a new phenomenological parameter but use the derived form[1)].

There are some subtleties in the calculations that we will address in the text.

This paper is organized as follows. In Sections 2 and 3, we introduce the generalized Breit-Fermi Hamiltonian and the Schördinger equation for the $\rm c\bar{c}$ bound states: $\rm J/\psi$, $\chi_{\rm c0}(1P)$, $\chi_{\rm c1}(1P)$, $\eta_{\rm c}(2S)$ and $\psi(2S)$. Then we numerically solve the eigen-equations for these bound states and fix the parameters. In Section 4, the Lamb shift is concerned and another set of the parameters is given to improve our predictions. The last section is devoted to our conclusion and discussion.

2 The Schördinger equation and the generalized Breit-Fermi Hamiltonian

For the $\rm c\bar{c}$ meson, the generalized Breit-Fermi Hamiltonian was given in Refs. [2, 10] as

$$H=H_0+H_1+..., \tag{1a}$$

and,

$$H_0=\frac{p^2}{m}+S(r)+V(r), \tag{1b}$$

1) It is noted that for a formula that is derived in the field theory, one can separate the contributions corresponding to different orders as long as there are no phenomenological parameters involved, and that is the case we deal with for the Lamb shift. Please see the text for details.

$$H_1 = H_{\rm sd} + H_{\rm si}, \tag{1c}$$

$$H_{\rm sd} = H_{\rm ls} + H_{\rm ss} + H_{\rm t} = \frac{1}{2m^2 r}(3V' - S')\vec{L}\cdot(\vec{S}_1 + \vec{S}_2) + \frac{2}{3m^2}\vec{S}_1\cdot\vec{S}_2\nabla^2 V(r) + \frac{1}{12m^2}\left(\frac{1}{r}V' - V''\right)S_{12}, \tag{1d}$$

$$H_{\rm si} = -\frac{p^4}{4m^3} + \frac{1}{4m^2}\left\{\frac{2}{r}V'(r)\cdot\vec{L}^2 + [p^2, V - rV'] + 2(V - rV')p^2 + \frac{1}{2}\left[\frac{8}{r}V'(r) + V'' - rV'''\right]\right\} \tag{1e}$$

where V and S stand for the vector and scalar potentials and $H_{\rm si}$ and $H_{\rm sd}$ represent the spin-independent and spin-dependent pieces, respectively. For the confinement piece, we adopt the Cornell type linear potential [11]. Thus the total potential at lowest order reads

$$U(r) = V(r) + S(r) = -aC_{\rm F}\frac{\alpha_{\rm s}}{r} + b\kappa^2 r, \tag{2a}$$

where

$$\begin{cases} V(r) = -c\ C_{\rm F}\alpha_{\rm s}/r + d\kappa^2 r \\ S(r) = -(a-c)C_{\rm F}\alpha_{\rm s}/r + (b-d)\kappa^2 r. \end{cases} \tag{2b}$$

With the Hamiltonian (1) and the potential (2), one can solve the Schördinger equation,

$$(E - 2m)\Psi(r) = H\Psi(r) = (H_0 + H_1)\Psi(r). \tag{3}$$

If we define the radial wave function as $R(x)$ with the dimensionless variable, $x = \kappa r$, then the radial equation is written as[1)]

$$\frac{{\rm d}^2}{{\rm d}x^2}u(x) = A(x)u(x) \tag{4a}$$

where,

$$A(x) = -\tilde{m}\left(\tilde{E} - 2\tilde{m} - \tilde{U}(x) - \tilde{H}_1'\right) + \frac{l(l+1)}{x^2} - \frac{1}{4}\left(\tilde{E} - 2\tilde{m} - \tilde{U}(x)\right)^2 \tag{4b}$$

with

$$\begin{cases} \tilde{m} = m/\kappa, \quad \tilde{E} = E/\kappa, \\ \tilde{H}_1' = H_1'/\kappa,\ \tilde{U}(x) = U(x)/\kappa, \\ \text{and},\ H_1 = H_1' - \dfrac{p^4}{4{\rm m}^3}. \end{cases} \tag{4c}$$

