

Two loop low temperature corrections to electron self energy

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Abstract: We recalculate the two loop corrections in the background heat bath using real time formalism. The procedure of the integrations of loop momenta with dependence on finite temperature before the momenta without it has been followed. We determine the mass and wavefunction renormalization constants in the low temperature limit of QED, for the first time with this preferred order of integrations. The correction to electron mass and spinors in this limit is important in the early universe at the time of primordial nucleosynthesis as well as in astrophysics.

Key words: renormalization, finite temperature field theory, electron self energy, two-loop corrections

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1 Motivations

The quantum field theory at finite temperature, Thermal Field Theory (TFT) has been used for more than 50 years. The techniques for calculations in many body systems were initially developed in condensed matter [1], and they are now also used to describe a large ensemble of multi-interacting particles in a thermal background. Since the late seventies, TFT has been used to study the phase transitions in cosmology and quantum field theories. It was adopted in particle physics to study the many particle systems. These methods are used not only in particle physics but also to study the various aspects of nuclear matter and plasma physics. The TFT methods are extensively used to describe the phase transitions due to symmetry breaking after the Hot Big Bang and in tracing the history of the early universe.

The main applications of interest in this context are in the following:

Cosmology: The early universe provides a very good example of the studies of hot plasmas. When the Big Bang occurred, the universe was impossibly hot and dense. It rapidly expanded and cooled. At $t = 200$ s, $T = 10^8$ K, it was cool enough for neutrons and protons to combine to form Deuterium, then He-

lium and traces of Lithium (primordial nucleosynthesis). For the next few 10^5 years, it was too hot for electrons to form atoms. The universe was filled with hot plasma of electrons and nuclei, bathed in photons constantly interacting with both, like the interior of a star.

Astrophysics: There is a series of different types of fusion reactions in stars leading to luminous supergiants. When helium fusion ceases in the core, gravitational compression increases the core's temperature above 6×10^8 K, at which carbon can fuse into neon and magnesium. As the core reaches 1.5×10^8 K, oxygen begins fusing into silicon, phosphorous, sulfur, and others. At 2.7×10^8 K, silicon begins fusing into iron. This process essentially stops with the creation of iron and a catastrophic implosion of the entire star begins. When the high mass stars exhaust their He fuel, they have enough gravitational energy to heat up to 6×10^8 K. Cores of neutron stars, red giants and white dwarfs are composed of extremely dense plasmas ($\rho = 10^6 - 10^{15}$ g/cm³). The neutrino and axion emission rates in these stars require TFT [2]. A tremendous amount of energy is released in a supernova. The only supernova in modern time, visible to the naked eye, was detected on February 23, 1987 and it is known as SN1987A. It emitted more than 10^{10}

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times as much visible light as the Sun for over one month, and temperatures as high as 2×10^{11} K were reached.

Heavy Ion Collisions: The quark gluon plasma is the form of matter at transition temperatures $T_c = 100 - 200$ MeV. The hot and dense environment in quark gluon plasma and the studies of its prospective reproduction in nucleus-nucleus collisions require the TFT methods for detailed explanation. With the increased feasibility of the creation of quark gluon plasma in heavy ion collisions, the methods developed in this theory achieved their specific relevance in QCD at finite temperature as well.

We are specifically interested in the first two applications here.

2 Finite temperature effects

The main idea of TFT is to use the approach of the usual quantum field theory. Matsubara [3] was the first to develop thermal field theory by incorporating a purely imaginary time variable into the evolution operator. In Euclidean space, the covariance breaks and time is included as an imaginary parameter. The imaginary time domain is finite and periodic, because of which the energy integrations are converted into summations over the discrete Matsubara frequencies. The presence of discrete energies along with the particle distribution functions destroys the covariance of the theory.

The important contributions by Schwinger [4], Mills [5], and Keldysh [6] led to the development of a formalism based upon the choice of a contour in the complex plane. This is called the real time formalism. Here, an analytical continuation of the energies along with Wick's rotation restores covariance in Minkowski space at the expense of Lorentz invariance. The breaking of Lorentz invariance leads to the non-commutative nature of the gauge theories [7]. The covariance is incorporated through the 4-component velocity of the background heat bath $u^\mu = (1, 0, 0, 0)$. In a heat bath, the particles are in constant interaction with the thermal surroundings. Implementing these interactions is straightforward, as is done in vacuum field theory. The temperature is included through the statistical distribution functions of the particles.