The approximation

$$p^2 \approx m\,(E - 2m - U(r)) \tag{4d}$$

is used in (3). This approximation is only applied to the terms such as the L–S coupling, S–S coupling and relativistic correction, etc., which are small compared with the leading one and in the common treatment are taken as a perturbation to the solution of the Schrödinger equation. Therefore, one can expect that the changes caused by the approximation are not significant. However, as is known, the perturbation method does not work perfectly for the charmonia system, so this approximation might induce some larger errors when convolution is carried out. Since this approximation has been widely adopted in the literature and greatly simplifies the calculation, we still keep it in this work. We will further investigate its legitimacy in our coming work.

3 The energy gap function of the $c\bar{c}$ charmonia and the numerical results

The radial Eq. (4) can be solved in terms of the method called "the iterative numerical process", which is introduced in the literature (for example, see [12, 13]). We have improved this method, and then fix the parameters a, b, c, d by fitting the well measured spectra of $c\bar{c}$ charmonia: J/ψ; $\chi_{c0}(1P)$; $\chi_{c1}(1P)$; $\eta_c(2S)$ and $\psi(2S)$. Instead of directly fitting the masses, we construct a series of relations that should be fitted,

$$\begin{cases} m[\psi(2S)] - m[\chi_{c1}(1P)] = E[2^3S_1] - E[1^3P_1]; \\ m[\psi(2S)] - m[{\rm J}/\psi(1S)] = E[2^3S_1] - E[1^3S_1]; \\ m[\psi(2S)] - m[\eta(2S)] = E[2^3S_1] - E[2^1S_0]; \\ m[\psi(2S)] - m[\chi_{c0}(1P)] = E[2^3S_1] - E[1^3P_0]. \end{cases} \tag{5}$$

where $E[n_r^{2S+1}l_j]$ represents the eigen-values of the radial Eq. (4) with various quantum numbers n_r, j, l, and s. Because the parameters a, b, c and d are involved in the potential (2), $E[n_r^{2S+1}l_j]$ must be functions of these parameters. m[meson] are the masses of the individual states, which are shown in Table 1 [14]. Sequentially, the parameters a, b, c and d are obtained by solving Eq. (5). By means of Newton's iterative method, we have achieved as (the details about the numerical method can be found in Ref. [15])

$$a = 1.1715,\ b = 1.2250,\ c = 0.8087,\ d = 0.5291 \tag{6}$$

Here, we set $\alpha_{\rm s} = 0.36$ and $\kappa = 0.42$ GeV, which seem somehow different from the values given in the

1) The standard form of the radial equation can be easily found in Ref. [12], and the method to make it dimensionless is given in Ref. [13].

literature [16–18]. But as noticed, the deviation may be included in the phenomenological parameters a, b, c and d.

Table 1. The experimental central values of the spectra of the $c\bar{c}$ charmonia states.

state	m/GeV	state	m/GeV
$J/\psi(1^3S_1)$	3.0969	$\eta_c(2^1S_0)$	3.6370
$\chi_{c0}(1^3P_0)$	3.4148	$\psi(2^3S_1)$	3.6861
$\chi_{c1}(1^3P_1)$	3.5107		

A few words are about our choice of the input. In principle, any five well measured states of the charmonia can be used as the input. However, unfortunately, the relationship between the $E[n_r^{2S+1}l_j]$ and the parameters in (5) is complicated, taking the central values of the masses of J/ψ; $\chi_{c0}(1P)$; $\chi_{c1}(1P)$; $\eta_c(2S)$ and $\psi(2S)$ as the inputs, one can obtain reasonable solutions, otherwise, Eq. (5) doesn't render solutions for a, b, c and d. The reason is due to the experimental errors.

Given a, b, c and d in (6), the masses of the charmonia states can be written as

$$M_1(n_r^{2S+1}L_J) = E\left[n_r^{2S+1}L_J\right] + E_0 \tag{7}$$

where E_0 is the zero-point energy,

$$E_0 = m[J/\psi] - E[1^3S_1] \tag{8}$$

and the final result is shown in Table 2.

Table 2. The mass spectra for the charmonia states (in GeV), with $m_c = 1.84$ GeV. The mass of the EXP is the value given in PDG [14].

meson	EXP	prediction	meson	EXP	prediction	meson	EXP	prediction
$\eta_c(1^1S_0)$	2.9803	3.0189	$\chi_{c1}(1^3P_1)^{\rm fit}$	3.5107	3.5107	$\eta_c(2^1S_0)^{\rm fit}$	3.6370	3.6370
$J/\psi(1^3S_1)^{\rm fit}$	3.0969	3.0969	$h_c(1^1P_1)$	3.5259	3.5100	$\psi(2^3S_1)^{\rm fit}$	3.6861	3.6861
$\chi_{c0}(1^3P_0)^{\rm fit}$	3.4148	3.4148	$\chi_{c2}(1^3P_2)$	3.5562	3.5564	$\psi(3^3S_1)$	/	4.1164

Explicitly, in the process, the masses of J/ψ, χ_{c0}, $\psi(2^3S_1)$ and $\chi_{c1}(1^3P_1)$ are taken as inputs to obtain the parameters and then the masses of other states in the family: $\eta_c(1S)$, $h_c(1P)$, $\chi_{c2}(1P)$ and $\psi(3S)$. The numbers are predicted.

4 The mass spectrum as the Lamb shift is taken into account

As is well known, the Lamb shift is due to the vacuum fluctuation and may cause sizable effects on the meson spectra. Indeed, the QED Lamb shift may not be very significant because of the smallness of the fine structure constant α, but for the QCD case, the situation will be different.

On the other hand, the Breit-Fermi Hamiltonian used in Section 3 does not include the effects of the Lamb shift in the eigen-energy (4). In this section, we will take the Lamb shift into account. However, we do not introduce the Hamiltonian induced by the Lamb shift into the differential equation because the corresponding pieces are very complicated and it is not necessary to do so. Instead, we simply add the estimated values of the effects to the binding energies of various states. Repeating the procedure carried out in last sections and adding the Lamb shift effects to the spectra, we re-fit the data to obtain a, b, c and d again and predict the mass spectra of the rest resonances.

Namely, we set the mass of a bound state as

$$2m_c + E_b + \Delta E_{LS} = M^{\rm exp}, \tag{9}$$

where E_b is the solution of the eigen-equation, ΔE_{LS} is the energy caused by the Lamb shift. Solving the equation, one can obtain the parameters again.

The authors of Refs. [8, 9] gave the theoretical expressions for the binding energies, which involve contributions of the Lamb shift. When we only concern the Lamb shift, we must single it out from the general formulas. It is not difficult, as a matter of fact, because the Lamb shift starts at $O(\alpha_s^3)$ [19]. The Lamb shift can be written as

$$\begin{aligned}\Delta E[n,j,l,s] =\ & m[\Delta E(\alpha_s^3)+\Delta E(\alpha_s^4)+\Delta E(\alpha_s^5)\\ & +\Delta E(\alpha_s^6)+\ldots]\end{aligned} \tag{10a}$$

For readers' convenience, let us directly copy Titard's formulas [9] below, where we dropped the tree-level terms and the relativistic corrections, and we have

$$\Delta E(\alpha_s^3) = -\alpha_s^3\frac{C_F^2}{8\pi n^2}\left(2\beta_0\gamma_E + 4a_1\right); \tag{10b}$$

$$\begin{aligned}\Delta E(\alpha_s^4) =\ & -\alpha_s^4\frac{C_F^2}{4n^2\pi^2}\left\{\left(a_1+\gamma_E\frac{\beta_0}{2}\right)^2\right.\\ & \left.+2\left[\gamma_E\left(a_1\beta_0+\frac{\beta_1}{8}\right)\right.\right.\end{aligned}$$

$$+\left(\frac{\pi^2}{12}+\gamma_{\rm E}^2\right)\frac{\beta_0^2}{4}+b_1\Bigg]\Bigg\}. \tag{10c}$$