Umezawa and coworkers [8] independently worked on a different approach, called Thermo-Field Dynamics, that also gives the same results. In this formalism, the propagators are taken in the form of 2×2 matrices. Field theory at finite temperature is renormaliz-

able, if the vacuum theory is so since the presence of the Boltzmann factor in the thermal corrections cuts off any ultraviolet divergence. Choosing the suitable counter terms as in vacuum can eliminate them. The infrared divergences are inherent in almost all perturbation theories, whether at zero or finite temperature. KLN theorem [9] demonstrates that singularities appearing at intermediate stages of the calculation cancel in the final state physical result.

Quantum Electro Dynamics (QED) is the simplest and most successful gauge theory. The behavior of QED at finite temperatures serves as a model for the determination of background effects in other physical theories - the electroweak theories as well as Quantum Chromo Dynamics. In real time formalism, the tree level fermion propagator in Feynman gauge in momentum space is [10]

$$S_\beta(p) = (\not{p} - m) \left[\frac{i}{p^2 - m^2 + i\epsilon} - 2\pi\delta(p^2 - m^2)n_F(E_p) \right], \quad (1)$$

where

$$n_F(E_p) = \frac{1}{e^{\beta(p \cdot u)} + 1}, \quad (2)$$

is the Fermi-Dirac distribution function with $\beta = \frac{1}{T}$.

The boson propagator is taken as

$$D_\beta^{\mu\nu}(p) = \left[\frac{i}{k^2 + i\epsilon} - 2\pi\delta(k^2)n_B(k) \right], \quad (3)$$

with

$$n_B(E_k) = \frac{1}{e^{\beta(k \cdot u)} - 1}. \quad (4)$$

3 One loop corrections

At the one loop level, Feynman diagrams are calculated in the usual way by substituting these propagators in place of the usual ones in vacuum. The Lorentz invariance breaking and conserving terms remain separate at the one loop level since the propagators comprise temperature dependent (hot) terms added to temperature independent (cold) terms. This effect has been studied in detail and established at the one-loop level [11]. The renormalization of QED in this scheme [12] has already been checked in detail at the one loop level for all temperatures and chemical potentials.

The thermal background effects are incorporated through radiative corrections. At finite temperature, electrodynamics electric fields are screened due to the interaction of the photon with the thermal background of charged particles. The physical processes take place in a heat bath comprising hot particles and

antiparticles instead of a vacuum. The exact state of all of these background particles is unknown since they continually fluctuate between different configurations. The net statistical effects of the background fermions and bosons enter the theory through the fermion and boson distributions, respectively.

The electric permittivity and the magnetic susceptibility of the medium are modified by incorporating the thermal background effects. At low temperatures, i.e., $T \ll m$ (m is the electron mass), the hot fermion contribution in the background is suppressed and only the hot photons contribute from the background heat bath. The vacuum polarization tensor in order α does not acquire any hot corrections from the photons in the heat bath. This is because of the absence of the self-interaction of photons in QED.

The thermal mass is generated radiatively. The mass shift that enters physical quantities acts as a kinematical cut-off, in the production rate of light weakly coupled particles from the heat bath. The effective mass corresponds to the fact that in the heat bath, the propagation of particles is influenced by their continuous interactions with the medium.

4 Higher order corrections

The higher order loop corrections are required to get predictions on perturbative behavior at finite temperature. At the higher-loop level, the loop integrals involve a combination of cold and hot terms that appear due to the overlapping propagator terms in the matrix element. In such situations, specific techniques are needed, even at the two loop level, to solve them. Higher loops become analytically even more complicated. In the hot terms, there appear overlapping divergent terms. The removal of such divergences is already shown at the two-loop level [13] for electron self energy.

We restrict ourselves to the low temperatures to prove the renormalizability of QED at the two-loop level through the order by order cancellation of singularities. The results have been shown [14] to depend on the order of carrying out the hot and cold integrations. The justification of this specific order is the fact that the temperature dependent part corresponds to the contribution of real background particles on mass-shell and incorporates thermal equilibrium. The breaking of Lorentz invariance changes these conditions for the cold integrals. We have checked that the renormalization can only be proven with the preferred order of integrations, i.e., if covariant hot integrals are evaluated before the cold ones.