Hoang et al. estimated the contribution of higher orders $O(\alpha_{\rm s}^5)$ and $O(\alpha_{\rm s}^6)$ to the binding energies [8]. Phenomenologically, these high-order terms can be attributed to the effects of the Lamb shift,

$$\begin{aligned}\Delta E(\alpha_{\rm s}^5) =\ & \alpha_{\rm s}^5\lg\alpha_{\rm s}\frac{C_{\rm F}^2}{4\pi n^2}\Bigg\{\frac{C_{\rm A}}{3}\Bigg[\frac{C_{\rm A}^2}{2}+\frac{4C_{\rm A}C_{\rm F}}{n(2l+1)}\\ &+\frac{2C_{\rm F}^2}{n}\left(\frac{8}{2l+1}-\frac{1}{n}\right)\Bigg]\\ &+\frac{3\delta_{\rm l0}C_{\rm F}^2}{2n}(C_{\rm A}+2C_{\rm F})-\frac{7C_{\rm A}C_{\rm F}^2\delta_{\rm l0}\delta_{\rm s1}}{3n}\\ &-\frac{C_{\rm A}C_{\rm F}^2(1-\delta_{\rm l0}\delta_{\rm s1})}{4nl(l+1)(2l+1)}(4X_{ljs}+\langle S_{12}\rangle_{ljs})\Bigg\};\end{aligned} \tag{10d}$$

$$\begin{aligned}\Delta E(\alpha_{\rm s}^6) =\ & \alpha_{\rm s}^6\lg^2\alpha_{\rm s}\frac{C_{\rm F}^2}{4\pi^2n^2}\Bigg\{\frac{\delta_{\rm l0}C_{\rm F}^2}{6n}\Bigg[\beta_0\left(\frac{13C_{\rm A}}{2}-C_{\rm F}\right)\\ &+\frac{C_{\rm A}}{3}(25C_{\rm A}+22C_{\rm F})\Bigg]-\frac{C_{\rm A}C_{\rm F}^2\delta_{\rm l0}\delta_{\rm s1}}{6n}[5\beta_0\\ &+7C_{\rm A}]-\frac{C_{\rm A}C_{\rm F}^2(1-\delta_{\rm l0})\delta_{\rm s1}}{8nl(l+1)(2l+1)}\Bigg[\beta_0\Bigg(2X_{ljs}\\ &+\frac{1}{2}\langle S_{12}\rangle_{ljs}\Bigg)+C_{\rm A}(2X_{ljs}+\langle S_{12}\rangle_{ljs})\Bigg]\Bigg\}.\end{aligned} \tag{10e}$$

n in Ref. [10] stands for the principal quantum number as $n=n_r+l$, where n_r and l are defined in Section 3. All of the constants as a_1, a_2, b_1, β_i (i=1, 2, 3) are given in Ref. [20] (also see Refs. [9, 21–24]).

The Lamb shift $\Delta E[n,j,l,s]$ depends on the coupling constant α_s (see Eq. (10)) [9] as

$$\begin{aligned}\alpha_{\rm s}(\mu^2)=&\frac{2\pi}{\beta_0\ln\mu/\Lambda}\Bigg\{1-\frac{\beta_1}{\beta_0^2}\frac{\ln(\ln\mu^2/\Lambda^2)}{\ln\mu^2/\Lambda^2}\\ &+\frac{\beta_1^2\ln^2(\ln\mu^2/\Lambda^2)-\beta_1^2\ln(\ln\mu^2/\Lambda^2)-\beta_1^2+\beta_2\beta_0}{\beta_0^4\ln^2\mu^2/\Lambda^2}\Bigg\}.\end{aligned} \tag{11}$$

It is noted that unlike the others in the full Hamiltonian, which can be written in the pure operator form, the contributions of the Lamb shift to the spectrum energies are always associated with the concrete states.

Using the formulas given above, one can evaluate the Lamb shift of the charmonia states. The scheme of renormalization is suggested by Pineda et al [9, 20]. Actually, there is a term $\ln\left[\frac{na\mu}{2}\right]$ in the theoretical expression of the energy (see Refs. [8, 9]), where $a(\mu^2)$ stands for the Bohr radius and μ is the renormalization scale,

$$a(\mu^2)=\frac{2}{mC_{\rm F}\tilde{\alpha}_{\rm s}(\mu^2)} \tag{12a}$$

where,

$$\begin{aligned}\tilde{\alpha}_{\rm s}(\mu^2)=\alpha_{\rm s}\Bigg\{1+&\left(a_1+\frac{\gamma_{\rm E}\beta_0}{2}\right)\frac{\alpha_{\rm s}}{\pi}\Bigg[\gamma_{\rm E}\left(a_1\beta_0+\frac{\beta_1}{8}\right)\\ &+\left(\frac{\pi^2}{12}+\gamma_{\rm E}^2\right)\frac{\beta_0^2}{4}+b_1\Bigg]\frac{\alpha_{\rm s}^2}{\pi^2}\Bigg\}.\end{aligned} \tag{12b}$$

If one defines [20]

$$\mu=\frac{2}{na}, \tag{13}$$

this choice of μ will cancel the terms related to $\ln\left[\frac{na\mu}{2}\right]$ in the spectrum energy.

The value of the parameter Λ is near 0.30 GeV [20]. Here, we choose it as 0.275 GeV. The reason is that, at this point, $\alpha_{\rm s}^{n=2}=0.36$, just the same as the value of $\alpha_{\rm s}$ that we used in section 3.

It is obviously different from the conventional renormalization scheme we commonly use. A consequence is that the coupling constant $\alpha_{\rm s}$ is different for different quantum numbers n,

$$\alpha_{\rm s}^{n=1}=0.31,\ \alpha_{\rm s}^{n=2}=0.36,\ \alpha_{\rm s}^{n=3}=0.43. \tag{14}$$

Simply adding the Lamb shift to the total binding energy is like changing the zero-point energy for each state. We still select masses of J/ψ, $\chi_{\rm c0}(1P)$, $\chi_{\rm c1}(1P)$, $\eta_{\rm c}(2S)$ and $\psi(2S)$ as inputs, and solve the Eq. (5) again as we did in the last Section. But the value of $\alpha_{\rm s}$ in (5) is taken as the value given in Eq. (14), which depends on n. The new solutions of a, b, c, and d are

$$\begin{cases}a^{(1)}=1.3943,\ b^{(1)}=1.4057,\\ c^{(1)}=0.6243,\ d^{(1)}=0.9910;\end{cases} \tag{15}$$

$$\begin{cases}a^{(2)}=1.4191,\ b^{(2)}=1.3292,\\ c^{(2)}=0.6459,\ d^{(2)}=0.9438.\end{cases} \tag{16}$$

where expression (15) is the solution when the Lamb shift is taken up to order $O(\alpha_{\rm s}^3)$

$$\Delta E[n,j,l,s]=m[\Delta E(\alpha_{\rm s}^3)]$$

and (16) is for the Lamb shift,

$$\begin{aligned}\Delta E[n,j,l,s] =\ & m[\Delta E(\alpha_{\rm s}^3)+\Delta E(\alpha_{\rm s}^4)+\Delta E(\alpha_{\rm s}^5)\\ &+\Delta E(\alpha_{\rm s}^6)]\end{aligned}$$