At the higher loop level, the vacuum polarization contribution is non zero, even at low temperature. The calculations are simplified if the temperature dependent integrations are performed before the temperature independent ones. The temperature independent loops can then be integrated using the standard techniques of Feynman parametrization and dimensional regularization, as in vacuum [15].

The second order in α corrections to the electron self energy at low temperature has been calculated [4] to be equal to

$$\Sigma_{\beta}(p) = \Sigma_{\beta}(p, T=0) - \frac{\alpha^2}{4\pi^3} \left[\frac{1}{\varepsilon} [(\not{p} + 6m)I_A - 2 \not{I}] - 3(\not{p} + 4m)I_A - 4 \not{I} - \frac{8\pi T^2}{3m^2} (\not{p} - m) \right], \quad (5)$$

where the singularity

$$\frac{1}{\varepsilon} = \frac{1}{\eta} - \gamma - \ln \left(\frac{4\pi\mu^2}{m^2} \right), \quad (6)$$

is the usual ultraviolet divergence in vacuum field theory in the MS bar scheme of renormalization with γ as the Euler-Mascheroni constant and

$$I_A = 8\pi \int \frac{dk}{k} n_B(k), \quad (7)$$

$$\frac{I^0}{E} = \frac{2\pi^3 T^2}{3E^2 v} \ln \frac{1+v}{1-v}, \quad (8)$$

$$\frac{\mathbf{I} \cdot \mathbf{p}}{|\mathbf{p}|^2} = \frac{2\pi^3 T^2}{3E^2 v^3} \left[\ln \frac{1+v}{1-v} - 2v \right], \quad (9)$$

with $v = \frac{|\mathbf{p}|}{p_0}$ [10, 11]. The divergences of the form I_A and $\frac{1}{\varepsilon}$ in Eqs. (6) and (7) cancel on addition of the appropriate counter terms. The finite contribution then leads to the physical mass of electron,

$$m_{\text{phys}}^2 = m^2 \left(1 + \frac{2\alpha\pi T^2}{3m^2} + \frac{4\alpha^2 T^2}{3m^2} \right). \quad (10)$$

5 Results

The electron mass and wave function renormalization can be obtained from the two loop self-energy for electrons calculated in the previous section. From Eq. (10), the change in the electron mass due to finite temperature up to the order α^2 relative to the cold electron mass is

$$\frac{\delta m}{m} = \frac{T^2}{m^2} (\alpha\pi + 4\alpha^2). \quad (11)$$

The renormalizability of the self mass of the electron is rechecked through the order by order cancel-

lation of singularities at both loop levels. It can also be noted that the second order term is much smaller than the first order term.

Using the standard procedure, the wave function renormalization constant comes out to be

$$Z_2^{-1} = 1 + \frac{\alpha}{4\pi} \left(4 - \frac{3}{\varepsilon}\right) - \frac{\alpha}{4\pi^2} \left(I_A - \frac{I^0}{E}\right) - \frac{\alpha^2}{4\pi^2} \left(3 + \frac{1}{\varepsilon}\right) I_A + \frac{2\alpha^2 T^2}{3\pi^2 m^2}. \quad (12)$$

In Fig. 1, a plot of Z_2^{-1} vs T/m is given for $O(\alpha^2)$ to demonstrate the affect of temperature in two-loops. The results here and in Ref. [14] are an explicit proof of the renormalizability of QED up to the two-loop level. They also estimate the temperature dependent modification in the electromagnetic properties of a medium. This helps to evaluate the decay rates and the scattering crossections of particles in such a

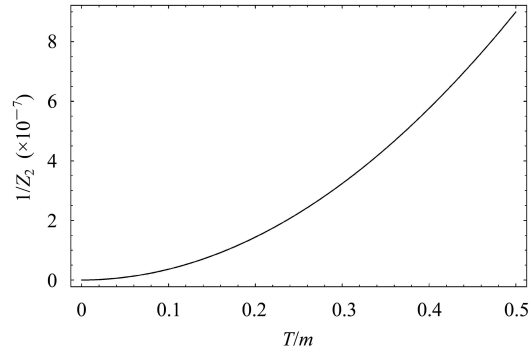


Fig. 1. A plot of wavefunction renormalization constant Z_2^{-1} vs T/m at two-loop level.

medium. These results can be applied to check the abundances of light elements in primordial nucleosynthesis, baryogenesis and leptogenesis. If the background magnetic fields are also incorporated, one can look for applications to neutron stars, supernovae, red giants, and white dwarfs.

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