Table 3. The charmonia mass spectra with contributions of the Lamb shift being taken into account. ΔE stands for the contribution of the Lamb shift, $M_{1,2}$ are the eigen-energies of the (4) and $\left(M'_{1,2}=M_{1,2}+\Delta E^{(1,2)}\right)$ are the real masses of charmonia. The indices (1,2) in M and M' correspond to the parameters set in Eq. (15) or Eq. (16) respectively.

meson	$\Delta E^{(1)}$	M_1	M'_1	$\Delta E^{(2)}$	M_2	M'_2
$\eta_c(1^1S_0)$	−0.0674	3.0820	3.0146	−0.1196	3.1612	3.0416
$J/\psi(1^3S_1)^{\rm fit}$	−0.0674	3.1643	3.0969	−0.1245	3.2215	3.0969
$\chi_{c0}(1^3P_0)^{\rm fit}$	−0.0310	3.4458	3.4148	−0.0799	3.4946	3.4148
$\chi_{c1}(1^3P_1)^{\rm fit}$	−0.0310	3.5417	3.5107	−0.0802	3.5909	3.5107
$h_c(1^1P_1)$	−0.0310	3.5593	3.5283	−0.0803	3.6063	3.5260
$\chi_{c2}(1^3P_2)$	−0.0310	3.6079	3.5769	−0.0804	3.6552	3.5748
$\eta_c(2^1S_0)^{\rm fit}$	−0.0310	3.6680	3.6370	−0.0714	3.7084	3.637
$\psi(2^3S_1)^{\rm fit}$	−0.0310	3.7171	3.6861	−0.0728	3.7589	3.6861
$\psi(3^3S_1)$	−0.020	4.1460	4.1260	−0.0531	4.1746	4.1215

up to the order $O(\alpha_s^6)$. With these two solutions, our predictions are given in Table 3.

5 Conclusion and discussion

In this work, we study the role of scalar potential in the spectra of charmonia. Our strategy is that the scalar and vector potentials have different fractions, which manifest in their coefficients (in the text, they are a, b, c and d for the Coulomb and confinement pieces, respectively). By fitting some members of the charmonia family, we can fit them. Then with the obtained parameters, we further predict the mass spectra of the rest resonances of charmonia. It is shown that unlike the QED case where the fraction of scalar potential is very small and negligible, the fraction of scalar potential is of the same order of magnitude as the vector potential. This is consistent with the conclusion of Ref. [25], and this is not surprising. As we indicated, for the vector-like coupling theories QED and QCD, the scalar potential can only appear at loop level or is induced by non-perturbative effect (QCD only). Thus it is loop-suppressed. However, for QCD, the coupling is sizable and the non-perturbative effects are somehow significant, so one can expect the fraction of scalar potential to be large.

Moreover, the Lamb shift is induced by the vacuum fluctuation and only appears at the loop level; indeed the leading contribution is at $O(\alpha_s^3)$. Therefore, for the QED case, it is hard to observe the Lamb shift (observation of the Lamb shift is a great success for theory and experiment indeed). However, for QCD, the effects are not ignorable. By taking into account the Lamb shift, we re-fit the model parameters and find that they are obviously distinct from those without considering the Lamb shift.

In this work, by studying charmonia spectra, we investigate the contribution of higher orders of α_s and non-perturbative QCD effects. However, to distinguish between them, one needs to do more theoretical research. This result helps us to get a better understanding of QCD, especially the non-perturbative effects. Even though it is only half-quantitative, it is an insight into the whole picture.

When we take into account the contribution of the Lamb shift to the mass spectra, it is more obvious that higher order effects are important in QCD. Because the Lamb shift only appears at order $O(\alpha_s^3)$, its existence manifests higher order effects. Our calculations show that while higher orders up to $O(\alpha_s^6)$ are involved, the fitted values of a, b, c, d are different from those when only $O(\alpha_s^3)$ is considered.

The same strategy can be applied to the bottomia family and even the B_c resonances, where one can further test the theoretical framework and investigate the higher order QCD behaviors. That will be the focus of our next work.

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